20S-PHYSICS1B-2 Midterm 2

[redacted]

TOTAL POINTS

102 / 102

QUESTION 1

Question 1 30 pts

1.1 part (a) 10 / 10

✓ **- 0 pts** Correct

- 5 pts Like charges repel.

- 5 pts Charges flow off of the kid to ground.

1.2 part (b) 10 / 10

✓ **- 0 pts** Correct

- 3 pts The inner surface of the conductor has charge -q.

- 2 pts Charges on the inner surface are nonuniform. More charges are concentrated on the side that is closer to the point charge.

- 5 pts The outer surface of the conductor is neutral.

1.3 part (c) 10 / 10

✓ **- 0 pts** Correct

- 5 pts The absolute potential difference between A and B decreases.

- 5 pts The capacitance increases upon inserting the conducting material.

QUESTION 2

Question 2 30 pts

2.1 part (a) 10 / 10

✓ **- 0 pts** Correct

- 5 pts The potential due to an infinitesimal area of charge.

- 5 pts The total potential due to the whole annular ring.

- 3 pts Incorrect/Incomplete calculations

2.2 part (b) 10 / 10

✓ **- 0 pts** Correct

- 5 pts The potential in the limit |z| >> ra, |z| >> rb.

- 5 pts The potential in the limit |z| << ra, |z| << rb.

- 3 pts Incorrect/Incomplete Calculations

2.3 part (c) 10 / 10

✓ **- 0 pts** Correct

- 5 pts Calculation of Ez. Note: -2 pts if the mistake is due to that a wrong potential is inherited from part (b).

- 5 pts Calculation of frequency. Note: -2 pts if the relation Fz = - k*z is obtained but the spring constant k is incorrect.

- 2 pts Incorrect due to incorrect answer in part b/Calculation error/Incomplete

QUESTION 3

Question 3 42 pts

3.1 part (a) 6 / 6

✓ **- 0 pts** Correct.

- 3 pts Write the correct integral for the total charge.

- 3 pts Calculate the integral to obtain the correct results.

- 6 pts No volume integral over \rho = no points

3.2 part (b) 8 / 8

✓ **- 0 pts** Correct

- 2 pts The electric field in the region r < a. Note: \rho not constant so is E not linear... Also, this part is not related to (a) since Q_enc

- 2 pts The electric field in the region a < r < b. *a

- 1 pts The electric field in the region b < r < c.

- 2 pts The electric field in the region r > c.

- 1 pts Plot

3.3 part (c) 6 / 6

✓ **- 0 pts** **Correct. Note: all quantities are constant**

- 2 pts sigma_c*SA = Q_tot + Q_c, solve for Q_c

- 2 pts The charge density on the inner surface. Note: full credit could be given if the mistake is due to an incorrect calculation in part (a).

- 2 pts The charge density on the outer surface.

+ 1 pts work

3.4 part (d) 8 / 8

✓ **- 0 pts** **Correct. Note: V is not just the anti-derivative of E. If you don't integrate in from infinity then you're not approaching the problem correctly and will miss terms.**

- 2 pts The electric potential in the region $r < a$.

- 2 pts The electric potential in the region $a < r < b$.

- 2 pts The electric potential in the region $b < r < c$.

- 2 pts The electric potential in the region $r > c$.

+ 3 pts If all incorrect: work (i.e. actually attempted to integrate E) & plot

+ 1 pts N/A

3.5 part (e) 7 / 7

✓ **- 0 pts** **Correct. The 3 pts are given to everyone as a base credit. The other 4 pts depend on your work (partial credit may be given).**

- 2 pts Conducting shell, or missing terms; plus...

- 2 pts Integral on insulating sphere. Note: if you didn't perform the integral, zero points.

3.6 part (f) 7 / 7

✓ **- 0 pts** **Correct. The 3 pts are given to everyone as a base credit. The other 4 pts depend on your work (partial credit may be given).**

- 2 pts E-field energy for each region;

- 2 pts Integrate/sum to get total E-field energy.

Name: [redacted]

Student ID number: [redacted]

Oral interview time (see below): [redacted]

Discussion section: [redacted]

By signing below, you affirm that you have neither given nor received unauthorized help on thi[redacted]

Signature: [redacted]

MIDTERM 2
PHYSICS 1B, SPRING 2020

MAY 22, 2020

READ THE FOLLOWING CAREFULLY:

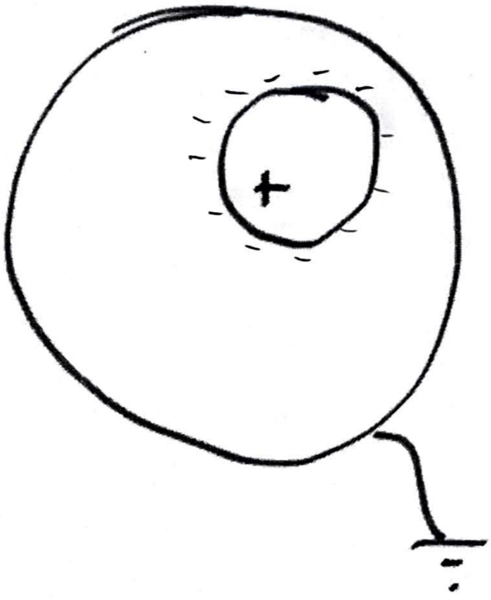
- ▷ Open book. Calculators will not be needed. Make sure that you work independently on the exam, i.e., no discussion with others.
- ▷ Oral interview time: choose a time period of 20 minutes between 11:59AM and 11:59PM on May 23 (Pacific Time) for possible oral interview (via the zoom office hour link). A random sample of students will be selected and be notified via email by 11:59AM on May 23 (Pacific Time). If all of the work on your submission is your own, you will have nothing to worry about.
- ▷ For non-response in the field of oral interview time, I may assess a score penalty.
- ▷ Make sure to submit your exam packet via GradeScope by 9AM May 23, Pacific Time. **No credit will be given for late submissions.**
- ▷ **You must make sure that your submission has exactly the same number of pages as the posted exam PDF (including this page).**
- ▷ **You must justify your answers to each question**. Simply giving the correct answer without proper justification (can be brief) will not result in full credit.
- ▷ Mistakes in grading: If you find a mistake in the grading of your exam, alert the instructor within one week of the exams being returned.

[1.] Short answer conceptual questions. Provide *concise* answers to the questions below; you should write enough to explain your answer, but an essay is not required (most can be answered in 2-3 sentences).

(a) **(10 pts)** During one of the in-class video demos, a kid stood on an insulator and placed her hands on the metal sphere of the Van der Graaf generator. This resulted in her hair standing on end (much like the below photo). Explain why this happened. If the kid wasn't standing on the insulator, would the same thing have happened? Why or why not?

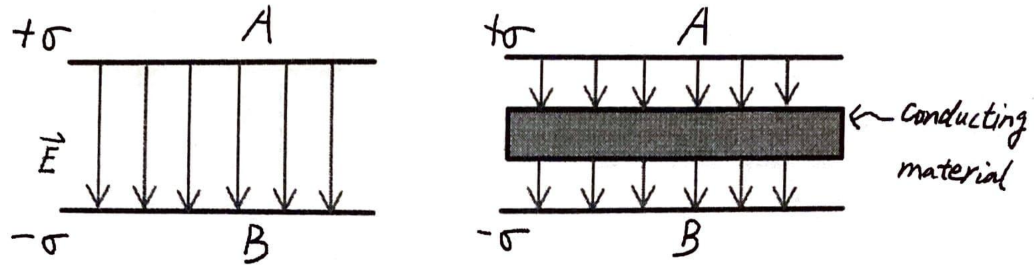
The generator creates static charge, which causes the hair strands to also gain a charge. The hairs start repelling each other and standing on end. If there was no insulator, the same thing would not have happened, because the electrons would leave the hair quickly.

(b) **(10 pts)** A solid conducting sphere has an internal spherical cavity which is offset from the center of the sphere. Inside the cavity, but offset from the center of the cavity, I place a positive point charge $+q$. The conducting sphere is connected to ground via a thin wire as shown. Describe (either by drawing or words) any charges that exist on the surfaces of the conductor; the interior surface around the cavity and the external surface. What is the total charge on the conducting sphere?



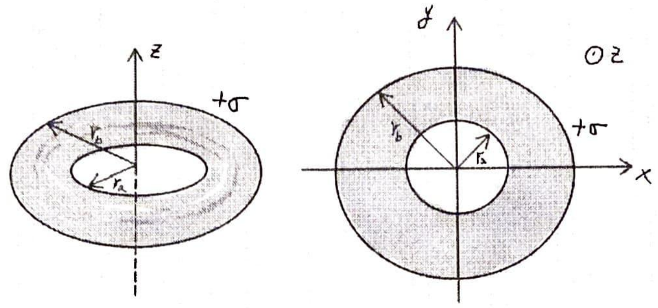
There is negative charge on the interior surface. The total charge on the conducting sphere is $-q$.

(c) **(10 pts)** I have a capacitor consisting of two large parallel plates A and B, one charged positively (surface charge density $+\sigma$) and the other charged negatively (surface charge density $-\sigma$). If I insert a slab of conducting material as shown below, how does the absolute value of the potential difference between the two parallel plates A and B change? How does the capacitance change?



The absolute value of the potential difference decreases, and the capacitance increases. This is because the new setup acts as two capacitors with distances between plates less than half of the distance in the original capacitor.

[2.] (30 pts) An annular ring with a uniform surface charge density $+\sigma$ sits in the xy plane, with its center at the origin of the coordinate. The annulus has an inner radius r_a and an outer radius r_b as shown below.



Side view   Top view

(a) What is the electric potential along the z-axis? Define the electric potential to be zero at infinity.

$$V(z) = \int_{r_a}^{r_b} \frac{k\sigma(2\pi r\,dr)}{\sqrt{r^2+z^2}} = \boxed{2\pi k\sigma\left[\sqrt{z^2+r_b^2} - \sqrt{z^2+r_a^2}\right]}$$

$$dV = \frac{k\,dQ}{d}$$

(b) From part (a), what is the electric potential in the limit $|z| \gg r_a$, $|z| \gg r_b$? What is the electric potential in the limit $|z| \ll r_a$, $|z| \ll r_b$? Hint: use the binomial expansion $(1+x)^\alpha \approx 1 + \alpha x$ for $|x| \ll 1$.

For $|z| \gg r_a$ and $|z| \gg r_b$, the term $\left[\sqrt{z^2+r_b^2} - \sqrt{z^2+r_a^2}\right]$ is equivalent to $z\sqrt{1+\frac{r_b^2}{z^2}} - z\sqrt{1+\frac{r_a^2}{z^2}}$. Using the provided binomial expansion, we get $z\left(1+\frac{r_b^2}{2z^2}\right) - z\left(1+\frac{r_a^2}{2z^2}\right)$ $= z\left(\frac{r_b^2}{2z^2} - \frac{r_a^2}{2z^2}\right) = \frac{1}{2z}\left(r_b^2 - r_a^2\right)$. The potential is $\boxed{\frac{\pi k \sigma}{z}\left[r_b^2 - r_a^2\right]}$

For $|z| \ll r_a$ and $|z| \ll r_b$, the term is also equivalent to $r_b\sqrt{\frac{z^2}{r_b^2}+1} - r_a\sqrt{\frac{z^2}{r_a^2}+1}$. Using the provided binomial expansion, this is equivalent to

$$r_b\left(1+\frac{1}{2}\cdot\frac{z^2}{r_b^2}\right) - r_a\left(1+\frac{1}{2}\frac{z^2}{r_a^2}\right)$$

$= r_b + \frac{z^2}{2r_b} - r_a - \frac{z^2}{2r_a}$. The potential is thus $2\pi k\sigma\left[r_b + \frac{z^2}{2r_b} - r_a - \frac{z^2}{2r_a}\right]$.

(c) A particle of mass m and negative charge $-q$ is constrained to move along the z-axis. If I kick the particle with a small displacement about the point $z = 0$, the particle will execute simple harmonic motion. Find the particle's frequency of oscillation. Hint: use the electric potential in the limit $|z| \ll r_a$, $|z| \ll r_b$ from part (b) and $E_z = -\partial V/\partial z$, where V is the electric potential.

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\left(2\pi k\sigma\left[r_b + \frac{z^2}{2r_b} - r_a - \frac{z^2}{2r_a}\right)\right.$$

$$= -2\pi k\sigma\left[\frac{z}{r_b} - \frac{z}{r_a}\right]$$

$$F = qE_z = 2\pi k\sigma\left[\frac{z}{r_a} - \frac{z}{r_b}\right]$$

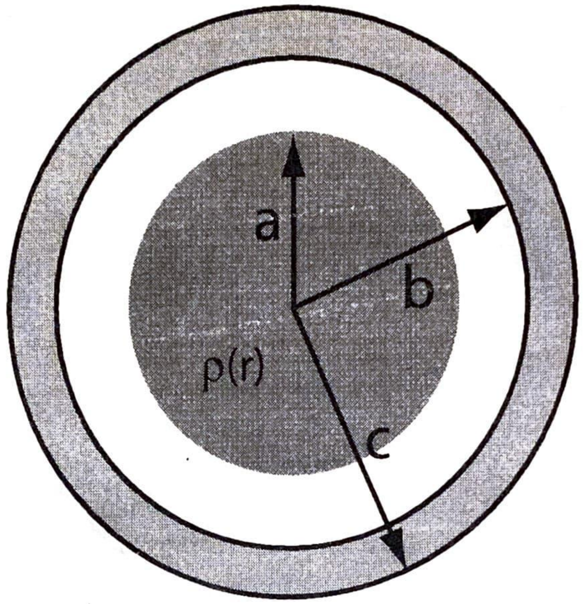
$$k_s = \frac{F}{z} = 2\pi k\sigma\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}} = \boxed{\frac{1}{2\pi}\sqrt{\frac{2\pi k\sigma}{m}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}}$$

[3.] **(40 pts)** An insulating sphere of radius a has an embedded nonuniform charge density:

$$\rho(r) = \frac{\rho_o}{2}\left(1 + \frac{r}{a}\right)$$

where ρ_o is a positive constant (this charge density is only valid for $r \leq a$). Surrounding the sphere, and concentric with it, is a conducting shell of inner radius $b > a$ and outer radius c which is charged (not neutral). I do not know what the charge on the conductor is, but I am told that a measurement of the radial electric field right on the outer surface of the conducting shell (at $r = c + \epsilon$ where we take the limit $\epsilon \to 0$), shows that it is positive and equal to E_o.



(a) What is the total charge contained in the insulating sphere of radius a?

$$dq = 4\pi r^2 dr \cdot \frac{\rho_0}{2}\left(1 + \frac{r}{a}\right) = 2\rho_0 \pi r^2 \left(1 + \frac{r}{a}\right) dr$$

$$= 2\rho_0 \pi \left(r^2 + \frac{r^3}{a}\right) dr$$

$$\int dq = \int_0^a 2\rho_0 \pi \left(r^2 + \frac{r^3}{a}\right) dr = 2\rho_0 \pi \left[\frac{1}{3}r^3 + \frac{1}{4a} r^4 \Big|_0^a \right]$$

$$= 2\rho_0 \pi \left(\frac{1}{3} a^3 + \frac{1}{4} a^3\right) = \boxed{\frac{7}{6} a^3 \rho_0 \pi}$$

(b) Find the electric field everywhere in space: for $r < a$, $a < r < b$, $b < r < c$ and $r > c$. Plot the electric field versus radius.

$r<a$:

$E = \frac{q}{\varepsilon_0} \cdot \frac{1}{4\pi r^2}$; $q = 2\rho_0 \pi \left[\frac{1}{3} d^3 + \frac{1}{4a} d^4 \Big|_0^r \right] = 2\rho_0 \pi \left(\frac{r^3}{3} + \frac{r^4}{4a} \right)$ from (a)

$E = \boxed{\frac{\rho_0}{\varepsilon_0}\left(\frac{r}{6} + \frac{r^2}{8a}\right)}$

$a<r<b$:

$E = \frac{q}{\varepsilon_0} \cdot \frac{1}{4\pi r^2}$; $q = \frac{7}{6} a^3 \rho_0 \pi$ from (a)

$E = \boxed{\frac{\rho_0}{\varepsilon_0} \cdot \frac{7a^3}{24 r^2}}$

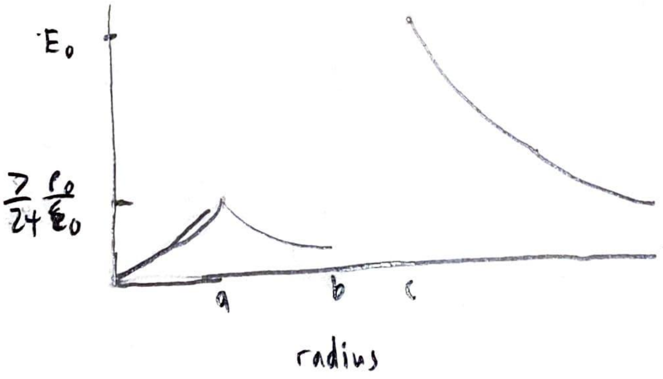
$b<r<c$:

$E = 0$ because it is inside a conductor

$r>c$:

$E = \frac{q}{\varepsilon_0} \cdot \frac{1}{4\pi r^2} = E_0$ @ $r = c + \epsilon$ where $\epsilon \to 0$

$E = E_0 \frac{c^2}{r^2}$



note: relative magnitudes of $\frac{7}{24}\frac{\rho_0}{\varepsilon_0}$ and E_0 are unknown

(c) What is the total charge on the conducting shell in terms of the given information? What is the charge density on the inner and outer surfaces?

$$E_0 = \frac{q_{total}}{\varepsilon_0} \cdot \frac{1}{4\pi c^2} \rightarrow q_{total} = E_0 \varepsilon_0 \cdot 4\pi c^2$$

$$q_{shell} = q_{total} - q_{sphere} = \boxed{4\pi E_0 \varepsilon_0 c^2 - \frac{7}{6}\pi a^3 \rho_0}$$

$$q_{inner} = -q_{sphere} = -\frac{7}{6}\pi a^3 \rho_0$$

$$\sigma_{inner} = \frac{q_{inner}}{4\pi b^2} = -\frac{7/6\, \pi a^3 \rho_0}{4\pi b^2} = \boxed{-\frac{7a^3\rho_0}{24b^2}}$$

$$q_{outer} = q_{total} - q_{inner} - q_{sphere} = q_{total} = E_0 \varepsilon_0 \cdot 4\pi c^2$$

$$\sigma_{outer} = \frac{q_{outer}}{4\pi c^2} = \boxed{E_0 \varepsilon_0}$$

(d) Find the electric potential everywhere in space: for $r < a$, $a < r < b$, $b < r < c$ and $r > c$. Plot the electric potential versus radius. Define the electric potential to be zero at infinity.

$r > c$:

$$V = -\int_\infty^r E_{r>c}\,dr = E_0 \frac{c^2}{r}\Big|_\infty^r = \boxed{E_0 \frac{c^2}{r}}$$

$b < r < c$:

$$V = -\int_\infty^c E_{r>c}\,dr - \int_c^r E_{b<r<c}\,dr = \boxed{E_0 c}$$

$a < r < b$:

$$V = -\int_\infty^c E_{r>c}\,dr - \int_c^b E_{b<r<c}\,dr - \int_b^r E_{a<r<b}\,dr$$

$$= E_0 c + \left[-\frac{\rho_0}{\varepsilon_0}\frac{7a^3}{24r}\Big|_b^r\right] = E_0 c + \frac{\rho_0}{\varepsilon_0}\frac{7a^3}{24r} - \frac{\rho_0}{\varepsilon_0}\frac{7a^3}{24b}$$

$$= \boxed{E_0 c + \frac{\rho_0}{\varepsilon_0}\frac{7a^3}{24}\left[\frac{1}{r} - \frac{1}{b}\right]}$$

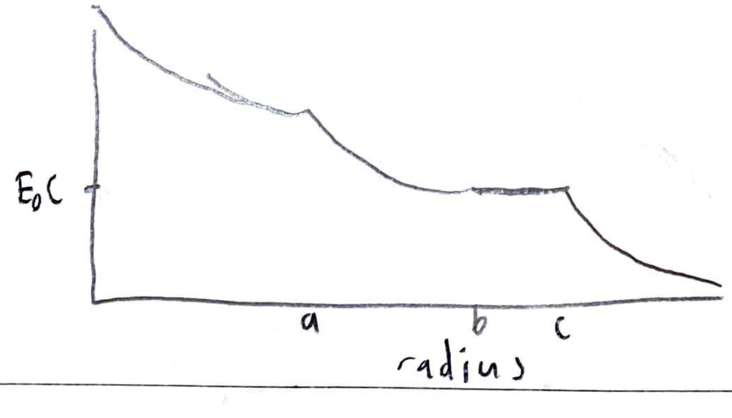
$r < a$:

$$V = -\int_\infty^c E_{r>c}\,dr - \int_c^b E_{b<r<c}\,dr - \int_b^a E_{a<r<b}\,dr - \int_a^r E_{r<a}\,dr$$

$$= E_0 c + \frac{\rho_0}{\varepsilon_0}\frac{7a^3}{24}\left[\frac{1}{a} - \frac{1}{b}\right] - \left[\frac{\rho_0}{\varepsilon_0}\left(\frac{1}{12}r^2 + \frac{r^3}{24a}\right)\Big|_a^r\right]$$

$$= E_0 c + \frac{\rho_0}{\varepsilon_0}\frac{7a^3}{24}\left[\frac{1}{a} - \frac{1}{b}\right] - \left[\frac{\rho_0}{\varepsilon_0}\left(\frac{1}{12}r^2 + \frac{r^3}{24a}\right) - \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{12}a^2 + \frac{1}{24}a^2\right)\right]$$

$$= \boxed{E_0 c + \frac{\rho_0}{\varepsilon_0}\frac{7a^3}{24}\left[\frac{1}{a} - \frac{1}{b}\right] - \frac{\rho_0}{\varepsilon_0}\left[\frac{1}{12}r^2 + \frac{r^3}{24a} - \frac{1}{8}a^2\right]}$$



(e) What is the work that I do to assemble charges in this configuration, i.e., the total electric potential energy of this configuration? Use your answer to part (d), and the relationship between the electric potential energy and electric potential for a collection of charges.

Potential energy for q outer:

$V_{outer} = \frac{1}{2} q_{outer} V(c) = \frac{1}{2} E_0 \varepsilon_0 \cdot 4\pi c^2 \cdot E_0 c = 2\pi c^3 E_0^2 \varepsilon_0$

Potential energy for q inner:

$V_{inner} = \frac{1}{2} q_{inner} V(b) = -\frac{7}{12}\pi a^3 \rho_0 \cdot E_0 c = -\frac{7}{12}\pi E_0 a^3 c \rho_0$

Potential energy for sphere:

$V_{sphere} =$

$\frac{1}{2}\int dq \cdot V(r) = \frac{1}{2}\int_0^a V(r) \cdot 2\rho_0 \pi \left(r^2 + \frac{r^3}{a}\right) dr$

$= \frac{1}{2}\int_0^a \left(E_0 c + \frac{\rho_0}{\varepsilon_0}\frac{7a^3}{24}\left[\frac{1}{a} - \frac{1}{b}\right] - \frac{\rho_0}{\varepsilon_0}\left[\frac{1}{12}r^2 + \frac{r^3}{24a} - \frac{1}{8}a^2\right]\right)$

$\left(2\rho_0 \pi \left(r^2 + \frac{r^3}{a}\right)\right) dr$

$= \frac{\pi a^3 \rho_0 \left(-1715 a^3 \rho_0 + 2012 a^2 b \rho_0 + 5880 b \varepsilon_0 E_0 c\right)}{100800\, b\, \varepsilon_0}$

$V_{total} = V_{outer} + V_{inner} + V_{sphere}$

$= \boxed{\frac{\pi\left(-1715 a^6 \rho_0^2 + 2012 a^5 b \rho_0^2 + 20160 E_0^2 b c^3 \varepsilon_0^2\right)}{10080\, b\, \varepsilon_0}}$

(f) What is energy stored in the electric field? Use your answer to part (b), and the relationship between energy density and electric field.
Compare the electric field energy in this part with the total electric potential energy in part (e). Are they the same or not?

$r>c$; $\int_c^{\infty} \frac{1}{2} \varepsilon_0 E(r)^2 \cdot 4\pi r^2 dr = \int_c^{\infty} \frac{1}{2} \varepsilon_0 E_0^2 \frac{c^4}{r^4} 4\pi r^2 dr$

$= \int_c^{\infty} 2\pi \varepsilon_0 E_0^2 \frac{c^4}{r^2} dr = -2\pi \varepsilon_0 E_0^2 c^4 \left[\frac{1}{r} \Big|_c^{\infty} \right]$

$= 2\pi \varepsilon_0 E_0^2 c^3$

$b<r<c$; $\int_b^c \frac{1}{2} \varepsilon_0 E^2 \cdot 4\pi r^2 dr = 0$

$a<r<b$; $\int_a^b \frac{1}{2} \varepsilon_0 E^2 \cdot 4\pi r^2 dr = \int_a^b \frac{1}{2} \varepsilon_0 \cdot 4\pi r^2 \cdot \frac{\rho_0^2}{\varepsilon_0^2} \left(\frac{7a^3}{24 r^2} \right)^2$

$= \frac{(\frac{7}{6} a^3 \rho_0 \pi)^2}{8\pi \varepsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{\frac{49}{36} a^6 \rho_0^2 \pi}{8 \varepsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{49 a^6 \rho_0^2 \pi}{288 \varepsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$

$r<a$; $\int_0^a \frac{1}{2} \varepsilon_0 E^2 \cdot 4\pi r^2 dr = \int_0^a \frac{1}{2} \varepsilon_0 \frac{\rho_0^2}{\varepsilon_0^2} \left(\frac{r}{6} + \frac{r^2}{8a} \right)^2 \cdot 4\pi r^2 dr$

$= \frac{\rho_0^2}{\varepsilon_0} \left(\frac{33\pi a^5}{1120} \right)$

Total; $\boxed{2\pi \varepsilon_0 E_0^2 c^3 + \frac{49 a^6 \rho_0^2 \pi}{288 \varepsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) + \frac{\rho_0^2}{\varepsilon_0} \left(\frac{33\pi a^5}{1120} \right)}$

$$= \frac{\pi(-1715 a^6 \rho_0^2 + 2012 a^5 b \rho_0^2 + 20160 E_0^2 b c^3 \varepsilon_0^2)}{10080 b \varepsilon_0}$$

Yes, it is equal to the total electric potential energy in (e)