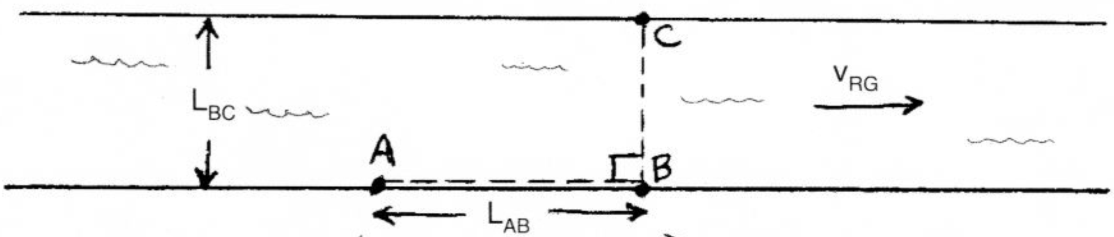
Physics 1A Winter 2022

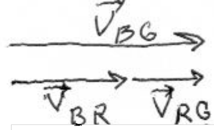
Midterm — Show all work, no credit for answers only

1. A fixed-speed boat travels on a river where the river speed with respect to the ground is $v_{RG} = 2.0$ km/hr. It travels from ports A to B in $t_{AB} = 30$ minutes, a distance $L_{AB} = 4.0$ km along the shore. It then travels the path B to C of length $L_{BC} = 3.0$ km that is perpendicular to the current.
   a) What is the speed of the boat in stationary water?
   b) What is the time to travel the path B to C?
   c) In crossing back to B the captain keeps the boat direction perpendicular to the shore. How far down the bank from B does it land? (30 points)



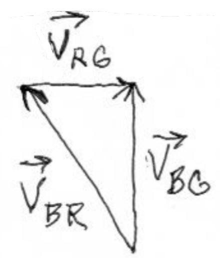
relative motion: $\vec{V}_{BG} = \vec{V}_{BR} + \vec{V}_{RG}$

a) $(V_{BG})_x = (V_{BR})_x + (V_{RG})_x$ $\quad L_{AB} = (V_{BG})_x t_{AB} = ((V_{BR})_x + (V_{RG})_x)\, t_{AB}$



$$\Rightarrow (V_{BR})_x = \frac{L_{AB}}{t_{AB}} - (V_{RG})_x = \frac{4.0\text{ km}}{0.5\text{ hr}} - 2.0\,\frac{\text{km}}{\text{hr}} = \boxed{6.0\,\frac{\text{km}}{\text{hr}}}$$

b)



right triangle

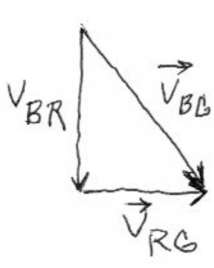
$$V_{BR}^2 = V_{BG}^2 + V_{RG}^2$$

$$\Rightarrow V_{BG} = \sqrt{V_{BR}^2 - V_{RG}^2} = \sqrt{(6.0\,\tfrac{\text{km}}{\text{hr}})^2 - (2.0\,\tfrac{\text{km}}{\text{hr}})^2}$$

$$= 5.7\,\frac{\text{km}}{\text{hr}}$$

$$t_{BC} = \frac{L_{BC}}{V_{BG}} = \frac{3.0\text{ km}}{5.7\,\frac{\text{km}}{\text{hr}}} = \boxed{0.53\text{ hr} = 32\text{ min}}$$

c)



$$\Delta t = \frac{L_{BC}}{V_{BR}} = \frac{3.0\text{ km}}{6.0\,\frac{\text{km}}{\text{hr}}} = 0.5\text{ hr}$$

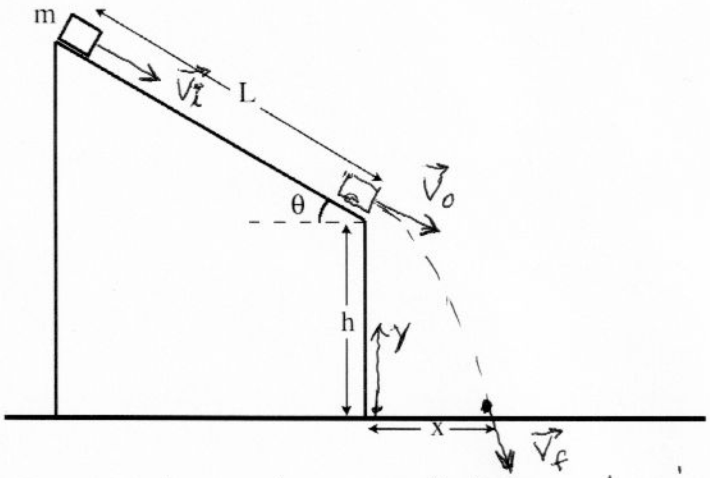
$$\Delta x = (V_{BG})_x \Delta t = V_{RG} \Delta t = (2.0\,\tfrac{\text{km}}{\text{hr}})(0.5\text{ hr})$$

$$= \boxed{1.0\text{ km}} \text{ to right of B}$$

2. A mass m = 1.0 kg sits at the top of an inclined plane with $\theta = 30°$, and the coefficient of kinetic friction between the plane and mass is 0.3. It is given a push as it is released so that its initial speed down the incline is v_i = 3.0 m/s. After traveling L = 6 m it arrives at a cliff of height h = 4 m.

a) What is the distance x from the base of the cliff where it hits the ground?

b) What is the (vector) velocity of the mass just before it hits the ground? (35 points)



acceleration down plane

$a = g\sin\theta - \mu_k g\cos\theta$

$v_0^2 = v_i^2 + 2(g\sin\theta - \mu_k g\cos\theta)L$

$v_0 = \sqrt{(3.0\frac{m}{s})^2 + 2(4.9\frac{m}{s^2} - (0.3)(9.8\frac{m}{s^2})(0.866))(6.\ldots)}$

$= 6.10\frac{m}{s}$

launches at $-30°$ below horizontal

$v_{0x} = v_0\cos\theta = 5.29\frac{m}{s}$

$v_{0y} = -v_0\sin\theta = -3.05\frac{m}{s}$

a) take t = time from launch at h to ground

$y = 0 = h + v_{0y}t - \frac{1}{2}gt^2$

$t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4(-\frac{1}{2}g)h}}{-g} = 0.644\,s, \cancel{-1.26\,s}$

$x = v_{0x}t = (5.29\frac{m}{s})(0.64s) = \boxed{3.40\text{ m}}$

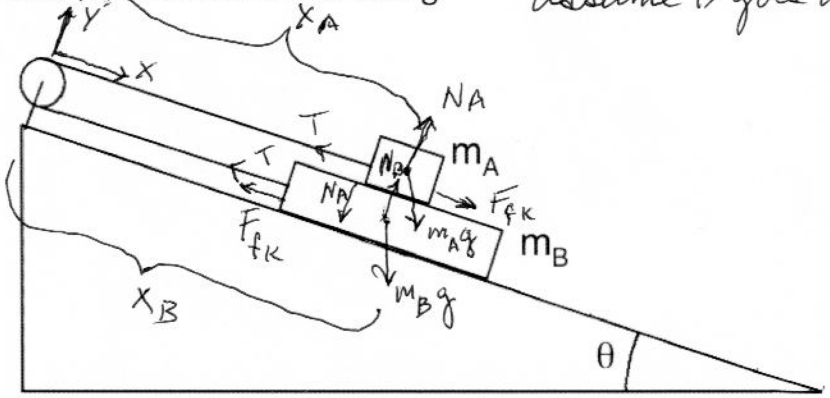
b) $v_{fx} = v_{0x} = \boxed{5.29\frac{m}{s}}$

$v_{fy} = v_{0y} - gt = -3.05\frac{m}{s} - (9.8\frac{m}{s^2})(0.64s)$

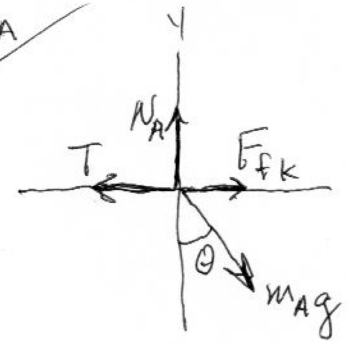
$= \boxed{-9.36\frac{m}{s}}$

or $\vec{v_f} = 5.29\frac{m}{s}\hat{i} - 9.36\frac{m}{s}\hat{j}$

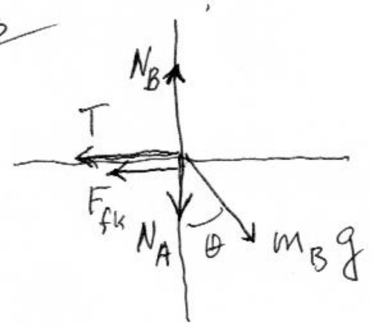
3. Masses m_A = 1.0 kg and m_B = 3.0 kg are connected with a massless string looping over a massless pulley as shown, on a θ = 30° incline. The coefficient of sliding friction of the surfaces between the two masses is 0.3, but the incline is frictionless. Find the acceleration of mass B and the tension in the string. *Assume B goes down, A goes up* (35 points



m_A



m_B



① $\Sigma F_{Ax} = m_A g \sin\theta + \mu_K N_A - T = m_A a_A$

② $\Sigma F_{Ay} = N_A - m_A g \cos\theta = 0$

string length l = const = $X_A + X_B$

take 2 time derivatives ⇒ $a_A = -a_B$ ⑤

F_{fk}, N_A are 3rd law reaction forces from A

③ $\Sigma F_{Bx} = m_B g \sin\theta - T - \mu_K N_A = m_B a_B$

④ $\Sigma F_{By} = N_B - N_A - m_B g \cos\theta = 0$

Four equations ① ② ③ ⑤ in four unknowns N_A, T, a_A, a_B

solve for

$$a_B = \frac{(m_B - m_A) g \sin\theta - 2\mu_K m_A g \cos\theta}{m_A + m_B}$$

$$= \boxed{1.18 \frac{m}{s^2}}$$

$$T = m_B (g \sin\theta - a_B) - \mu_K m_A g \cos\theta$$

$$= \boxed{8.61\ N}$$