Midterm 1, Phys. 1A, Winter 2015

40 pts tot. (each problem 10 points).
(Show your work ! No credit for just writing down a result. Write neatly !)

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1. 10

2. 10

3. 10

4. 10

Total: 40

1. The vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ have components: $\mathbf{V}_1 = (2, 1)$; $\mathbf{V}_2 = (1, 3)$.
Find the components of the vector $\mathbf{U} = \mathbf{V}_1 + \mathbf{V}_2$, the modulus $|\mathbf{U}|$, and the angle φ that $\mathbf{U}$ makes with the x-axis.
(You may leave numerical values of trigonometric functions indicated if you do not have a calculator).

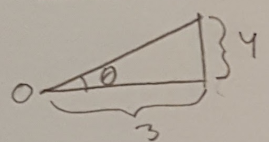
$\vec{U} = \vec{V}_1 + \vec{V}_2 = \langle 2, 1\rangle + \langle 1, 3\rangle = \langle 2+1, 1+3\rangle = \langle 3, 4\rangle$

$\therefore$ $\vec{U} = \langle 3, 4\rangle$ x-component=3 y-component=4

$|\vec{U}| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$|\vec{U}| = 5$

✓



$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

2. The acceleration of a particle has components: $\boldsymbol{a}(t) = (\alpha\, t\, , \beta)$. The particle starts from rest at the origin at $t = 0$.
Find the position of the particle at time $t = \beta / \alpha$.

$\vec{a}(t) = \langle \alpha t, \beta\rangle$

$\vec{v}(t) = \langle \frac{\alpha t^2}{2} + V_{ox}, \beta t + V_{oy}\rangle$

$V_{ox} = 0 \quad V_{oy} = 0 \therefore \vec{v}(t) = \langle \frac{\alpha t^2}{2}, \beta t\rangle$

$\vec{r}(t) = \langle \frac{\alpha t^3}{6} + r_{ox}, \frac{\beta t^2}{2} + r_{oy}\rangle$

$r_{ox} = 0 \quad r_{oy} = 0 \therefore \vec{r}(t) = \langle \frac{\alpha t^3}{6}, \frac{\beta t^2}{2}\rangle$

$t = \frac{\beta}{\alpha}$

$r\left(\frac{\beta}{\alpha}\right) = \left(\frac{\alpha\left(\frac{\beta}{\alpha}\right)^3}{6}, \frac{\beta\left(\frac{\beta}{\alpha}\right)^2}{2}\right) = \left(\frac{\alpha(\beta^3)}{6(\alpha^3)}, \frac{\beta^3}{2\alpha^2}\right)$

$r\left(\frac{\beta}{\alpha}\right) = \left(\frac{\beta^3}{6(\alpha^2)}, \frac{\beta^3}{2(\alpha^2)}\right)$ ✓

3. A cannon ball is fired from ground level with initial velocity V_0 (at some angle $0 < \varphi < \pi/2$ above the horizontal) and strikes the ground at a distance L from the starting point. Where does the ball strike the ground if it is fired (at the same angle) with velocity $2V_0$?

For V_0

$\vec{v}(t_1) = (V_0\cos\theta,\ V_0\sin\theta - gt_1)$

$\vec{r}(t_1) = (V_0\cos\theta t_1,\ V_0\sin\theta t_1 - \frac{gt_1^2}{2})$

$V_0\sin\theta t_1 - \frac{gt_1^2}{2} = 0 \Rightarrow 2V_0\sin\theta t_1 = gt_1^2$

$t_1 = \frac{2V_0\sin\theta}{g}$ = time to hit the ground

$X_1 = V_0\cos\theta\left(\frac{2V_0\sin\theta}{g}\right) = \frac{2V_0^2\cos\theta\sin\theta}{g} = L$

For $2V_0$: $Y_2 = 2V_0\sin\theta t_2 - \frac{gt_2^2}{2}$

$2V_0\sin\theta t_2 = \frac{gt_2^2}{2}$

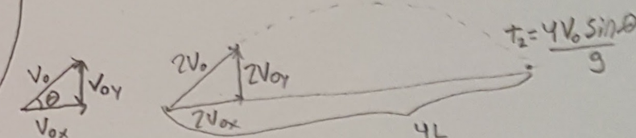
$t_2 = \frac{4V_0\sin\theta}{g}$ = time to hit the ground

$X_2 = 2V_0\cos\theta t_2 = 2V_0\cos\theta\left(\frac{4V_0\sin\theta}{g}\right)$

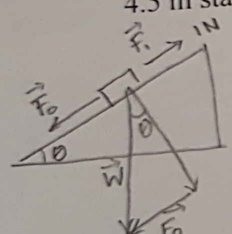
$X_2 = \frac{8V_0^2\cos\theta\sin\theta}{g} = 4\left(\frac{2V_0^2\cos\theta\sin\theta}{g}\right)$

$\therefore X_2 = 4L$ ✓

The ball strikes the ground at a distance 4L from the starting point.



4. A block of mass $m = 1$ kg slides down an incline (making an angle $\varphi = 30°$ with the horizontal); applied to the block there is a constant force $F_1 = 1$ N in the direction opposite to the motion (in addition to the force of gravity and the force exerted by the incline, which has no friction).
Calculate the magnitude of the velocity of the block after it slides down a distance L = 4.5 m starting from rest.



$\sin\theta = \frac{\vec{F_0}}{\vec{W}}$

$\vec{F_0} = \vec{W}\sin\theta = mg\sin(30°)$

$\vec{F}_{tot} = \vec{F_0} - \vec{F_1} = mg\sin(30°) - 1\ kg(m/s^2)$

$\vec{F}_{tot} = ma = (1\ kg)a \quad \therefore \quad a = \frac{(1\ kg)\ g\ \sin(30°) - 1\ kg(m/s^2)}{1\ kg} = g\sin(30°) - 1\ m/s^2$

$\vec{V_f}^2 = 2aL = 2(g\sin(30°) - 1\ m/s^2)(4.5\ m)$

$\vec{V_f} = \sqrt{2(g\sin(30°) - 1\ m/s^2)(4.5\ m)} = \sqrt{2((9.8\ m/s^2)\sin(30°) - 1\ m/s^2)(4.5\ m)}$

$\vec{V_f} = \sqrt{35.1}\ m/s \approx 5.92\ m/s$ ✓