Midterm 2

Physics 1B (Lec 5)

Name: Solutions

ID number:

Discussion section:

Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	total

Problem 1

Sound wave with a frequency 200 Hz and amplitude 0.25 mm moves through gas. The wavelength is 2 m.

(a) Find the speed of the sound wave

$$v = \lambda f = 400\ m/s$$

(b) Find the maximal speed of a gas particle oscillating in this wave

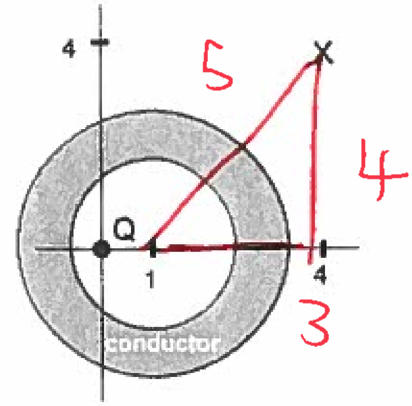
$$y = A\cos(\omega t)$$

$$v_y = -A\omega\sin(\omega t)$$

$$V_{max} = A\omega = A2\pi f = 0.314\ m/s$$

Problem 2

A positive point charge Q=3 × 10^{-9} C is placed at the origin (0,0). A conducting spherical shell, carrying zero net charge, with the inner and the outer radii $R_i = 1.5\ m$ and $R_o = 2.5\ m$, respectively, is centered at a point with coordinates (1,0), as shown. All coordinates are in meters. *[Hint: does the charge density on the outer surface depend on the location of charge Q?]*



(a) Calculate the charge on the inner surface of the sphere. (Justify your answer.)

$-Q \quad (-3\times10^{-9} C)$

(b) Calculate the electric field at point X with coordinates (4,4).

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{3\times10^{-9} C}{4\pi\epsilon_0 (5)^2} = 1.08\ N/C$$

(c) Calculate the surface charge density on the outer surface of the sphere.

$$\sigma = \frac{Q}{4\pi r^2} = \frac{3\times10^{-9} C}{4\pi (2.5\ m)^2} = 3.82\times10^{-11} \frac{C}{m^2}$$

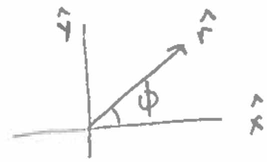
Problem 3

A thin thread carrying a constant charge density $\lambda = 4 \times 10^{-9}$ C/m is shaped as 3/4 of a circle. Calculate the electric field at the center of the circle O.

$\vec{E}$ will have $\hat{x}$ and $\hat{y}$ compontents:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl}{r^2}\,\hat{r}$$

(+4)



$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$

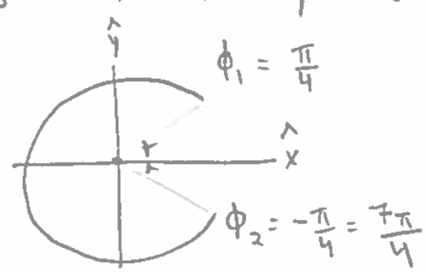
$r = R = 1, \quad dl = R\,d\phi, \quad \vec{r} = R(-\hat{r})$

(+4)

$$\vec{E} = \frac{\lambda R}{4\pi\epsilon_0 R^2}\int d\phi\,(-\hat{r}) = \frac{-\lambda}{4\pi\epsilon_0}\int(\hat{x}\cos\phi + \hat{y}\sin\phi)\,d\phi = \frac{\lambda}{4\pi\epsilon_0}(\hat{x}\sin\phi - \hat{y}\cos\phi)\Big|_{\phi_1}^{\phi_2}$$

Can cancel a component by making the ring symmetric along $\hat{x}$ or $\hat{y}$ (magnitude will be the same)

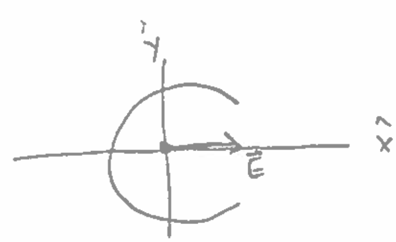
Along $\hat{x}$:



$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0}(\hat{x}\sin\phi - \hat{y}\cos\phi)\Big|_{\frac{\pi}{4}}^{\frac{7\pi}{4}} = \frac{\lambda\sqrt{2}}{4\pi\epsilon_0}\,\hat{x}$$

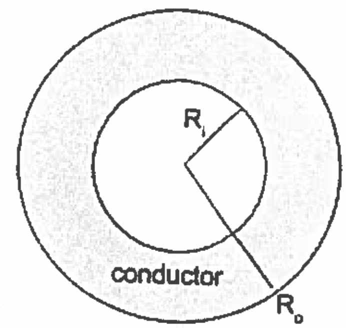
(+4) (+4) (+4)

Points radially outward



Problem 4

A sphere of radius R_i carrying the charge density $\rho = \rho_0(R_i/r)$, $r < R_i$, is surrounded by a conducting spherical shell with the inner and the outer radii R_i and R_o, respectively. There is no net charge on the conducting shell, and no charge outside R_o.



(a) [8 pts] Calculate the electric field $E(r)$ for $r < R_i$

$$Q = \int_0^r \rho_0 \frac{R_i}{s} 4\pi s^2 \, ds$$

(5)

$$= \rho_0 R_i 2\pi r^2$$

$$E = \frac{Q}{\varepsilon_0 A} = \frac{2\pi r^2 \rho_0 R_i}{4\pi r^2 \varepsilon_0} = \frac{\rho_0 R_i}{2\varepsilon_0}$$

(b) [4 pts] Calculate the electric field $E(r)$ for $R_i < r < R_o$

$$\because \sum Q_{enclosed} = 0 \Rightarrow \vec{E} = 0.$$

(c) [8 pts] Calculate the electric field $E(r)$ for $r > R_o$

$$Q = \int_0^{R_i} \rho_0 \cdot \frac{R_i}{s} \cdot 4\pi s^2 \, ds = 2\pi \rho_0 R_i^3.$$

$$E = \frac{Q}{\varepsilon_0 A} = \frac{2\pi \rho_0 R_i^3}{\varepsilon_0 \cdot 4\pi r^2} = \frac{\rho_0 R_i^3}{2 r^2 \varepsilon_0}$$