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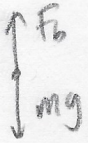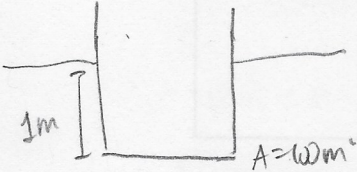**Problem 1.**

A rectangular flat-bottom barge with a bottom area $A = 100\,\text{m}^2$ is loaded so that the bottom is at $H = 1\text{m}$ below the surface. The density of water is $\rho = 10^3 \text{kg/m}^3$, and the water surface is perfectly still.

10 (a) Calculate the mass of the barge.

$\rho = 10^3 kg/m^3$

$\sum F_y = 0 = F_b - mg = \rho_{H2O}\, g V - mg$ ✓

$\rho g V - mg = 0 \Rightarrow mg = \rho g V \Rightarrow m = \rho V$

$m_{barge} = \rho_{water}\, V_{displaced} = 10^3 kg/m^3 \times (100 m^2 \times 1m)$

$= 1.0 \times 10^5 kg$ ✓

8 (b) A round hole with radius $r = 2$ cm is made in the bottom of the barge, and the water starts leaking in. When the water level reaches $h = 5$ cm, a bilge alarm will alert the barge operator. How long will it take for the water to reach the level 5 cm? (Assume that the Bernoulli's equation is applicable.) →
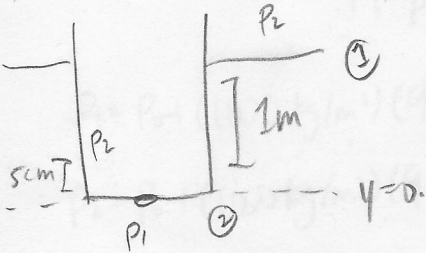
$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 \overset{0}{=} P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$P_1 = P_2 = P_{atm}$

$\cancel{P_1} + \cancel{P_2} + \rho g h = P_2 + (10^3 kg/m^3)(9.8 m/s^2)(1m) = P_2 + 9800.$

$(P_2 + 9800) + \rho g y_1 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$\rho g y_1 - \rho g y_2 + 9800 = \frac{1}{2}\rho v_2^2$

$v_2 = \sqrt{\dfrac{2\rho g(y_1 - y_2) + 2(9800)}{\rho}} = \sqrt{2g(y_1 - y_2) + \dfrac{2}{\rho}(9800)}$

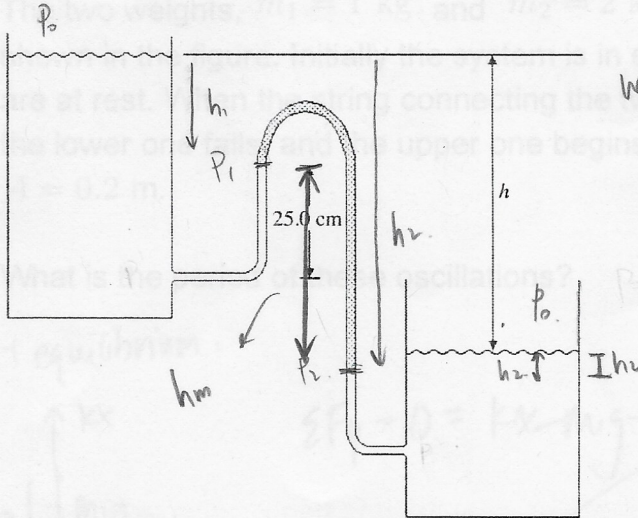$= \sqrt{19.6 + 19.6} \Rightarrow v = 6.26\, m/s.$

$\dfrac{dV}{dt} = A \cdot v \Rightarrow dV = A \cdot v\, dt. = A \cdot \dfrac{dh}{dt} = A \cdot H$

$= (100 m^2)(0.05 m) = 5.$

$5 = A \cdot v\, dt.$

$dt = \dfrac{5}{100 m^2 (6.26 m/s)} =$ 0.008 s.

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Problem 2

The two water reservoirs shown in the figure are open to the atmosphere, and the water has density 1000 kg/m³. The manometer contains incompressible oil with a density of 820 kg/m³. What is the difference in elevation h if the manometer reading m is 25.0 cm?

water $\rho = 1000 \, kg/m^3$

oil $\rho = 820 \, kg/m^3$

$$P_1 = P_2 + \rho g h \implies P_1 = P_2 + 820 \, kg/m^3 (9.8 \, m/s)(0.25m)$$

$$P_1 = P_2 + 2009.$$

$$P_1 = P_0 + \rho g h_1 \qquad\qquad P_2 = P_0 + \rho g h_2$$

$$P_0 = P_1 - \rho g h_1 \qquad\qquad P_0 = P_2 - \rho g h_2$$

$$P_1 - \rho g h_1 = P_2 - \rho g h_2 \implies P_1 - P_2 = \rho g h_1 - \rho g h_2 = \rho g (h_1 - h_2).$$

$$= 2009.$$

$$P_1 = P_0 + (1000 \, kg/m^3)(9.8 \, m/s^2)(h_1).$$

$$P_2 = P_0 + (1000 \, kg/m^3)(9.8 \, m/s^2)(h_2)$$

$$P_1 - \rho g h_1 = P_2 - \rho g h_2.$$

$$P_1 - P_2 = \rho_w g (h_1 - h_2).$$

$$h_1 - h_2 = \frac{P_1 - P_2}{\rho_w g} = \frac{2009}{(1000 \, kg/m^3)(9.8)} = 0.205m.$$

Height $h = h_1 + h_m - h_2$

$(h_1 - h_2) + h_m$

$\implies 0.205m + 0.25m$

$h = 0.455 m$ ✗

4.5 cm

Problem 3
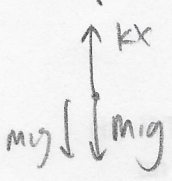
The two weights, $m_1 = 1 \text{ kg}$ and $m_2 = 2 \text{ kg}$ hang on the spring as shown in the figure. Initially the system is in equilibrium, and the weights are at rest. When the string connecting the two weights is cut at point X, the lower one falls, and the upper one begins to oscillate with an amplitude $A = 0.2$ m.
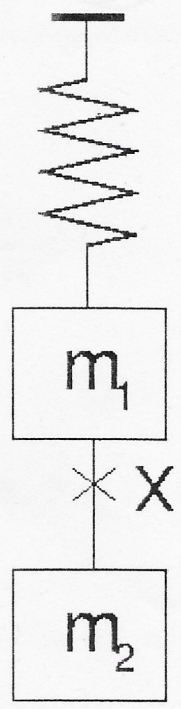
What is the period of these oscillations?

At equilibrium:

↑ kx

$m_1 g \downarrow \quad \downarrow m_2 g$

$\Sigma F_y = 0 = kx - m_1 g - m_2 g$

~~$\Rightarrow k = \dfrac{m_1 g + m_2 g}{x}$~~

when m_2 is cut:

$$T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow 2\pi \sqrt{\frac{m_1}{}}$$

$$= 2\pi \sqrt{\frac{m_1}{\frac{g(m_1 + m_2)}{x}}} = 2\pi \sqrt{\frac{(1 \text{ kg})}{\frac{(9.8 \text{ m/s}^2)(1+2\text{kg})}{0.2\text{m}}}} = 0.518 \text{ s.} = 0.5 \text{ s.}$$

✓

$120 \text{ cm} = \ell$
$= 1.2 \text{ m}.$

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Problem 4

A simple pendulum has a length of 120 cm.

(a) What is its period of oscillations?

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{1.2 \text{ m}}{9.8 \text{ m/s}^2}} = 2.20 \text{ s}. \checkmark$$

(b) What is the period of oscillations inside an elevator moving up with an acceleration 1.2 m/s²

$$\text{Total acc.} = g - a = 9.8 - 1.2 \text{ m/s}^2 = 8.6 \text{ m/s}^2$$

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{1.2 \text{ m}}{8.6 \text{ m/s}^2}} = 2.35 \text{ s.}$$

(c) What is the period of the same pendulum on Mars, where the acceleration of gravity is about 0.37 that on Earth?

$$T = 2\pi \sqrt{\frac{L}{g_\circ}} \quad , g_\circ = 0.37 g.$$

$$= 2\pi \sqrt{\frac{1.2 \text{ m}}{(9.8 \text{ m/s}^2)(0.37)}} = 3.6 \text{ s} \checkmark$$

Problem 5

Two violinists are trying to tune their instruments in an orchestra. One is producing the desired frequency of 440.0 Hz. The other is producing a frequency of 448.4 Hz. By what percentage should the out-of-tune musician change the tension in his string to bring his instrument into tune at 440.0 Hz?

$f_1 = 440 \text{ Hz.}$ $f_2 = 448.4 \text{ Hz.}$

string fixed at both ends:

$$f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

same $L, \mu \Rightarrow \dfrac{f_a}{f_b} = \sqrt{\dfrac{F_a}{F_b}}$

$$\frac{440}{448.4} = \sqrt{\frac{F}{F_1}}$$

$$(2Lf)^2\mu = F = 4L^2f^2\mu.$$

$f_1:$ $F = (2L \cdot 440)^2 \mu.$ $f_2:$ $F' = (2 \cdot L \cdot 448.4)^2 \mu.$

$\quad\quad = 774400 L^2\mu$ $\quad\quad = 804250.24 L^2\mu.$

$(F'-F) \times 100 = 96.29\%$ $100 - 96.29\% = 3.71\%$

(decrease)

The out-of-tune musician should change the tension in his string by about 3.71%.

Problem 6

Standing waves of frequency 50 Hz are produced on a string that has mass per unit length 0.025 kg/m. With what <u>tension</u> must the string be stretched between two supports if adjacent nodes in the standing wave are to be 0.8 m apart?

$$f = 50 \text{ Hz} \qquad \mu = 0.025 \text{ kg} \frac{m}{m}. \qquad f = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{v}{2L}$$

Adjacent nodes $\frac{\lambda}{2} = 0.8 \text{ m}. \Rightarrow \lambda = 1.6 \text{ m}$

$$F = (2Lf)^2 \mu.$$

$$v = \lambda f = (1.6 \text{ m})(50 \text{ Hz})$$
$$= 80 \text{ m/s}.$$

$$\longrightarrow \lambda_n = \frac{2L}{n}. \quad L = n\frac{\lambda}{2}. \quad f = \frac{v}{2L}$$
$$n = \frac{2L}{\lambda}. \quad 2L = n\lambda.$$

$$f = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{v}{2L} \Rightarrow v = \sqrt{\frac{F}{\mu}}. \qquad f = \frac{v}{\lambda}.$$
$$\downarrow$$
$$v = \lambda f$$

$$F = v^2\mu = (80 \text{ m/s})^2 (0.025 \text{ kg/m})$$
$$= 160 \text{ N}.$$

✓

1	18
2	18
3	15
4	17
5	20
6	20
TOT	108