Midterm 1

Physics 1B (Lec 4)

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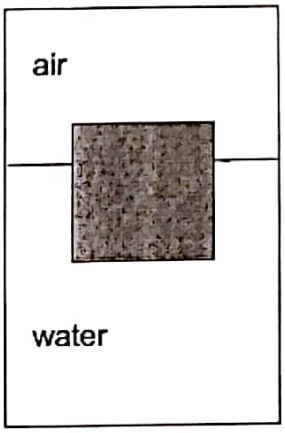
Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	16	2	18	3	20	4	20	5	20	6	20	total 114

Problem 1. A block with dimensions 1.00 m x 1.00 m x 1.00 m and mass $m_b = 900\,\text{kg}$ floats on the surface of the water ($\rho_w = 1.00\,\text{g/cm}^3$). $\rho_{obj} = \frac{900\,\text{kg}}{1\,\text{m}^3}$



10 (a) What is the height of the block above the water line?

$$\%\ \text{submerged} = \frac{\rho_{obj}}{\rho_W} = \frac{900\,\text{kg/m}^3}{1000\,\text{kg/m}^3} = 90\%$$

height above water line = 10% of 1m

= 10 cm

6 (b) What is the total force F of pressure acting on the bottom of the block, including the atmospheric pressure ($p_0 = 1.01 \times 10^5$ Pa) and the contribution from the water?

$F = pa$

$F_{atm} = p_0 A = 1.01 \times 10^5\,\text{Pa} \cdot 1\,\text{m}^2 = 1.01 \times 10^5\,\text{N}$

$F_W = F_B = V_{disp}\,\rho_W\,g = 0.9\,\text{m}^3\,(1000\,\text{kg/m}^3)\,(9.81\,\text{m/s}^2) = 8829\,\text{N}$

$F_{tot} = F_W =$ 8829N

Problem 2. Water with density 1.0 g/cm^3 comes out of a garden hose with speed of $v = 2.0$ m/s. Neglecting friction and viscosity, what is the gauge pressure inside the hose, far from the opening?

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const}$$

gauge pressure $= P - P_0$

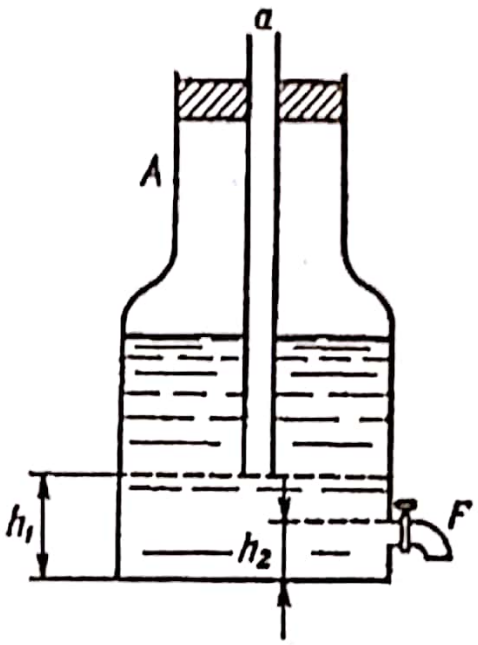
$$P + \frac{1}{2}\rho(0) + \rho g(0) = P_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

$$P - P_0 = \frac{1}{2}\rho v^2 = \frac{1}{2}(1000\ \text{kg/m}^3)(2.0\ \text{m/s})$$

gauge pressure: 1000 Pa

+18

20 **Problem 3.** A vessel filled with water communicates with the atmosphere through a glass tube (a), as shown in the figure. Assuming that $h_1 = 10\,\text{cm}$ and $h_2 = 5\,\text{cm}$, what is the speed of the fluid coming out of the faucet F?



$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{cnst.}$$

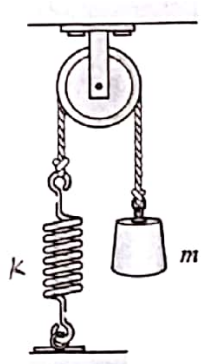
$$p_0 + \rho g h_1 + \frac{1}{2}\rho (0)^2 = p_0 + \rho g h_2 + \frac{1}{2}\rho v^2$$

$$v = \sqrt{2g(h_1 - h_2)}$$

$$= \sqrt{2(9.8\,\text{m/s}^2)(0.1\,\text{m} - 0.05\,\text{m})}$$

$$= \boxed{0.990\ \text{m/s}}$$

Problem 4. A mass m=1 kg is attached to a spring with a spring constant k=50 N/m as shown in the figure. The mass is kicked from the position of equilibrium with the downward velocity 3 m/s. Find the amplitude of the resulting small oscillations.



$t=0 \qquad V_0 = 3\,m/s \qquad y = 0$

$y(t) = A\cos(\omega t + \varphi)$

$A\cos\varphi = 0$

$\varphi = \pm\frac{\pi}{2}$

check $-\frac{\pi}{2}$ ⟵ ↓

$-A\omega\sin\varphi = V_0$ → $V_0 = A\omega$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{1}} = \sqrt{50}$

$A = \frac{V_0}{\omega} = \frac{3}{\sqrt{50}}$

$= \boxed{0.424\ m}$

+20

Problem 5. A piston of an engine undergoes vertical harmonic oscillations with amplitude 10 cm and some frequency $f = 1/T$. A washer rests on top of the piston. As the motor speed is slowly increased, at what minimal value of f will the washer no longer stay in contact with the piston? *(Hint: can the washer move downward with an acceleration greater than g?)*

$A = 0.1\text{m}$ $\quad T = \frac{2\pi}{\omega}$ $\quad \omega = 2\pi f$

$v(t) = -A\omega \sin(\omega t + \varphi)$

$a(t) = -A\omega^2 \cos(\omega t + \varphi)$

$a(t) = -A(2\pi f)^2 \cos(2\pi f t + \varphi) > g$

$\cos(\ldots) = -1$ since we want max accel

take -1 to remove negative sign

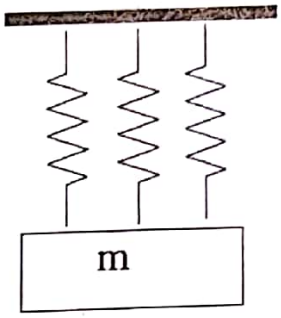
$A(2\pi f)^2 > g$

$f > \frac{1}{2\pi}\sqrt{\frac{g}{A}}$

$f > \boxed{1.5772 \text{ Hz}}$

washer will not stay in contact since piston will accelerate faster than g.

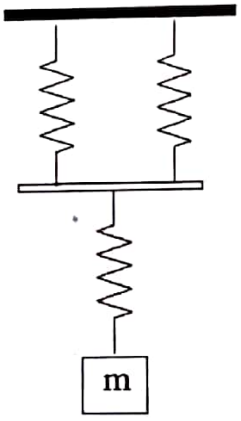
Problem 6. (a) Three identical springs with spring constants k=60 N/m are connected to the mass m=1 kg as shown in the figure. Find the period of small oscillations.



eq to having one spring w/ $k = 180$ N/m

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1\,kg}{180\,N/m}} = \boxed{0.468\,s}$$

(b) Three identical springs with k=60 N/m are connected to the mass m=1 kg as shown as shown in the figure. The horizontal bar is massless. What is the period of small oscillations?



top two springs eq to having one spring w/ $K_1 = 120$ N/m, $K_2 = 60$ N/m

$$T = 2\pi\sqrt{\frac{m}{K_{eff}}} = 2\pi\sqrt{\frac{m(K_1+K_2)}{K_1K_2}} = 2\pi\sqrt{\frac{1(180)}{7200}} = \boxed{0.993\,s}$$

for springs in series: $\frac{1}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2}$

$$\frac{1}{K_{eff}} = \frac{K_1+K_2}{K_1K_2}$$