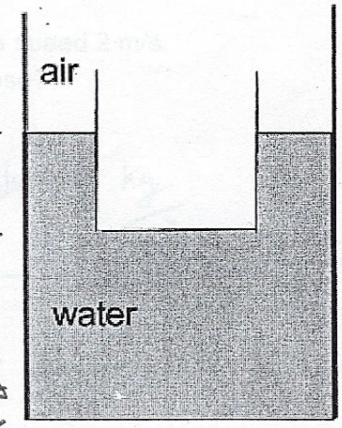
Problem 1. An open metal box with dimensions 1.00 m x 1.00 m x 1.00 m and mass 800 kg floats on the surface of the water ($\rho_w = 1.00\,\mathrm{g/cm^3}$).

$\rho_w = 1000\ \mathrm{kg/m^3}$

h_{above}

h_{below}



(a) What is the height of the box above the water line?

$\uparrow F_B$

$\downarrow mg$

Archimedes said so.

$mg = F_B$ // since it's floating

$800(9.8) = \rho_w V_d g$ // I use 9.8 as g

$V_{displace} = Area * h_{below}$

$= (1\times1) * h_{below}$

$800(9.8) = 1000(1\times1\times h)\,9.8$

$h_{below} = 0.8\,m$

$h_{above} = 1 - 0.8 = 0.2\,m$

(b) What is the total force F of pressure acting on the bottom of the box, including the atmospheric pressure ($p_0 = 1.01 \times 10^5$ Pa) and the contribution from the water?

$h_{below} = 0.8\,m$

$\rho_{fluid} = \rho_w = 1000\ \mathrm{kg/m^3}$

$A_{rea} = 1\,m^2$

$P = \frac{F}{A}$ //def of pressure

$F_{fluid} = P A_{rea}$

$F_{fluid} = (\rho_{fluid} g h_{below}) A$

$= (1000)(9.8)(0.8)(1)$

$= 7840\ N$

$P_{absolute} = P_{atmosphere} + \underbrace{\rho g h_{below}}_{gauge}$ // I use 9.8 as g

$P_{abs} = 1.01\times10^5 + 1000(9.8)(0.8)$

$P = \frac{F}{A}$ // def. of pressure

$F = PA = P_{abs}(1) = P_{abs} = 108840\ N$

But this is kinda dumb cuz it's really just experiencing (gauge pressure * area)

Problem 2. A paint with density 1.2 g/cm^3 comes out of a paint gun with a speed 2 m/s. Neglecting friction and viscosity, what is the gauge pressure inside the hose?

$\rho_{paint} = \frac{1.2\,\cancel{g}}{cm^3} \cdot \frac{1\,kg}{1000\,\cancel{g}} \cdot \frac{1{,}000{,}000\,cm^3}{1\,m^3} = 1{,}200\ \frac{kg}{m^3}$

Bernoulli

$(P_{atm} + P_{gauge}) + \cancel{\rho g h} + \cancel{\tfrac{1}{2}\rho v^2} = \cancel{P_{atm}} + \cancel{\rho g h} + \tfrac{1}{2}\rho v^2$

$P_{gauge} = \frac{1}{2}\rho v^2 = \frac{1}{2}(1{,}200)(2)^2 = 2{,}400$ Pascals

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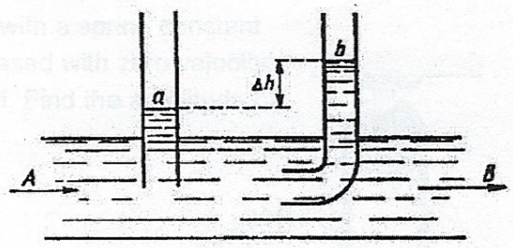
Problem 3. Water flows along a horizontal pipe AB, as shown in the figure. The difference between the levels of the liquid in tubes a and b is $\Delta h = 1$ cm. The diameters of tubes a and b are the same. Water density is $\rho = 1$ g/cm^3 . Determine the velocity of the water flowing along the pipe AB.



$\rho_{water} = \frac{1g}{cm^3} = \frac{1000\,kg}{m^3}$

$\Delta h = 0.01m$

Bernoulli: $\cancel{P} + \rho g h + \frac{1}{2}\rho v^2 = \cancel{P} + \cancel{\rho g h} + \frac{1}{2}\rho v^2$

$\cancel{\rho} g \Delta h = \frac{1}{2}\cancel{\rho} v^2$

$g\Delta h = \frac{1}{2} v^2$

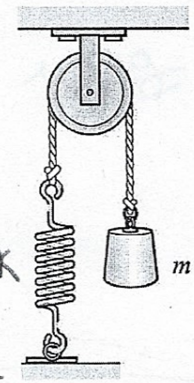
$\sqrt{2g\Delta h} = v$

// I use 9.8 as g

$v = 0.4427$ m/s

✓

Problem 4. A mass m=10 kg is attached to a spring with a spring constant k=300 N/m as shown in the figure. The mass is released with zero velocity from the position in which the spring was unstretched. Find the amplitude of the resulting small oscillations.



gravity

K

— equilibrium

$F_{spring} = -kx$ // Hooke's law

in this case, the force is mg since we are dropping a mass on earth.

that's counteracting spring force

$mg = kx$

$\frac{mg}{k} = x_{max}$ // $x_{max\ displacement} = A_{mplitude}$

$\frac{mg}{k} = A$ // I use 9.8 as g

$A = 0.32666\ m$

in terms of mass frequency

SHM

Problem 5. A horizontal platform vibrates horizontally with an amplitude 10 cm and a frequency $f = 0.5$ Hz. When a small block is placed on top of the platform, the frequency and the amplitude remain the same. What is the minimum value μ that the coefficient of static friction must have for the block to oscillate with the platform without sliding?
(Hint: the force of friction on the block of mass m cannot exceed $(\mu\, mg)$.)

Isolate K:

$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

$2\pi f = \sqrt{\frac{K}{m}}$

$4\pi^2 f^2 = \frac{K}{m}$

$4\pi^2 f^2 m = K$

$A = 0.1\,m$

$f = 0.5\ Hz$

$T = \frac{1}{f} = 2$ sec.

$F_{static} = \mu F_N = \mu mg$

Max force experienced by small block $= Kx = KA$ // Hooke's law.
Because that is the max force that is ever applied to the horizontal platform.

Force friction static must $=$ max Force applied to the platform over the oscillations (Force of spring), which is encountered at the ends of the oscillations, when displacement $=$ Amplitude (max displacement)

$F_{static} = F_{spring}$

$\mu mg = KA$

subs!

$\mu \cancel{m} g = (4\pi^2 f^2 \cancel{m})A$

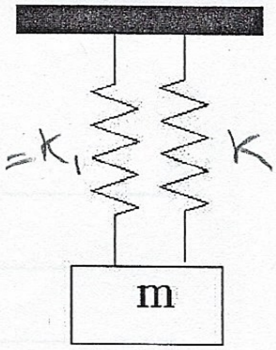
$\mu = \dfrac{4\pi^2 f^2 A}{g}$

// I use 9.8 as g

$\mu = 0.1007$

✓

Problem 6. (a)Two identical springs with a spring constant k=30 N/m are connected as shown in the figure. The horizontal bar is massless. A mass m=1 kg is attached as shown. Find the period of small oscillations.



Let $k_1 = k = 30$ N/m

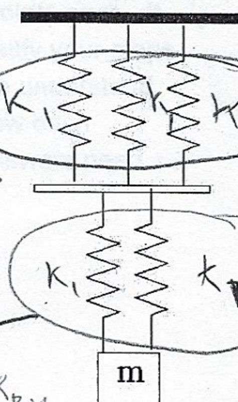
$30 = k_1$ $\quad k_1$

$$\text{Period} = T = 2\pi\sqrt{\frac{m}{K_{Total}}}$$

$$K_{Total} = k_1 + k_1 \quad \text{// since it is parallel}$$
$$= 2k_1$$
$$= 60$$

$$T = 2\pi\sqrt{\frac{1}{60}} = 0.811 \text{ sec}$$

(b) Find the period of small oscillations for five identical springs with k= 30 N/m connected to mass m=1 kg as shown. (The horizontal bar is massless.)



Oh wow, what fun :)

$3k_1 = k_{Top}$ $\quad k_1 \quad k_1 \quad k_1$

$k_1 \quad k_1$ $\quad 2k_1 = k_{Bottom}$

since springs are in parallel

$$K_{\text{Top three springs}} = k_1 + k_1 + k_1 = 3k_1 = 90$$
$$K_{\text{Bottom Two springs}} = k_1 + k_1 = 2k_1 = 60$$

$$K_{\text{All 5 springs}} = \frac{1}{\frac{1}{K_{Top}} + \frac{1}{K_{Bottom}}}$$

// since springs are in series

// Treat K_{Top} & K_{Bottom} as 2 springs in series

$$= \frac{1}{\frac{1}{90} + \frac{1}{60}} = \frac{1}{\frac{2}{180} + \frac{3}{180}} = \frac{1}{\frac{5}{180}} = \frac{180}{5}$$

$$K_{\text{all 5 springs}} = 36$$

subs.

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$T = 2\pi\sqrt{\frac{1}{36}} = 1.047 \text{ seconds}$$

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