Problem 1

Sound wave with a frequency 200 Hz and amplitude 0.25 mm moves through gas. The wavelength is 2 m.

(a) Find the speed of the sound wave

$V = \lambda f$

$= (2m)(200/s)$

$= \boxed{400\ m/s}$

✓

(b) Find the maximal speed of a gas particle oscillating in this wave

$y(x,t) = A\sin(kx - \omega t + \varphi)$

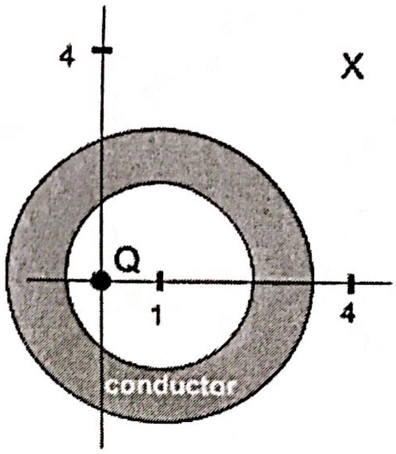
$V(x,t) = \frac{dy}{dt} = -A\omega\cos(kx - \omega t + \varphi)$

$V_{max} = |-A\omega| = (0.25\times10^{-3}\ m)(2\pi)(200/s)$

$= \boxed{0.314\ m/s}$

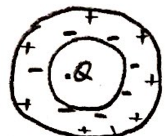
Problem 2

A positive point charge Q=3 × 10^{-9} C is placed at the origin (0,0). A conducting spherical shell, carrying zero net charge, with the inner and the outer radii $R_i = 1.5\ m$ and $R_o = 2.5\ m$, respectively, is centered at a point with coordinates (1,0), as shown. All coordinates are in meters. *[Hint: does the charge density on the outer surface depend on the location of charge Q?]*



(a) Calculate the charge on the inner surface of the sphere. (Justify your answer.)

charge is -3×10^{-9}C. shell carries 0 net charge. shell is a conductor, so the electrons move to the inner surface freely in response to Q.



net charge 0.

(b) Calculate the electric field at point X with coordinates (4,4).

$EA = q_{enc}/\epsilon_0$

(1,0) to (4,4)

dist= $\sqrt{3^2+4^2} = 5$

$$E = \frac{(3\times10^{-9})}{4\pi(5)^2\epsilon_0} = \boxed{1.08\ N/C}$$

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(c) Calculate the surface charge density on the outer surface of the sphere.

charge density doesn't depend on location of Q.

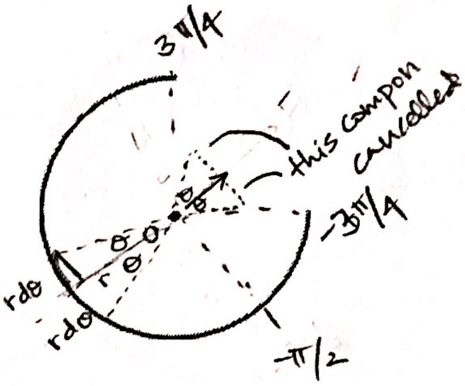
$$E = \frac{kq}{r^2} = \frac{q_{enc}}{4\pi r^2\epsilon_0} = \frac{3\times10^{-9}}{4\pi(2.5)^2\epsilon_0} = 4.316\ N/C = \frac{\sigma}{\epsilon_0}$$ (surface of conductor)

$$\sigma = E\cdot\epsilon_0 = \frac{q_{enc}}{4\pi r^2} = \boxed{3.82\ C/m^2}$$

-1.

Problem 3

A thin thread carrying a constant charge density $\lambda = 4 \times 10^{-9}$ C/m is shaped as 3/4 of a circle. Calculate the electric field at the center of the circle O.



$$E = \int k \frac{dq}{r^2}, \quad dq = \lambda r d\theta$$

$$E = \int_{-3\pi/4}^{3\pi/4} \frac{k \lambda r d\theta}{r^2} \cos\theta \quad (E\sin\theta \text{ cancelled})$$

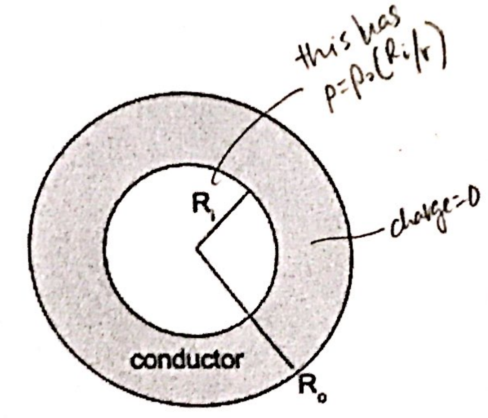
$$= \int_{-3\pi/4}^{3\pi/4} \frac{k\lambda \cos\theta}{r} d\theta$$

$$= \frac{k\lambda}{r}\left[\sin\theta \Big|_{-3\pi/4}^{3\pi/4}\right] = \frac{k\lambda}{r}\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right] = \frac{\sqrt{2}k\lambda}{r}$$

$$E = \frac{\sqrt{2}k(4\times10^{-9})}{1} = \boxed{50.86 \text{ N/C}}$$

Problem 4

A sphere of radius R_i carrying the charge density $\rho = \rho_0(R_i/r)$, $r < R_i$, is surrounded by a conducting spherical shell with the inner and the outer radii R_i and R_o, respectively. There is no net charge on the conducting shell, and no charge outside R_o.



(a) [8 pts] Calculate the electric field $E(r)$ for $r < R_i$

$$E(r) = \frac{q_{enc}}{\epsilon_0 \cdot A}$$

$$q_{enc} = \int_0^{r'} \rho_0 \left(\frac{R_i}{r'}\right) 4\pi r'^2 dr' = 4\pi \rho_0 R_i \int_0^{r} r' dr'$$

$$= 4\pi \rho_0 R_i \left[\frac{1}{2} r'^2 \Big|_0^r\right] = 2\pi \rho_0 R_i r^2$$

$$E(r) = \frac{\cancel{2}\cancel{\pi} \rho_0 R_i \cancel{r^2}}{2 \epsilon_0 \cdot \cancel{4\pi} \cancel{r^2}} = \boxed{\frac{\rho_0 R_i}{2\epsilon_0}}$$

(b) [4 pts] Calculate the electric field $E(r)$ for $R_i < r < R_o$

$E(r) = 0$. conducting shell.

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(c) [8 pts] Calculate the electric field $E(r)$ for $r > R_o$

$$E(r) = \frac{q_{enc}}{\epsilon_0 \cdot A}$$

$$q_{enc} = \int \rho \, dV = \int \rho 4\pi r^2 dr$$

$$V = \frac{4}{3}\pi r^3$$

$$dV = 4\pi r^2 dr$$

$$q_{enc} = \int_0^{R_i} \rho_0 \left(\frac{R_i}{r}\right) 4\pi r^2 dr$$

$$= 4\pi \rho_0 R_i \int_0^{R_i} r\,dr = 4\pi \rho_0 R_i \left[\frac{1}{2} r^2 \Big|_0^{R_i}\right] = 2\pi \rho_0 R_i^3$$

$$E(r) = \frac{2\pi \rho_0 R_i^3}{\epsilon_0 \cdot 4\pi r^2} = \boxed{\frac{\rho_0 R_i^3}{2 \epsilon_0 r^2}}$$