Midterm 1

Physics 1B (Lec 5)

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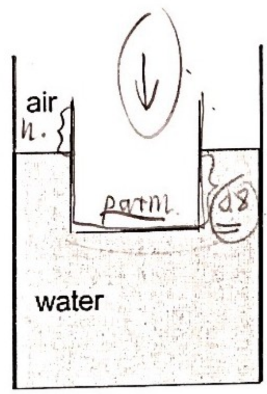
Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	6	total
20	20	20	20	20	20	120

Problem 1. An open metal box with dimensions 1.00 m x 1.00 m x 1.00 m and mass 800 kg floats on the surface of the water ($\rho_w = 1.00\,\text{g/cm}^3$).

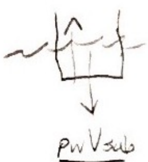


(a) What is the height of the box above the water line?

$$\Rightarrow \quad \rho_w V_{sub} = \rho_w A h_{below} = m_{block}.$$

$$h_{below} = \frac{m_{block}}{\rho_w A}$$

IF floating: $m_{block} \cancel{g} = \rho_w V_{sub} \cancel{g}$



$$\Rightarrow \frac{800\,\text{kg}}{(1000\,\frac{\text{kg}}{\text{m}^3})(1.00\,\text{m} \cdot 1.00\,\text{m})}$$

$$= 0.8\,\text{m}$$

$$(0.8)(1000) = \underline{800\ \text{kg}}$$

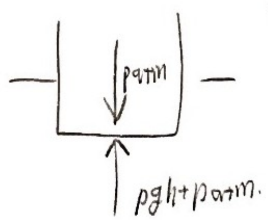
$$h_{above} = 1.00 - 0.8 = \boxed{0.20\,\text{m}}$$

DC: $\underline{0.80}$

(b) What is the total force F of pressure acting on the bottom of the box, including the atmospheric pressure ($p_0 = 1.01 \times 10^5$ Pa) and the contribution from the water?

$$\Rightarrow \quad F_{pressure\,bottom} = \underbrace{(p_{atm} + \rho_w g (0.8)) A_{side}}_{P_{total} A_{outside}} - \underbrace{(p_{atm}) A_{side}}_{P_{total} A_{inside}}$$

$$\Rightarrow (\cancel{1.01 \times 10^5} + (1000)(9.8)(0.8))(1.00)^2 - (\cancel{1.01 \times 10^5})(1.00)^2$$



$$\approx \boxed{7840\,\text{N}}$$

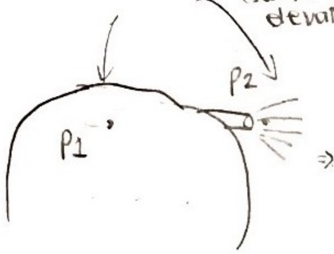
DC: $7840 = m_{block}\, g$, which allows it to float in equilibrium.

Problem 2. A paint with density 1.2 g/cm³ comes out of a paint gun with a speed 2 m/s. Neglecting friction and viscosity, what is the gauge pressure inside the hose?

Pick pts:

(same elevation)

$\rho_{paint} = 1.2 g/cm^3 = 1200 kg/m^3$



$A_1 v_1 = A_2 v_2 \quad v_1 = \frac{A_2}{A_1} v_2$

- Since $A_2 \ll A_1$

$v_1 \approx 0.$

$\Rightarrow \quad P_{hose} + \cancel{\frac{1}{2}\rho v_1^2} = P_{atm} + \frac{1}{2}\rho v_2^2$

$P_{hose} - P_{atm} =$

$\Rightarrow. \quad P_{gauge} = P_{hose} - P_{atm} = \frac{1}{2}\rho v_2^2$

ρv

$\frac{1}{2}(1200)(2)^2$

$\approx \boxed{2400 \text{ Pa}}$

20/20

$\Rightarrow$

$\Rightarrow.$

$\frac{1.2 g}{cm^3}$

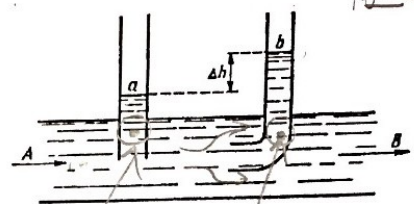
$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$p_{atm} + h_a = p_1 + \frac{1}{2}\rho v_1^2$

$p_{atm} + \ = p_1 +$

$\rho g \Delta h =$

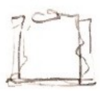
Problem 3. Water flows along a horizontal pipe AB, as shown in the figure. The difference between the levels of the liquid in tubes a and b is $\Delta h = 1$ cm. The diameters of tubes a and b are the same. Water density is $\rho = 1\ \mathrm{g/cm^3}$. Determine the velocity of the water flowing along the pipe AB.



⇒.

let p_1 be

p_2 be



$p_{atm} + \rho g h_1 = p_1 \rightarrow p_2 - p_1 = \rho g(h_2 - h_1) = \rho g(\Delta h)$

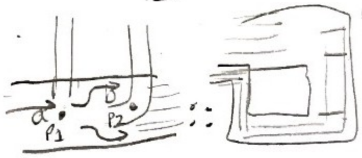
$p_{atm} + \rho g h_2 = p_2$

Pick pts

since $h_2 > h_1$

$p_2 > p_1$

and $v_1 \gg v_2$.



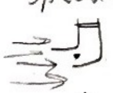
$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \cancel{\frac{1}{2}\rho v_2^2}$

$\frac{1}{2}\rho v_1^2 = p_2 - p_1 = \rho g \Delta h.$

let be

the

assume a pitot tube like setup where the speed of fluid around the



neck of tube b is actually $\underline{0}$

⇒ (The tube curves into the pipe & the level of water in it is stable so there is technically no "flow" here).

⇒ $\frac{1}{2}\rho v_1^2 = \rho g \Delta h$

$v_1 = \sqrt{2g\Delta h}$

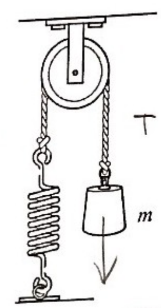
⇒ $\sqrt{2(9.8)(0.01)}$

$= \boxed{0.4427\ \mathrm{m/s}}$

✓

0.4

Problem 4. A mass m=10 kg is attached to a spring with a spring constant k=300 N/m as shown in the figure. The mass is released with zero velocity from the position in which the spring was unstretched. Find the amplitude of the resulting small oscillations.



assume massless string uniform tension so that $T = kx$

$mg - kx$

$mg - kx$

will spring compress? No! otherwise string goes slack!

Flip the system:

spring

$\frac{mg}{k}$

mg

$mg - kx$

when this system is @ equilibrium:
$mg = kx$

$x_{equil.} = \frac{mg}{k}$

system is released @ unstretched length; $\frac{mg}{k}$ above equilibrium. so $x_{init} = \frac{mg}{k}$
not moving

$\cancel{\frac{1}{2}mv^2} + \frac{1}{2}k x_{init}^2 = \frac{1}{2}kA^2$

$\therefore x_{init} = A$ where $x_{init} = \frac{mg}{k}$

Apply conservation of energy to this vertical spring system w/ respect to new equilibrium

the amplitude is

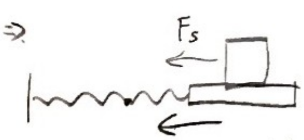
so

$\boxed{0.327m} \Leftarrow \frac{mg}{k}$

$\frac{10(9.8)}{300}$

Problem 5. A horizontal platform vibrates horizontally with an amplitude 10 cm and a frequency $f = 0.5$ Hz. When a small block is placed on top of the platform, the frequency and the amplitude remain the same. What is the minimum value μ that the coefficient of static friction must have for the block to oscillate with the platform without sliding?
(Hint: the force of friction on the block of mass m cannot exceed (μmg).)

$\sqrt{\frac{k}{m}} = \omega \quad \omega^2 = \frac{k}{m}$



$A = 0.1$ cm
$f = 0.5$ Hz

$\omega = 2\pi f$

the block is on the verge of sliding

when $\mu_s mg = F_{max}$ from spring

$\mu_s mg = kA$

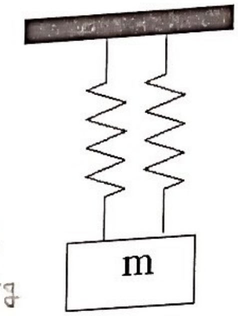
$\mu_s \geq 0.10071$

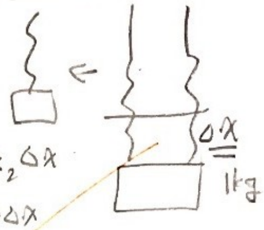
amp. =

$\mu_s = \frac{kA}{mg} \approx \frac{\omega^2 A}{g} = \frac{(2\pi f)^2 A}{g} \simeq \boxed{0.10071}$

$= \frac{(2\pi(0.5))^2(0.10)}{9.8} \simeq$

Problem 6. (a)Two identical springs with a spring constant k=30 N/m are connected as shown in the figure. The horizontal bar is massless. A mass m=1 kg is attached as shown. Find the period of small oscillations.





$\Rightarrow$ $k_{eff} = 2(30) = 60$ · because

$F = k_1\Delta x + k_2\Delta x$

$= (k_1+k_2)\Delta x$

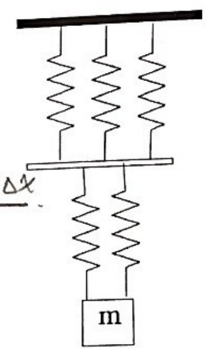
$\Rightarrow (2k)\Delta x$

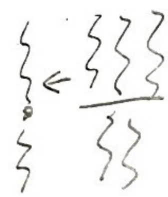
$\Rightarrow$

$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$

$= 2\pi\sqrt{\frac{1}{60}} \approx \boxed{0.811\ s}$

(b) Find the period of small oscillations for five identical springs with k= 30 N/m connected to mass m=1 kg as shown. (The horizontal bar is massless.)





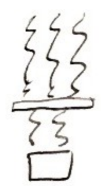
k_{eff} for top 3: $k+k+k = 3k$

$\Rightarrow F = k\Delta x + k\Delta x + k\Delta x = \underline{3k\Delta x}$

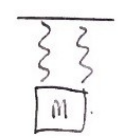
k_{eff} for bottom 2: $k+k = 2k$

$F = k\Delta x + k\Delta x = 2k(\Delta x)$

$\Rightarrow$



k_{eff} For whole system:



$\frac{1}{k_{eff}} = \frac{1}{3k} + \frac{1}{2k}$

$\frac{5k}{6k^2} \Rightarrow \frac{5}{6k}$

$k_{eff} = \underline{\underline{\frac{6}{5}k}}$

$F = 3k\Delta x_1 = 2k\Delta x_2$ so $\Delta x_1 = \frac{2k}{3k}\Delta x_2$

$\frac{2}{3}\Delta x_2$

$F = k_{eff}(\Delta x_1 + \Delta x_2)$

$k_{eff}(\frac{5}{3}\Delta x_2)$

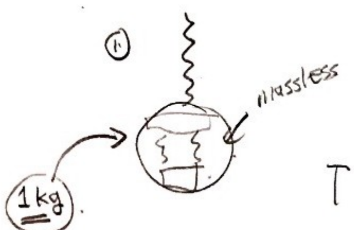
$= 2k\Delta x_2$

$\Rightarrow k_{eff} = \underline{\underline{\frac{6}{5}k}}$

↓

simplify as

①



② simplify as

$T = 2\pi\sqrt{\frac{m}{\frac{6}{5}k}} \Rightarrow 2\pi\sqrt{\frac{5m}{6k}} \approx \boxed{1.047\ s}$

$= \frac{m}{\frac{6k}{5}}$ $\underline{\underline{\frac{5m}{6k}}}$

20/20