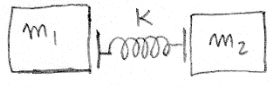
Final Exam, Phys. 1A, Spring 2020

60 pts tot. (each problem 10 points)
Show your work ! Don't just write down a result. However, you may state formulas for moments of inertia without proof, but state clearly to what axis of rotation the formula you use refers to. Also, write your name on your sheets.

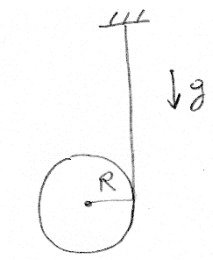
1. A uniform cylinder of mass *m* and radius *R* rolls down a slope without slipping. If the cylinder starts from rest at the top of the slope, and its center of mass has dropped by a vertical distance *h* when it is at the bottom of the slope, calculate the translational velocity of the center of mass of the cylinder at the bottom of the slope.

2. The (mass-less) spring, of spring constant *k* , is compressed by Δ , with the masses pressed against the plates, then released. The masses are then free to move on the frictionless surface. Find the final velocities of the two masses (v_1 and v_2), and the ratio of the kinetic energies (E_1/E_2) .



3. A uniform disk (mass *M* , radius *R*) is set up as a frictionless turntable, spinning with angular velocity ω_0 around a vertical axis through its center. A gecko of mass *m* is initially crouching at the rim of the disk, then starts walking (without slipping: geckos don't slip) towards the center of the disk.
a) Calculate the final angular velocity of the disk, when the gecko has reached the center of the disk.
b) What is the work done by the gecko?

4. A yo-yo consists of a uniform disk (mass *m*, radius *R*) and a (massless) string wound around it. Holding on to the end of the string, you let the yo-yo go. The acceleration of gravity is g.



a) Find the acceleration of the center of mass of the yo-yo (assume the string is tight and the yo-yo does not slip).
b) If at the bottom of the run the center of mass of the yo-yo has dropped by a vertical distance *h*, and assuming the yo-yo comes back up to the same height (no losses), find the vertical impulse (force integrated over time) you must be giving to the yo-yo (with your hand, through the string) when it reverses the direction of motion at the bottom of the run.

5. A particle of mass m is moving on the following trajectory:

$$\begin{cases} x(t) = R\ cos(\alpha t^2) \\ y(t) = R\ sin(\alpha t^2) \\ z(t) = 0 \end{cases}$$

Calculate the angular momentum $\vec{L}(t)$ of the particle, and the torque $\vec{\tau}(t)$ acting on the particle (both referred to the origin of the coordinate system).

6. Where do you have to hit a billiard ball so that it starts immediately to roll without slipping? The ball is a uniform sphere of radius R, and you hit it with the cue held horizontally. Give your answer in terms of the angle θ shown in the drawing.