Midterm 1

Physics 1B (Lec 5)

Name: Solutions

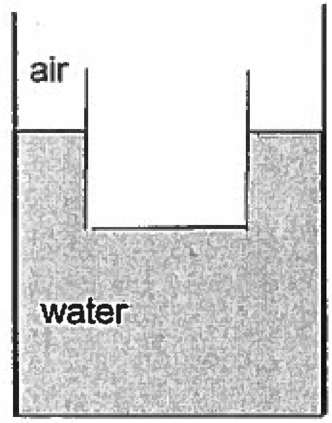
ID number:

Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	6	total

Problem 1. An open metal box with dimensions 1.00 m x 1.00 m x 1.00 m and mass 800 kg floats on the surface of the water ($\rho_w = 1.00\,\text{g/cm}^3$).



(a) What is the height of the box above the water line?

$mg = \rho g h \rightarrow h = \frac{m}{\rho A} = 80\,\text{cm}$

Above water line: $100\,\text{cm} - 80\,\text{cm} = \boxed{20\,\text{cm}}$

(b) What is the total force F of pressure acting on the bottom of the box, including the atmospheric pressure ($p_0 = 1.01 \times 10^5$ Pa) and the contribution from the water?

$mg = \rho g V = 7840\,\text{N}$

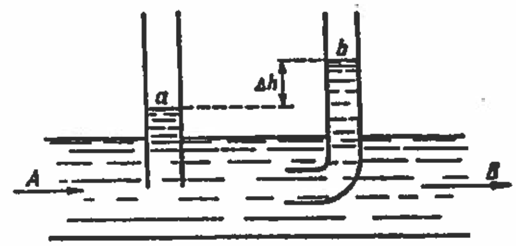
Including $F = P_0 A \rightarrow 1.09 \times 10^5\,\text{N}$

Problem 2. A paint with density 1.2 g/cm³ comes out of a paint gun with a speed 2 m/s. Neglecting friction and viscosity, what is the gauge pressure inside the hose?

$P_{gauge} = \frac{1}{2}\rho v^2 = \frac{1}{2} \times 1.2 \times 10^3 \times 2^2 = 2.4 \times 10^3$ Pa.

If student use significant figure $\Rightarrow 2 \times 10^3$ Pa.

Problem 3. Water flows along a horizontal pipe AB, as shown in the figure. The difference between the levels of the liquid in tubes a and b is $\Delta h = 1$ cm. The diameters of tubes a and b are the same. Water density is $\rho = 1\ g/cm^3$. Determine the velocity of the water flowing along the pipe AB.



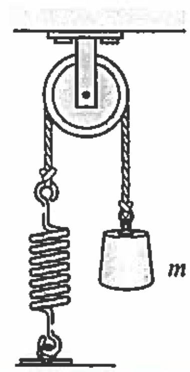
$P_1 = P_0 + \rho g h_1 \qquad P_2 = P_0 + \rho g h_2$

$P_1 + \frac{1}{2}\rho v^2 = P_2$

$\frac{1}{2}\rho V^2 = P_2 - P_1 = \rho g \Delta h$

$V = \sqrt{2g\Delta h} \simeq 0.44\ m/s$

Problem 4. A mass m=10 kg is attached to a spring with a spring constant k=300 N/m as shown in the figure. The mass is released with zero velocity from the position in which the spring was unstretched. Find the amplitude of the resulting small oscillations.

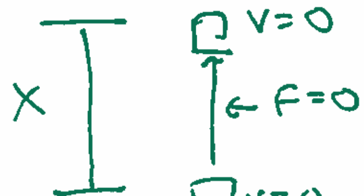


Two methods:

1. Energy $\frac{1}{2}kx^2 = mgx$

$\Rightarrow x = \frac{2mg}{k}$.

But this is not the amplitude!



We set v = 0 at both points $\Rightarrow$ these are the endpoints of the osc. The amplitude is the distance between equilibrium and the endpoints, half the end-to-end distance.

$A = \frac{1}{2}x = \frac{mg}{k}$

2. Force.

$mg = kx \Rightarrow x = \frac{mg}{k}$. This is correct because it's the distance from the endpoint ○ to the equilibrium point.

$A = 32.7\text{ cm} = .327\text{ m}$

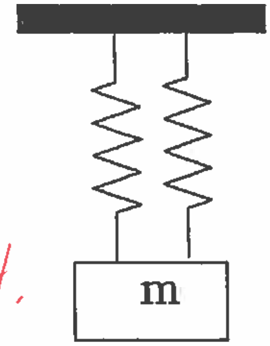
Problem 5. A horizontal platform vibrates horizontally with an amplitude 10 cm and a frequency $f = 0.5$ Hz. When a small block is placed on top of the platform, the frequency and the amplitude remain the same. What is the minimum value μ that the coefficient of static friction must have for the block to oscillate with the platform without sliding?
(Hint: the force of friction on the block of mass m cannot exceed (μmg).)

$$m a_{max} \leq \mu m g$$

$$a_{max} = \omega^2 A = (2\pi \times 0.5)^2 (0.1)$$

$$\Rightarrow \mu \geq \frac{a_{max}}{g} \simeq 0.1$$

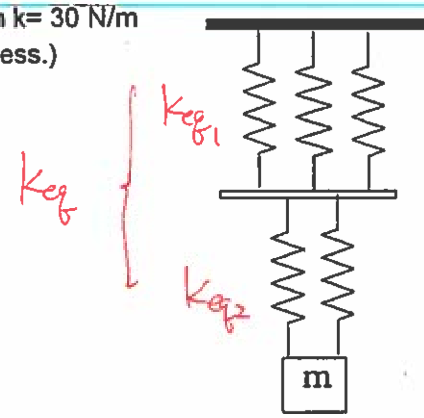
Problem 6. (a)Two identical springs with a spring constant k=30 N/m are connected as shown in the figure. The horizontal bar is massless.
A mass m=1 kg is attached as shown. Find the period of small oscillations.



$$K_{eq} = 60\ N/m.$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{60}} = \frac{\pi}{\sqrt{15}} = 0.81\ sec. \quad \#.$$

(b) Find the period of small oscillations for five identical springs with k= 30 N/m connected to mass m=1 kg as shown. (The horizontal bar is massless.)



$$K_{eq_1} = 3\times 30 = 90\ N/m$$

$$K_{eq_2} = 2\times 30 = 60\ N/m$$

$$\frac{1}{K_{eq}} = \frac{1}{K_{eq_1}} + \frac{1}{K_{eq_2}}$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{m\cdot\left(\frac{1}{K_{eq_1}} + \frac{1}{K_{eq_2}}\right)} = 2\pi\sqrt{m\cdot\frac{K_{eq_1}+K_{eq_2}}{K_{eq_1}\cdot K_{eq_2}}}$$

$$= 2\pi\sqrt{\frac{90+60}{90\times 60}} = 2\pi\sqrt{\frac{\cancel{150}^{\cancel{3}\ 1}}{\cancel{5400}_{\cancel{108}\ 36}}} = \frac{\pi}{3} = 1.05\ sec \quad \#$$