Midterm 1

Physics 1B (Lec 5)

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Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	6	total
20	20	20	20	20	20	120

Problem 1

Sound wave with a frequency 200 Hz and amplitude 0.25 mm moves through gas. The wavelength is 2 m.

(a) Find the speed of the sound wave

$v_{wave} = \lambda f = (200)(2)$

$= \boxed{400 \text{ m/s}}$

wave moves λ in period $T = \frac{1}{f}$

so that $v = \frac{\lambda}{T} = \lambda f$

(b) Find the maximal speed of a gas particle oscillating in this wave

let wave funct. be $y(x,t) = A\cos(kx - \omega t)$, $k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$

For any pt. @ fixed (equilib.) position x: $\frac{\partial y}{\partial t} = v(x,t) = -A\omega \sin(kx - \omega t)$

so that $v_{max} = |-A\omega| = A\omega$ (when $\sin(kx - \omega t) = -1$)

since $-1 \leq \sin(kx - \omega t) \leq 1$

$\Rightarrow A(2\pi f)$

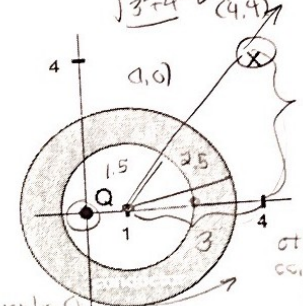
$\Rightarrow (0.25 \times 10^{-3} \text{ m})(2\pi)(200 \text{ Hz})$

$= \boxed{0.314 \text{ m/s}}$

Problem 2

A positive point charge Q=3 × 10^{-9} C is placed at the origin (0,0). A conducting spherical shell, carrying zero net charge, with the inner and the outer radii $R_i = 1.5\ m$ and $R_o = 2.5\ m$, respectively, is centered at a point with coordinates (1,0), as shown. All coordinates are in meters. *[Hint: does the charge density on the outer surface depend on the location of charge Q?]*

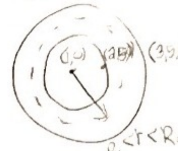
$\sqrt{3^2+4^2}$ ✓ (4,4)



otherwise, the conductor would not be in equilibrium.

(a) Calculate the charge on the inner surface of the sphere. (Justify your answer.)

Draw Gaussian sphere inside shell, $R_i < r < R_o$



E inside the conducting shell must be 0, If we draw a Gaussian sphere (centered with shell) that crosses through the shell, E through it must be 0 and by Gauss's Law ($R_i < r < R_o$)

$\Phi = \oint \vec{E} \cdot d\vec{A} = \oint 0 \cdot d\vec{A} = 0 = \frac{Q_{enc}}{\varepsilon_0}$ so that Q_{enc} must be 0. Therefore the charge on the inner surface must be $\boxed{-3 \times 10^{-9}\ C}$ to cancel out the enclosed positive charge.

(b) Calculate the electric field at point X with coordinates (4,4).

Draw Gaussian surface centered w/ sphere w/ edge passing thru point (4,4).

Let r = distance from center to X: $\sqrt{(4-1)^2 + 4^2} = 5$

$\Phi = \oint \vec{E} \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$

Q on outside surface is evenly distributed, $Q = 3 \times 10^{-9}$ C.

$\Phi = EA = E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$

Therefore when $r > R_o$ we can treat the shell like a point charge of $Q = +3 \times 10^{-9}$ @ (1,0).

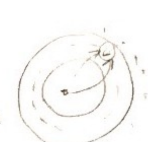
$E = \frac{Q_{ot}}{4\pi\varepsilon_0 r^2}$

$E = \frac{3 \times 10^{-9}}{\varepsilon_0 (4\pi (5)^2)}$

$= \boxed{1.079\ N/C}$

(pointing in direction $3\hat{i} + 4\hat{j}$)

(c) Calculate the surface charge density on the outer surface of the sphere.



↑ charge distribution inside/at surface of cavity produces Ø net electric field & does not affect outer surface distribution

Since the inner surface has charge -3×10^{-9} C, the outer surface must have charge $+3 \times 10^{-9}$ to maintain the shell's Ø net charge.

The charge distribution on the inside of the shell does not affect the charge distribution on the outside. The positive charge on the outer surface will distribute itself evenly, yielding uniform charge density

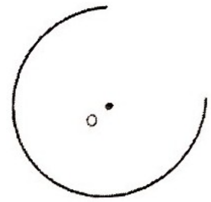
as the like charges will want to get as far away as possible from one another

$\frac{Q}{4\pi (R_o)^2} = \frac{+3 \times 10^{-9}}{4\pi (2.5)^2} = \boxed{+3.82 \times 10^{-11}\ C/m^2}$

$r = 1\,m$

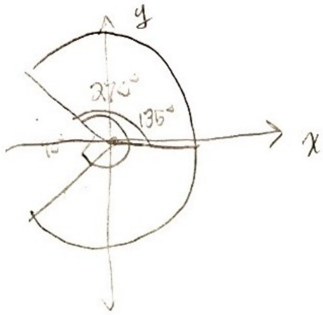
Problem 3

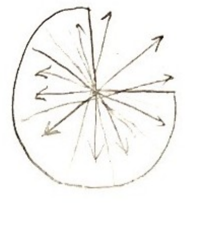
A thin thread carrying a constant charge density $\lambda = 4 \times 10^{-9}$ C/m is shaped as 3/4 of a circle. Calculate the electric field at the center of the circle O.



①

Align w/ x axis as follows:

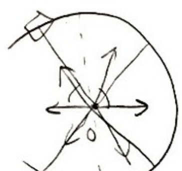




net field should point 45° above x-axis, since $\lambda > 0$

∴ more $d\vec{E}$ elements non canceled. have components components pointing in that direction than the opposite direction

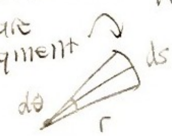
(Field line diagram)



⇒ All vertical components (in this orientation) of $d\vec{E}$ cancel out due to symmetry.

$$\vec{E} = \int d\vec{E} = \int \frac{dQ}{4\pi\epsilon (r^2)}$$

Take dQ to be small arc segment



⇒ let s = arc length

$$dQ = \lambda\, ds = \lambda r\, d\theta$$

$$ds = r\, d\theta$$

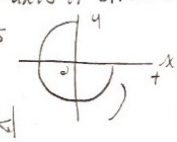
$$|\vec{E}| = \left(\lambda \int_{-3\pi/4}^{3\pi/4} \frac{r\, d\theta}{4\pi\epsilon (r^2)} \cos\theta \right)$$

all dQ elements are distance r from center

for horizontal component only.

$$E = \boxed{50.865\ N/C}$$

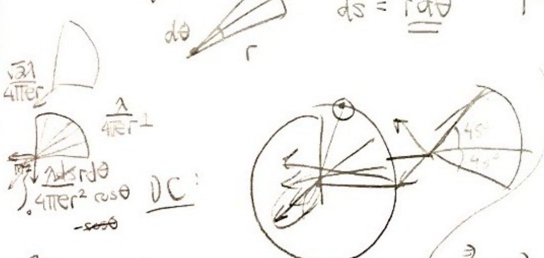
(pointing 45° above x-axis if orientation is



)

⇒ $\frac{\sqrt{2}\lambda}{4\pi\epsilon_0 r} = \frac{\sqrt{2}(4\times10^{-9})}{4\pi\epsilon(1)}$

≈ 50.865



$\int_0 \frac{\lambda r\, d\theta}{4\pi\epsilon r^2} \cos\theta$ DC

$-\sin\theta$

By superposition $\vec{E}_{\text{whole circle}} = \vec{0}$

$\vec{E}_{\text{net}} =$

$\vec{E}$ from ◺ : $2\lambda \int_0^{\pi/4} \frac{r\, d\theta}{4\pi\epsilon (r^2)} \cos\theta$

$\frac{\sqrt{2}\lambda}{4\pi\epsilon r} + \vec{E}_{3/4\text{circle}}$ $\frac{\sqrt{2}\lambda}{4\pi\epsilon r}$ (down) $\frac{\sqrt{2}\lambda}{4\pi\epsilon r}$ $\sin\theta\Big|_0^{\pi/4}$

so $\vec{E}_{3/4\text{circle}} = \frac{\sqrt{2}\lambda}{4\pi\epsilon r}$

⇒ $\vec{E} = \frac{\lambda}{r 4\pi\epsilon_0} \int_{-3\pi/4}^{3\pi/4} \cos\theta\, d\theta$

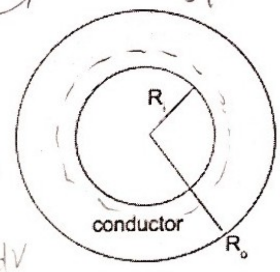
$= \frac{\lambda}{4\pi\epsilon_0 r} \left(\sin\theta \Big|_{-3\pi/4}^{3\pi/4} \right) = \frac{\lambda}{4\pi\epsilon_0 (r)} \sqrt{2} \cancel{\left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right)}$

$E(4\pi r^2) = \dfrac{\frac{\rho_0 R_i}{\varepsilon_0}\int_0^r \frac{r}{r} 4\pi r^2 \, dr}{\varepsilon_0 4\pi r^2}$ $\quad \dfrac{\rho_0 R_i r^2}{\varepsilon_0 2 r^2}$

only dependent on r so spherically symmetric.

Problem 4

A sphere of radius R_i carrying the charge density $\rho = \rho_0(R_i/r)$, $r < R_i$, is surrounded by a conducting spherical shell with the inner and the outer radii R_i and R_o, respectively. There is no net charge on the conducting shell, and no charge outside R_o.



(a) [8 pts] Calculate the electric field $E(r)$ for $r < R_i$

Gaussian surface inside sphere:

Gauss's Law: $\Phi = E(A) = \frac{Q_{enc}}{\varepsilon_0}$ where $Q_{enc} = \int_0^r \rho(r)\,dV$

$E(4\pi r^2) = \dfrac{\int_0^r \rho(r)\,dV}{\varepsilon_0} = \dfrac{\rho_0 \int_0^r \left(\frac{R_i}{r}\right) dV}{\varepsilon_0}$

$\Rightarrow R_i \rho_0 \int_0^r \frac{1}{r} 4\pi r^2 \, dr$

$4\pi R_i \rho_0 \int_0^r r \, dr$

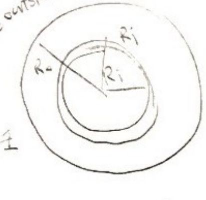
$\Rightarrow 4\pi R_i \rho_0 \left(\frac{1}{2} r^2 \Big|_0^r\right)$

$\Rightarrow \dfrac{4\pi R_i \rho_0 \left(\frac{r^2}{2}\right)}{\varepsilon_0}$

$\Rightarrow E(r) = \dfrac{\cancel{4\pi} R_i \rho_0 \cancel{r^2}}{2\varepsilon_0 (\cancel{4\pi r^2})}$, $r < R_i$

$\Rightarrow \boxed{\dfrac{\rho_0 R_i}{2\varepsilon_0}}$

no charge outside



$\uparrow \left(\frac{1}{r}\right)$

DC: $r=1$ $R=1$

$= \int_0^r \frac{\left(\frac{1}{r}\right)(4\pi r^2 dr)}{\varepsilon_0}$ $\quad \frac{1}{3}$?

$E(4\pi r^2)$ $\frac{r^2}{2r^2}$

$\Rightarrow$ spherically symmetric.

(b) [4 pts] Calculate the electric field $E(r)$ for $R_i < r < R_o$.

$R_i < r < R_o$

$\Rightarrow$ Inside conductor $\boxed{E(r) = 0}$ since conducting shell is in equilibrium; otherwise, the field would cause charge to move.

20/20

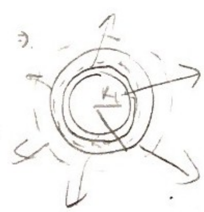
(c) [8 pts] Calculate the electric field $E(r)$ for $r > R_o$

$\frac{4\pi\rho_0 R_i^3}{\cancel{8}\cancel{2}} \int_0^{R_i} r\,dr$

$\Rightarrow \Phi = E(A) = \frac{Q_{enc}}{\varepsilon_0}$ $\qquad Q_{enc} = \int_0^{R_i} \rho_0 \left(\frac{R_i}{r}\right) 4\pi r^2 \, dr$

$\Rightarrow 4\pi\rho_0 R_i \int_0^{R_i} r \, dr$

$\Rightarrow$



Draw Gaussian surface where $r > R_o$ - spherically symmetric since $\rho(r)$ only dependent on r.

$\frac{4\pi\rho_0 R_i}{2}$ $\cancel{2\pi\rho_0 R_i R_i^2}$

$4\pi\rho_0 R_i \left(\frac{1}{2} r^2 \Big|_0^{R_i}\right)$

$\Rightarrow \dfrac{4\pi\rho_0 R_i (R_i^2)}{2} = 2\pi\rho_0 R_i^3$

$E(4\pi r^2) = \dfrac{2\pi\rho_0 R_i^3}{\varepsilon_0}$

$E = \dfrac{\cancel{2\pi}\rho_0 R_i^3}{\cancel{4\pi} r^2 \varepsilon_0}$ $\Rightarrow E(r) = \boxed{\dfrac{\rho_0 R_i^3}{2\varepsilon_0}\left(\dfrac{1}{r^2}\right)}$, $r > R_o$

DC: act like pt. charge when $r > R$