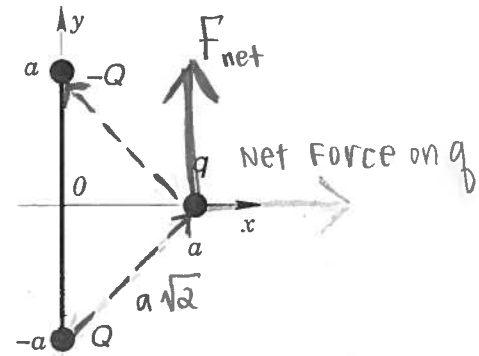
Midterm 2
Physics 1B

Name: Ramya Satish

ID number: [redacted]

Lecture 5. Section (or TA name): Justin Little

1. A dipole consists of a positive charge Q and a negative charge $-Q$ positioned at $y = -a$ and $y = a$ and connected by a rigid rod, as shown. A positive charge q is located on the x-axis at $x = a$.



10 (a) What is the direction of the force acting on the charge q? (Draw a vector showing the direction.)

The directionvector is the arrow labeled F_{net}. The direction is upward. (Magnitude not shown in arrow)

10 (b) What is the magnitude of the force acting on charge q?

$$F_{Qq} = \frac{kqQ}{(a\sqrt{2})^2} = \frac{kqQ}{2a^2}$$

$$F_{Qq,y} = \frac{kqQ\sqrt{2}}{2a^2\sqrt{2}}$$

$$F_{-Qq,y} = \frac{kqQ}{2a^2\sqrt{2}}$$

$$F_{net} = \frac{\cancel{2}kqQ}{\cancel{2}a^2\sqrt{2}} = \boxed{\frac{kqQ}{\sqrt{2}\,a^2}\ N}$$

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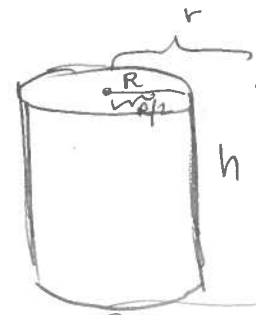
2. A very long, solid, non-conducting cylinder has radius R. The volume charge density depends on the distance r from the central axis as

$$\rho(r) = \rho_0 \left(\frac{R}{r}\right), \quad r \leq R;$$

$$\rho(r) = 0, \quad r > R.$$



10 a) What is the electric field at distance $r = R/2$ from the axis?

$$q_{encl} = \int dq = \int_0^{R/2} \rho_0 \left(\frac{R}{r}\right) dV$$

$$= \int_0^{R/2} \rho_0 \frac{R}{\cancel{r}} (2\pi \cancel{r} h)\, dr \qquad V = \pi r^2 h,\ dV = 2\pi r h\, dr$$

$$= \rho_0 2\pi R^2 h$$

Gauss Law: $\frac{\rho_0 \cancel{2\pi} R^2 \cancel{h}}{\cancel{2}\varepsilon_0} = EA = \frac{\cancel{2\pi} R \cancel{h} E}{\cancel{2}}, \quad \frac{\rho_0 R^2}{\varepsilon_0} = ER$

$$\boxed{E = \frac{\rho_0 R}{\varepsilon_0} \ \frac{N}{C}}$$

10 b) What is the electric field at distance $r = 3R$ from the axis?

$$q_{encl} = \int dq = \int_0^R \rho_0 \frac{R}{r}\, dV$$

$$= \int_0^R \rho_0 \frac{R}{\cancel{r}} (2\pi \cancel{r} h)\, dr$$

$$= \rho_0 2\pi R^2 h$$

Gauss Law: $\frac{\rho_0 2\pi R^2 h}{\varepsilon_0} = EA$

$$\frac{\rho_0 \cancel{2\pi} R^{\cancel{2}} \cancel{h}}{\varepsilon_0} = E(\cancel{2\pi})(3\cancel{R})(\cancel{h})$$

$$E = \boxed{\frac{\rho_0 R}{3\varepsilon_0} \ \frac{N}{C}}$$

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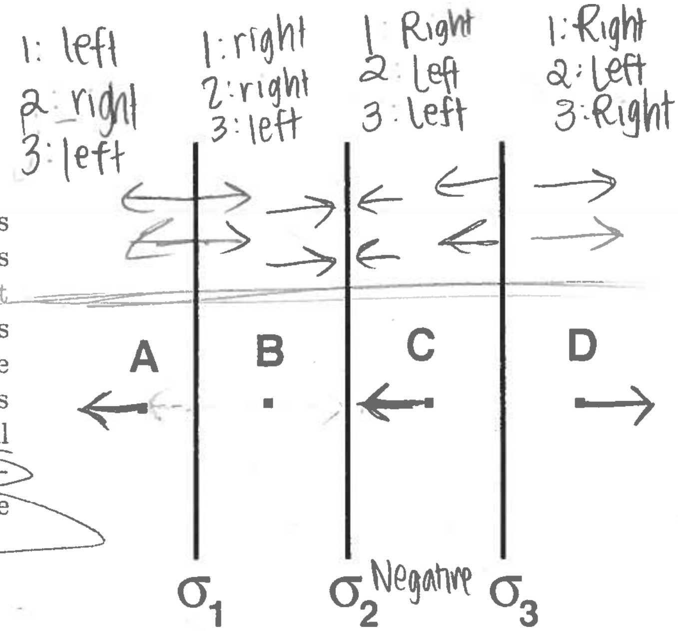
3. Three very large square planes are arranges parallel to each other, as shown (view from edge). The first plane is charged positively and has charge per unit area $\sigma_1 = \sigma$. The other two planes have charge densities $\sigma_2 = -2\sigma$ and $\sigma_3 = 3\sigma$. Express all results below in terms of σ. Show direction of the field by an arrow in the diagram.



magnitude E field of sheet of charge with uniform charge/unit area σ is $\frac{\sigma}{2\varepsilon_0}$

(a) What is the direction and the magnitude of the electric field at point A?

$$|E_1| = \frac{\sigma}{2\varepsilon_0} \quad |E_2| = \frac{-2\sigma}{2\varepsilon_0} = \frac{-\sigma}{\varepsilon_0} \quad |E_3| = \frac{3\sigma}{2\varepsilon_0}$$

$$E_{net} = \frac{-\sigma}{2\varepsilon_0} + \frac{\sigma}{\varepsilon_0} - \frac{3\sigma}{2\varepsilon_0} \quad \boxed{\frac{\sigma}{\varepsilon_0}\,\frac{N}{C}\text{ to the left}}$$

(b) What is the direction and the magnitude of the electric field at point B?

$$|E_1| = \frac{\sigma}{2\varepsilon_0} \quad |E_2| = \frac{\sigma}{\varepsilon_0} \quad |E_3| = \frac{3\sigma}{2\varepsilon_0}$$

$$E_{net} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{\varepsilon_0} - \frac{3\sigma}{2\varepsilon_0} = \boxed{0\,\frac{N}{C}}$$

(c) What is the direction and the magnitude of the electric field at point C?

$$|E_1| = \frac{\sigma}{2\varepsilon_0} \quad |E_2| = \frac{\sigma}{\varepsilon_0} \quad |E_3| = \frac{3\sigma}{2\varepsilon_0}$$

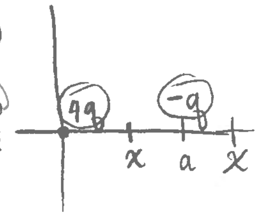
$$E_{net} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} - \frac{3\sigma}{2\varepsilon_0} = \boxed{\frac{2\sigma}{\varepsilon_0}\cdot\frac{N}{C}\text{ to the left}}$$

(d) What is the direction and the magnitude of the electric field at point D?

$$|E_1| = \frac{\sigma}{2\varepsilon_0} \quad |E_2| = \frac{\sigma}{\varepsilon_0} \quad |E_3| = \frac{3\sigma}{2\varepsilon_0}$$

$$E_{net} = \frac{3\sigma}{2\varepsilon_0} - \frac{\sigma}{\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}\,\frac{N}{C}\text{ to the right}}$$

4. Two particles are fixed to an x axis: particle 1 of charge $(4q)$ at $x = 0$ and particle 2 of charge $(-q)$ at $x = a$.



(a) Assuming the potential vanishes at infinity, at what finite coordinate on the axis is the net potential produced by the particles equal to zero? (Find all such points if there are more than one.)

$$V = k \sum_i \frac{q_i}{r_i}$$

Let x represent the position of the coordinate

$$\frac{4kq}{x} - \frac{kq}{a-x} = 0 \quad \text{OR} \quad \frac{4kq}{x} - \frac{kq}{x-a} = 0$$

$$\frac{4\cancel{kq}}{x} = \frac{\cancel{kq}}{a-x} \qquad \frac{4\cancel{kq}}{x} = \frac{\cancel{kq}}{x-a}$$

$$a - x = \frac{x}{4} \qquad \frac{x}{4} = x - a,$$

$$4a - 4x = x, \ 4a = 5x, \ \boxed{x = \tfrac{4}{5}a} \qquad x = 4x - \cancel{4a}, \ 4a = 3x, \ \boxed{x = \tfrac{4}{3}a}$$

(b) At what finite coordinate on the axis is the electric field equal to zero?

$$E_{net} = \frac{kq_1}{r_1^2} + \frac{kq_2}{r_1^2}$$

$$0 = \frac{k(4q)}{x^2} + \frac{-kq}{(x-a)^2}$$

$$\frac{\cancel{kq}}{(x-a)^2} = \frac{4\cancel{kq}}{x^2}$$

$$(x-a)^2 = \frac{x^2}{4}$$

$$x^2 - 2ax + a^2 = \frac{x^2}{4}$$

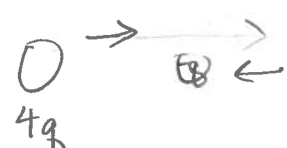
$$0.75x^2 - 2ax + a^2 = 0$$

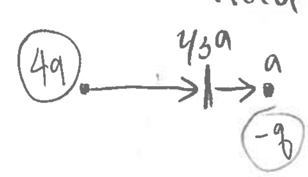
$$x = \frac{2a \pm \sqrt{4a^2 - 3a^2}}{1.5}$$

$$= \frac{2a \pm a}{1.5}$$

$$= \boxed{2a} \text{ or } \cancel{\tfrac{2}{3}a}$$

Does not work give direction of field





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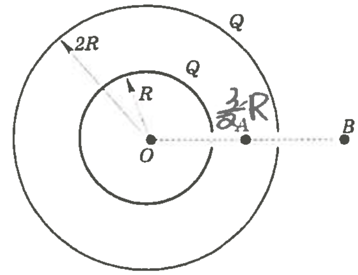
5. Two concentric thin non-conducting spherical shells have radii R and $2R$, and each has a small hole along the line OB, as shown. Each shell carries a positive charge Q, uniformly distributed with a constant charge density.



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(a) What is the electric potential at point A, at distance $(3R/2)$ from the center, assuming the potential is zero at infinity? (Any effect of the holes should be neglected.)

$$E_1(4\pi r^2) = \frac{Q}{\varepsilon_0} \qquad E_2(4\pi r^2) = \frac{2Q}{\varepsilon_0}$$

$$E_1 = \frac{Q}{4\pi r^2 \varepsilon_0} \qquad E_2 = \frac{Q}{2\pi r^2 \varepsilon_0}$$

$$V = \int_{\frac{3}{2}R}^{2R} \vec{E_1}\, d\vec{r} + \int_{2R}^{\infty} \vec{E_2}\, d\vec{r} \checkmark$$

$$= \int_{\frac{3}{2}R}^{2R} \frac{Q}{4\pi r^2 \varepsilon_0}\, dr + \int_{2R}^{\infty} \frac{Q}{2\pi r^2 \varepsilon_0}\, dr$$

$$= \frac{Q}{4\pi\varepsilon_0} \Big|_{3/2 R}^{2R} -\frac{1}{r} + \frac{Q}{2\pi\varepsilon_0} \Big|_{2R}^{\infty} -\frac{1}{r} = \frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{2R} + \frac{1}{1.5R}\right) + \frac{Q}{2\pi\varepsilon_0}\left(0 + \frac{1}{2R}\right)$$

$$= \frac{Q}{R24\pi\varepsilon_0} + \frac{Q}{4\pi R\varepsilon_0}$$

$$= \boxed{\frac{7Q}{24\pi R\varepsilon_0} \checkmark \ \frac{J}{C}}$$

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(b) A proton with charge q_p and mass m_p is released from point O in the direction of point B with an initial speed v_0. What is the speed of the proton at point B at distance $(3R)$ from the center? (Assume the motion is non-relativistic, and there are no forces other than the electrostatic forces.)

$$K_i + U_i = K_f + U_f \checkmark$$

$$\frac{1}{2} m_p V_0^2 + k\sum_i \frac{q_i}{r_i} = \frac{1}{2} m_p V_f^2 + U_B \rightarrow$$

$$U_B = q_p \int E_B\, dr \qquad E_B 4\pi r^2 = \frac{2Q}{\varepsilon_0}$$

$$U_B = q_p \int_{3R}^{\infty} \frac{2Q}{4\varepsilon_0 \pi r^2} = \frac{Q q_p}{2\pi\varepsilon_0}\left(\frac{1}{3R}\right) = \frac{Q q_p\, 2}{(4\pi R\varepsilon_0)(3)} \rightarrow \frac{2kQq_p}{3R} \checkmark$$

$$\frac{1}{2} m_p V_0^2 + \frac{kQq_p}{R} + \frac{kQq_p}{2R} \checkmark = \frac{1}{2} m_p V_f^2 + \frac{2kQq_p}{3R}$$

$$\frac{1}{2} m_p V_0^2 + \frac{3kQq_p}{2R} = \frac{1}{2} m_p V_f^2 + \frac{2kQq_p}{3R}$$

$$\frac{1}{2} m_p V_0^2 + \frac{5kQq_p}{6R} = \frac{1}{2} m_p V_f^2$$

$$m_p V_0^2 + \frac{5}{3}\frac{kQq_p}{R} = m_p V_f^2$$

$$V_f = \boxed{\sqrt{V_0^2 + \frac{5kQq_p}{3Rm_p}} \checkmark \ \frac{m}{s}}$$

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1	2	3	4	5	Total
20	20	20	20	20	100