Midterm 2

Physics 1B (Lec 5)

Name: [redacted]

ID number: [redacted]

Discussion section: [redacted]

Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	total
20	8	16	15	59

Problem 1

Sound wave with a frequency 200 Hz and amplitude 0.25 mm moves through gas. The wavelength is 2 m.

(a) Find the speed of the sound wave

$v = \lambda f = (200\,Hz)(2\,m) = \boxed{400\ m/s}$

(b) Find the maximal speed of a gas particle oscillating in this wave

$v_{max} = A\omega \qquad \omega = kv, \quad k = 2\pi/\lambda$

$\omega = \frac{2\pi}{\lambda} v$

$= \left(\frac{2\pi}{2m}\right)(400\,m/s) = 1256.637\ s^{-1}$

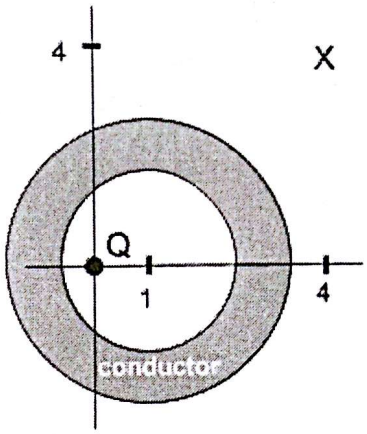
$v_{max} = (0.25\,mm)(1256.637\,s^{-1})$

$= 314.16\ mm/s$

$= \boxed{0.314\ m/s}$

Problem 2

A positive point charge Q=3 × 10^{-9} C is placed at the origin (0,0). A conducting spherical shell, carrying zero net charge, with the inner and the outer radii $R_i = 1.5\ m$ and $R_o = 2.5\ m$, respectively, is centered at a point with coordinates (1,0), as shown. All coordinates are in meters. *[Hint: does the charge density on the outer surface depend on the location of charge Q?]*



(a) Calculate the charge on the inner surface of the sphere. (Justify your answer.)

0, bic the charge will tend to go toward the outer surface.

1

(b) Calculate the electric field at point X with coordinates (4,4).

$\oint E \cdot dA = Q_{encl}/\varepsilon_0$

$4\pi r^2 E = Q/\varepsilon_0$

$E = \frac{Q}{4\pi r^2 \varepsilon_0}$, $r = 4\sqrt{2}$ m

$E = \boxed{0.842\ N/C}$

5

(c) Calculate the surface charge density on the outer surface of the sphere.

$dq = \sigma 2\pi r\, dr$

$\int_{R_i}^{R_o} dE = \int_{R_i}^{R_o} \frac{k\sigma 2\pi r\, dr}{(4\sqrt{2})^2}$

2

$E = \frac{k\sigma\pi}{16} \int_{R_i}^{R_o} r\, dr$

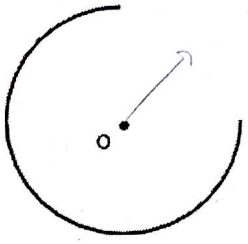
$= \sigma \left(\frac{k\pi}{16}\right)\left(\frac{1}{2}\right)\left(R_o^2 - R_i^2\right)$

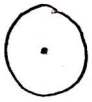
$0.842 = \sigma\,(3.58 \times 10^8)$

$\boxed{\sigma = 3.015 \times 10^{-8}\ C/m^2}$

Problem 3

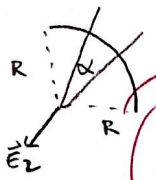
A thin thread carrying a constant charge density $\lambda = 4 \times 10^{-9}$ C/m is shaped as 3/4 of a circle. Calculate the electric field at the center of the circle O. $R = 1m$





Full circle: $\vec{E}_1 = \vec{0}$.

Quarter circle:



-4 ??? His?

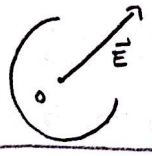
$$dE = \frac{K\lambda R\, d\alpha}{R^2} \Rightarrow E = \int_{-\pi/4}^{\pi/2} \frac{K\lambda R\, d\alpha}{R^2} \cos\alpha = \frac{\sqrt{2}\, K\lambda}{R}$$

$$= \frac{(\sqrt{2})(9\times10^9\ \frac{Nm^2}{C^2})(4\times10^{-9}\ C/m)}{1\ m}$$

$$= 50.91168825\ N/C$$

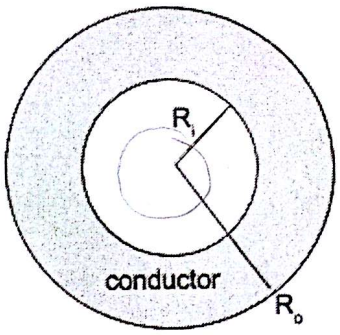
$\Rightarrow E = |E_1 - E_2| =$ **50.912 N/C** (in the opposite direction)

$$\vec{E} = 50.912\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) N/C$$



Problem 4

A sphere of radius R_i carrying the charge density $\rho = \rho_0(R_i/r)$, $r < R_i$, is surrounded by a conducting spherical shell with the inner and the outer radii R_i and R_o, respectively. There is no net charge on the conducting shell, and no charge outside R_o.



(a) [8 pts] Calculate the electric field $E(r)$ for $r < R_i$

$4\pi r^2 E = \frac{1}{\varepsilon_0}\int_0^r \rho(r)\, dV = \frac{1}{\varepsilon_0}\int_0^r \rho(R)\, 4\pi r^2 dr$

$r^2 E = \frac{1}{\varepsilon_0}\int_0^r \rho_0 \frac{R_i}{r} r^2 dr$

$= \frac{\rho_0 R_i}{\varepsilon_0}\int_0^r r\, dr = \frac{r^2}{2}\frac{\rho_0 R_i}{\varepsilon_0}$

$\Rightarrow E = \boxed{\frac{\rho_0 R_i}{2\varepsilon_0}}$ ✓

(b) [4 pts] Calculate the electric field $E(r)$ for $R_i < r < R_o$

$\vec{E} = 0$, there is no electric field within a conductor ✓

(c) [8 pts] Calculate the electric field $E(r)$ for $r > R_o$

$\oint E \cdot dA = \frac{Q_{encl}}{\varepsilon_0}$

$(4\pi r^2) E = \frac{1}{\varepsilon_0}\int_0^r \rho_0 \left(\frac{R_i}{r}\right) \frac{4}{3}\pi r^3 dr$

$4\pi r^2 E = \frac{1}{\varepsilon_0}\rho_0 R_i \frac{4\pi}{3}\int_0^r r^2 dr$

$r^2 E = \frac{1}{\varepsilon_0}\frac{\rho_0 R_i}{3}\frac{r^3}{3}$

$\boxed{E(r) = \frac{\rho_0 R_i r}{9\varepsilon_0}}$

15/20