Midterm exam 1

Physics 1B, Spring 2016

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Lectu[redacted]g time, or TA name): : Justin Little Wed 3-4

Please write solutions with some minimal derivation in the space provided below each problem; it is not sufficient to give just the final answer. The level of detail should be such that a grader, or your fellow classmate would understand how you solved the problem.

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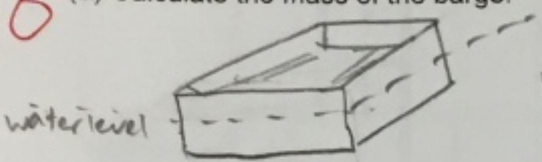
Problem 1.

A rectangular flat-bottom barge with a bottom area $A = 100\,\mathrm{m}^2$ is loaded so that the bottom is at $H = 1\mathrm{m}$ below the surface. The density of water is $\rho = 10^3\mathrm{kg/m}^3$, and the water surface is perfectly still.

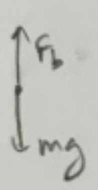
10 (a) Calculate the mass of the barge.



$A = 100\,m^2$

$H = 1\,m$

volume $= 100\,m^3$



$F_b - mg = 0$ ✓

$mg = F_b$

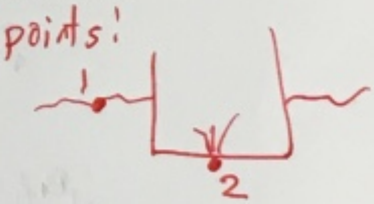
$F_b =$ weight displaced fluid $= (100\,m^3)\left(\frac{10^3\,kg}{m^3}\right)g = 10^5 g\,kg$

$mg = 10^5 g\,kg$

$m = 10^5\,kg$ ✓

5 (b) A round hole with radius $r = 2$ cm is made in the bottom of the barge, and the water starts leaking in. When the water level reaches $h = 5$ cm, a bilge alarm will alert the barge operator. How long will it take for the water to reach the level 5 cm? (Assume that the Bernoulli's equation is applicable.)

Choose these points!



$p + \rho g y + \frac{1}{2}\rho v^2 =$ constant

pressure $= \frac{\text{Force}}{\text{area}}$ Force buoyancy $= 10^5 g\,N \rightarrow \frac{10^5 g N}{100\,m^2} = 10^3 g\,Pa$

Apply Bernoulli's equation to the top and bottom of the 5cm layer of water (at hole & at surface) ✗

$10^3 g Pa + \rho g(.05) + \frac{1}{2}\rho v^2 = 0 + \rho g(\ 0\ ) + \frac{1}{2}\rho(0)$

open to air

take surface to be $y=0$

since area is so large, this becomes negligible

$\frac{1}{2}\rho v^2 = -98{,}490$

$|\rho v^2| = 196980$

$v^2 = 196.98$

$v = 14.03\,m/s$ (at bottom)

$\frac{dV}{dt} = \pi(.02\,m)^2 \cdot 14.03\,m/s \cdot 10^{-3}\,\frac{L}{m^3}$

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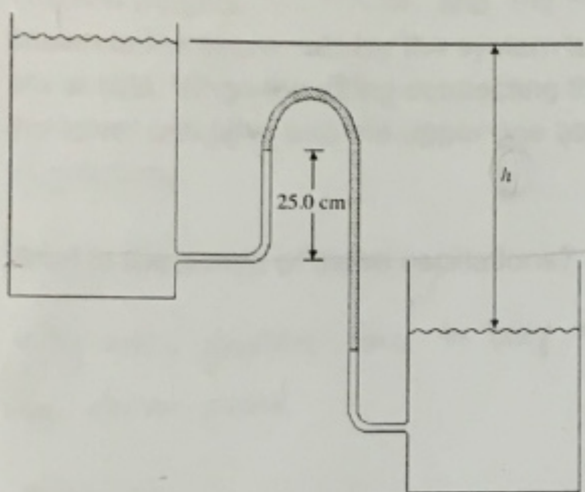
Problem 2

The two water reservoirs shown in the figure are open to the atmosphere, and the water has density 1000 kg/m³. The manometer contains incompressible oil with a density of 820 kg/m³. What is the difference in elevation h if the manometer reading m is 25.0 cm?



pressure on both surfaces is equal ✓

$p = p_0 + \rho g h$

$p = p_0 + \rho_{H_2O} g (25cm)$ oil pushed up 25 cm

$(-)\ p = p_0 + \rho_{oil}\ g(h)$

$\rho_w g h + \rho_o g m = \rho_w g m$

$0 = \rho_{H_2O} g (25cm) - \rho_{oil}\ g(h)$

$\rho_{oil}\ g(h) = \rho_{H_2O} g (25cm)$

$h = \frac{\rho_{H_2O}}{\rho_{oil}}(25cm) = \frac{1000 \frac{kg}{m^3}}{820 \frac{kg}{m^3}}(25cm) = 30.49\ cm$ ✗

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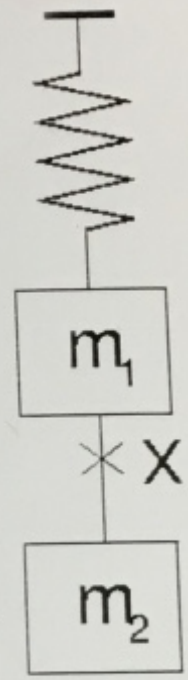
Problem 3

The two weights, $m_1 = 1$ kg and $m_2 = 2$ kg hang on the spring as shown in the figure. Initially the system is in equilibrium, and the weights are at rest. When the string connecting the two weights is cut at point X, the lower one falls, and the upper one begins to oscillate with an amplitude $A = 0.2$ m.



1kg

2kg

What is the period of these oscillations?

2 kg mass provides force to keep m_1 at .2 m away from equilibrium point

Hooke's Law

$F_x = -kx$

$2g = -k(.2)m \Rightarrow k_{effective} = 10g \frac{kg}{s^2}$

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1kg}{10g\frac{kg}{s^2}}} = 2\pi\sqrt{.01s^2} = 2\pi(.101)s \approx .63$ sec

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Problem 4

A simple pendulum has a length of 120 cm.

(a) What is its period of oscillations?

$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.2m}{9.8m/s^2}} = 2\pi(.35s) = .7\pi \text{ sec} \approx 2.2 \text{ sec}$

(b) What is the period of oscillations inside an elevator moving up with an acceleration 1.2 m/s^2

elevator moves up w/ $a = 1.2m/s^2$

w/in elevator, position stays same, so

apparent gravity $= 1.2m/s^2 + g = 11m/s^2$

$T = 2\pi\sqrt{\frac{1.2m}{11m/s^2}} = 2\pi(.33 \text{sec}) \approx 2.08 \text{ sec}$

(c) What is the period of the same pendulum on Mars, where the acceleration of gravity is about 0.37 that on Earth?

$T = 2\pi\sqrt{\frac{1.2m}{(9.8m/s^2)(.37)}} = 2\pi(.58 \text{sec}) \approx 3.615 \text{ sec}$

if in elevator, $T = 2\pi\sqrt{\frac{1.2m}{4.789m/s^2}} = 2\pi(.5s) \approx 3.15 \text{sec}$

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Problem 5

Two violinists are trying to tune their instruments in an orchestra. One is producing the desired frequency of 440.0 Hz. The other is producing a frequency of 448.4 Hz. By what percentage should the out-of-tune musician change the tension in his string to bring his instrument into tune at 440.0 Hz?

$$v = \sqrt{\frac{F}{\mu}} \leftarrow \text{tension} = \lambda f$$

$$f = \frac{1}{\lambda}\sqrt{\frac{F}{\mu}}$$

$$\frac{F}{\mu} = (\lambda f)^2$$

$$F_1 = \frac{(\lambda f)^2}{\mu} = \frac{(\lambda \cdot 440)^2}{\mu}$$

$$F_2 = \frac{(\lambda f)^2}{\mu} = \frac{(\lambda \cdot 448.4)^2}{\mu}$$

$$\frac{F_2}{F_1} = \frac{\cancel{\lambda^2}(448.4\,Hz)^2}{\cancel{\mu}} \cdot \frac{\cancel{\mu}}{\cancel{\lambda^2}(440\,Hz)^2} = \left(\frac{448.4\,Hz}{440\,Hz}\right)^2 = 1.039$$

Out-of-tune musician has a tension that is 3.9% higher than that of the in-tune musician

$$\frac{F_1}{F_2} = \left(\frac{440\,Hz}{448.4\,Hz}\right)^2 = .963$$

$1 - .963 = .037 \rightarrow$ reduce tension by 3.7% to match frequencies ✓

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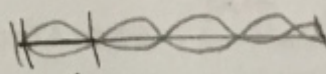
Problem 6

Standing waves of frequency 50 Hz are produced on a string that has mass per unit length 0.025 kg/m. With what tension must the string be stretched between two supports if adjacent nodes in the standing wave are to be 0.8 m apart?

$f = 50\ Hz$ Find F_T.

$\mu = .025\ \frac{kg}{m}$

$\lambda = 1.6m$



distance between two nodes $= \frac{1}{2}\lambda$

$\lambda = 2(.8m) = 1.6\ m$

const. velocity $= \sqrt{\frac{F}{\mu}} = \lambda f$

$\sqrt{\frac{F_T}{\mu}} = \lambda f$

$\frac{F_T}{\mu} = (\lambda f)^2$

$F_T = \mu(\lambda f)^2$

$= (.025\ \frac{kg}{m})(1.6m \cdot 50Hz)^2$

$= (.025\ \frac{kg}{m})(6400\ \frac{m^2}{s^2})$

$= \underline{160\ N}$ ✓

unit check

$\frac{kg}{m}(m \cdot Hz)^2 = \frac{kg}{m}\left(\frac{m^2}{s^2}\right) = \frac{kg}{s^2} = N$

1	15
2	11
3	20
4	20
5	20
6	20
TOT	106