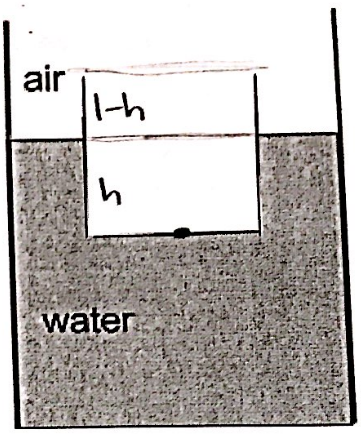
Problem 1. An open metal box with dimensions 1.00 m x 1.00 m x 1.00 m and mass 800 kg floats on the surface of the water ($\rho_w = 1.00\,g/cm^3$).



$1000\ kg/m^3$

(a) What is the height of the box above the water line?

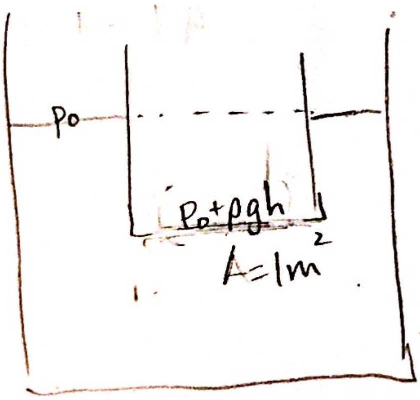
$$\frac{V_{submerged}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}} = \frac{800}{1000} = 0.8$$

$$V_{submerged} = (1\ m) \times (1m) \times h = 0.8 \times 1 m^3$$

$$h = 0.8 m$$

$$1 - h = \boxed{0.2m}$$

(b) What is the total force F of pressure acting on the bottom of the box, including the atmospheric pressure ($p_0 = 1.01 \times 10^5$ Pa) and the contribution from the water?



$$F = PA$$

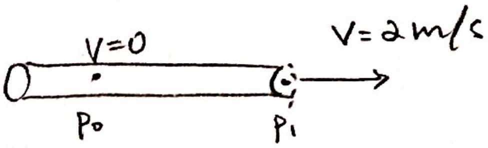
$$= (P_0 + \rho g h) A$$

$$= [1.01 \times 10^5 + 1000(9.8)(0.8)](1)$$

$$= \boxed{108840 N}$$

Problem 2. A paint with density 1.2 g/cm^3 comes out of a paint gun with a speed 2 m/s. Neglecting friction and viscosity, what is the gauge pressure inside the hose?

$$P_0 + \rho g h_0 + \frac{1}{2}\rho V_0^2 = P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2$$



$$h_0 = h_1$$

$$P_0 + \frac{1}{2}\rho V_0^2 = P_1 + \frac{1}{2}\rho V_1^2$$

$V_0 = 0$ $\quad P_1 = 0$ (gauge pressure)

$$P_0 = \frac{1}{2}\rho V_1^2 = \frac{1}{2}(1.2\,g/cm^3)(2\,m/s)^2$$

$$= \frac{1}{2}(1.2\times10^3\,kg/m^3)(2\,m/s)^2$$

$$= \boxed{2400\,Pa}$$

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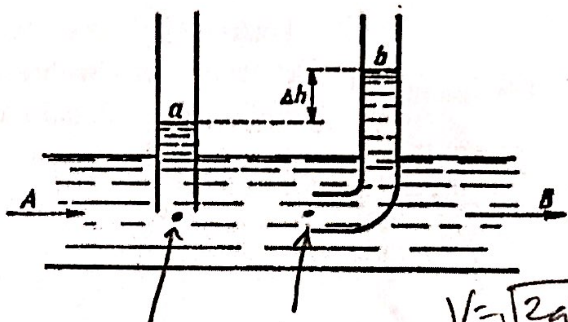
Problem 3. Water flows along a horizontal pipe AB, as shown in the figure. The difference between the levels of the liquid in tubes a and b is $\Delta h = 1$ cm. The diameters of tubes a and b are the same. Water density is $\rho = 1$ g/cm³. Determine the velocity of the water flowing along the pipe AB.



$V = \sqrt{2gh} = 0.44??$

$P_A + \rho g h_A + \frac{1}{2}\rho V_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho V_B^2$

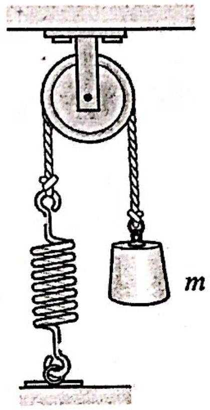
$P_A = P_B$ $\quad V_B = 0$

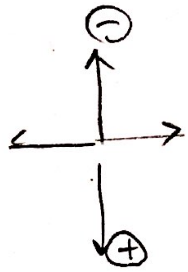
$\rho g h_A + \frac{1}{2}\rho V_A^2 = \rho g h_B$

$\frac{1}{2}\rho V_A^2 = \rho g \Delta h$

$V_A = \sqrt{2g\Delta h} = \sqrt{2(9.8)(.01)} = \boxed{0.44 \text{ m/s}}$

Problem 4. A mass m=10 kg is attached to a spring with a spring constant k=300 N/m as shown in the figure. The mass is released with zero velocity from the position in which the spring was unstretched. Find the amplitude of the resulting small oscillations.





$F = ma = -kx$

$= 10(9.8) = -300x$

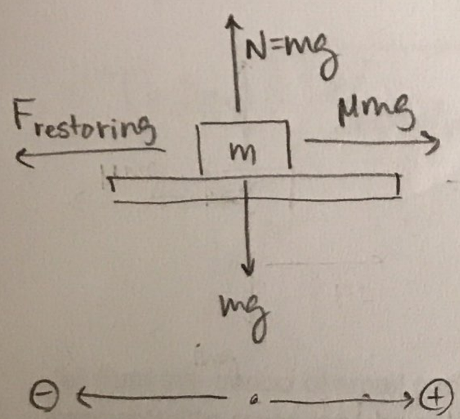
$x = -0.327\,m$

$A = 0.327\,m$

← simple harmonic motion

Problem 5. A horizontal platform vibrates horizontally with an amplitude 10 cm and a frequency $f = 0.5$ Hz. When a small block is placed on top of the platform, the frequency and the amplitude remain the same. What is the minimum value μ that the coefficient of static friction must have for the block to oscillate with the platform without sliding? *(Hint: the force of friction on the block of mass m cannot exceed $(\mu\, mg)$.)*

$A = 0.1\,m \quad f = 0.5$



relative to platform, the block must be "still"

$F_{rest} + \mu mg = 0$

When $x = A$, $F_r = -kA$

$-kA + \mu mg = 0$

$\mu mg = kA$

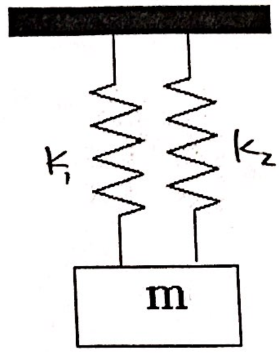
$\mu = \frac{kA}{mg}$ $\quad \frac{k}{m} = \omega^2 = (2\pi f)^2$

$\mu = (2\pi f)^2 \cdot \frac{A}{g}$

$\boxed{\mu = 0.101}$

✓

Problem 6. (a)Two identical springs with a spring constant k=30 N/m are connected as shown in the figure. The horizontal bar is massless. A mass m=1 kg is attached as shown. Find the period of small oscillations.



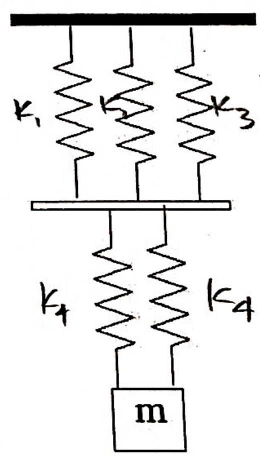
$T = 2\pi\sqrt{m/k_{eq}}$

$k_{eq} = k_1 + k_2 = 60\ N/m$

$T = 2\pi\sqrt{1\,kg / 60\,N/m} = \boxed{0.81s}$

✓

(b) Find the period of small oscillations for five identical springs with k= 30 N/m connected to mass m=1 kg as shown. (The horizontal bar is massless.)



$T = 2\pi\sqrt{m/k_{eq}}$

$$\frac{1}{k_{eq}} = \frac{1}{k_1+k_2+k_3} + \frac{1}{k_4+k_5}$$

$$= \frac{1}{90\,N/m} + \frac{1}{60\,N/m} = \frac{150}{5400\,N/m} = \frac{1}{k_{eq}}$$

$T = 2\pi\sqrt{1\,kg / 36\,N/m} = \boxed{1.05s}$

✓

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