Midterm 2

Physics 1B (Lec 4)

Name: [redacted]

ID number: [redacted]

Discussion section: Matteo 4pm Wed.

Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	total
20	20	20	20	80

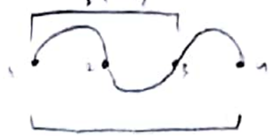
Problem 1

A standing wave is generated on a string with linear mass density $\mu = 2$ g/cm and the tension force $F = 100$ N. The distance between the first and the fourth nodes is 0.3 m.

7 (a) Find the wavelength of the standing wave

$\lambda_n = \frac{2}{n}L$

$\frac{2}{3}(.3)$



$\lambda = 0.3\left(\frac{2}{3}\right) = \boxed{0.2m}$

0.3m

7 (b) Find the frequency of the wave on the string

$f_n = \frac{v}{2L}n \qquad v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{100N}{0.2 \text{ kg/m}}} = 22.361 \text{ m/s}$

$\mu = \frac{2g}{cm} \cdot \frac{100 cm}{1m} \cdot \frac{1 kg}{1000g} = 0.2 \text{ kg/m}$

$\lambda = 0.2 = \frac{2}{n}L \rightarrow \frac{n}{L} = \frac{2}{0.2} = 10$

$f = \frac{v}{2} \cdot \frac{n}{L} = \frac{22.361}{2} \cdot 10 = \boxed{111.805Hz}$

6 (c) Find the wavelength of the sound wave generated by the string if the speed of sound in air is 340 m/s. *(Hint: the sound wave must have the same frequency as the string, but not necessarily the same wavelength.)*

$f = 111.805Hz$

$111.805 = \frac{v}{2L}n = \frac{340}{2L}(n)$

$\frac{L}{n} = 1.521 \text{ m}$

$\lambda_n = 2\frac{L}{n} = \boxed{3.041 \text{ m}}$

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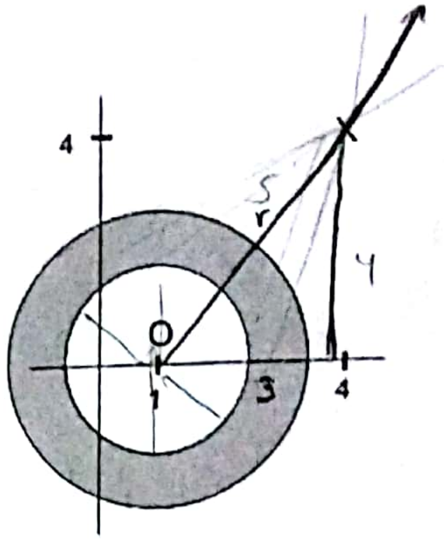
+20

Problem 2

A spherical shell with inner and outer radii $R_i = 1.5\ m$ and $R_o = 2.5\ m$, respectively, is centered at a point with coordinates (1,0), as shown. The shell carries a constant charge density $\rho = 2 \times 10^{-9}$ C/m³. All coordinates are in meters.



(a) Calculate the electric field at the center of the sphere O(1,0) (Justify your answer.)

$\vec{E} = 0$ because if you draw a Gaussian sphere anywhere inside the inner sphere ($r < 1.5m$), the charge enclosed is zero & therefore the field inside is zero. Another way to look at it is that all the radial vectors will cancel out with each other at the center since they have the same magnitude.

(b) Calculate the electric field at point X with coordinates (4,4)

can be treated as point charge

$E A = \frac{q_{en}}{\varepsilon_0} \rightarrow \vec{E} = \frac{q_{en}}{\varepsilon_0 A}$

$r = 5m$ (3-4-5 triangle)

$V_{shell} = \frac{4}{3}\pi(R_o^3 - R_i^3)$

$q_{en} = \frac{4}{3}\pi (R_o^3 - R_i^3)\rho$

Gauss sphere → $A = 4\pi r^2 \rightarrow 4\pi(5)^2 = 100\pi$

$E = \frac{\frac{4}{3}\pi(R_o^3 - R_i^3)\rho}{\varepsilon_0(100\pi)}$

$= \boxed{36.296\ \frac{N}{C}}$

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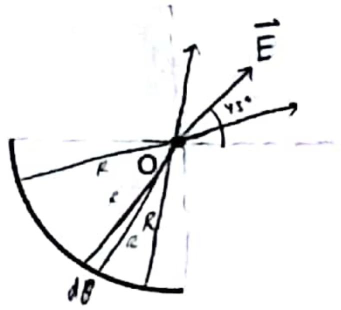
Problem 3

A thin thread carrying a constant charge density $\lambda = 4 \times 10^{-9}$ C/m is shaped as 1/4 of a circle. Calculate the electric field at the center of the circle O.



$R = 1\,m$

$dq = \lambda R d\theta$

components will cancel, only field left will be @ 45° angle.

$$E = \int_{-\pi/4}^{\pi/4} \frac{\overbrace{k\lambda R d\theta}^{dq}}{R^2} \cos\theta$$ ← project onto 45° line.

$$= \frac{k\lambda}{R}\int_{-\pi/4}^{\pi/4} \cos\theta\, d\theta = \frac{k\lambda}{R}\left(\sin\theta\Big|_{-\pi/4}^{\pi/4}\right)$$

$$= \frac{k\lambda}{R}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}\,k\lambda}{R} = \frac{\sqrt{2}\,(9\cdot 10^{9})(4\times 10^{-9})}{1}$$

$$= \boxed{36\sqrt{2}\ N/C}$$

$$= \boxed{50.912\ N/C}$$

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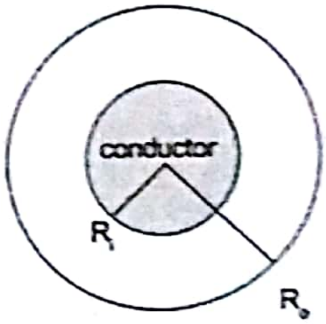
Problem 4



A filled conducting sphere with zero charge and radius R_i is surrounded by a spherical shell with the inner and the outer radii R_i and R_o, respectively. The shell carries the charge density $\rho(r) = \rho_0(R_i/r)$, $R_i < r < R_o$. There is no charge outside R_o.

(a) Calculate the electric field $E(r)$ for $r < R_i$

$\boxed{\vec{E} = 0}$ for $r < R_i$ because E inside conductors is always zero.

(b) Calculate the electric field $E(r)$ for $R_i < r < R_o$

r-prime to not confuse integration variable & radius.

$$AE = \frac{q_{en}}{\varepsilon_0} \rightarrow E = \frac{q_{en}}{\varepsilon_0 A} \qquad V = \frac{4}{3}\pi r^3 \quad dV = 4\pi r^2 dr$$

$$q_{en} = \int_{R_i}^{r} \rho\, dV = \int_{R_i}^{r} \rho_0 \left(\frac{R_i}{r'}\right) \cdot 4\pi r'^2 dr' = \rho_0 R_i 4\pi \int_{R_i}^{r} r' dr' = \rho_0 R_i 4\pi \left(\frac{r^2}{2} - \frac{R_i^2}{2}\right)$$

$A = 4\pi r^2$ (gaussian sphere)

$$= \rho_0 R_i \pi (2r^2 - 2R_i^2)$$

$$= 2\rho_0 R_i \pi (r^2 - R_i^2)$$

$$E = \frac{2\rho_0 R_i \pi (r^2 - R_i^2)}{\varepsilon_0 4\pi r^2} = \boxed{\frac{\rho_0 R_i (r^2 - R_i^2)}{2\varepsilon_0 r^2} = E(r)}$$

(c) Calculate the electric field $E(r)$ for $r > R_o$

$$AE = \frac{q_{en}}{\varepsilon_0} \rightarrow E = \frac{q_{en}}{\varepsilon_0 A}$$

$$q_{en} = \int_{R_i}^{R_o} \rho\, dV = \int_{R_i}^{R_o} \rho_0 \left(\frac{R_i}{r'}\right) 4\pi r'^2 dr' = \rho_0 R_i 4\pi \int_{R_i}^{R_o} r' dr'$$

$$= \rho_0 R_i 4\pi \left(\frac{R_o^2}{2} - \frac{R_i^2}{2}\right)$$

$A = 4\pi r^2$ (gaussian sphere)

$$= 2\rho_0 R_i \pi (R_o^2 - R_i^2)$$

$$E = \frac{2\rho_0 R_i \pi (R_o^2 - R_i^2)}{4\pi r^2 \varepsilon_0}$$

$$\boxed{E(r) = \frac{\rho_0 R_i (R_o^2 - R_i^2)}{2 r^2 \varepsilon_0}}$$