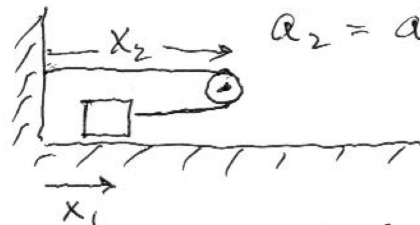
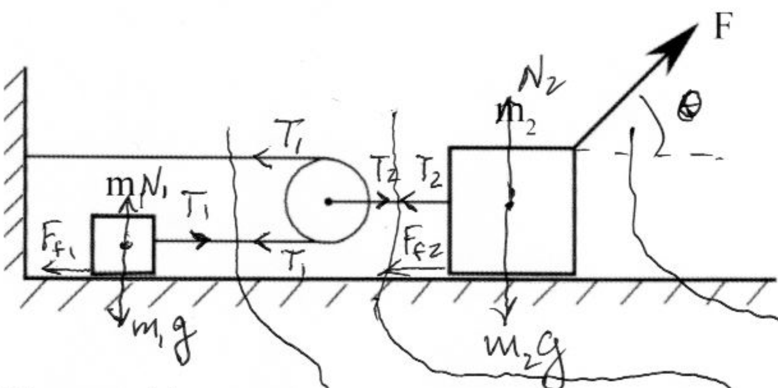
1. Two masses $m_1 = 1.00$ kg and $m_2 = 2.00$ kg are connected as shown by a massless pulley and massless strings, and a force F = 10.0 N is applied to m_2 at an angle $\theta = 45°$ to the horizontal. If the coefficient of kinetic friction is $\mu_k = 0.10$ for both blocks, find the accelerations of the blocks a_1 and a_2, and the force m_2 exerts on the floor. (35 pts.)



$a_2 = a_{pulley} = \ddot{x}_2$



string length $l = x_2 + (x_2 - x_1)$

$0 = 2\ddot{x}_2 - \ddot{x}_1 \Rightarrow a_1 = 2a_2$

$T_1 - \mu_k N_1 = m_1 a_1$

$N_1 - m_1 g = 0$

$T_2 - 2T_1 = m_p a_p = 0$

$\Rightarrow T_2 = 2T_1$

$F\cos\theta - T_2 - \mu_k N_2 = m_2 a_2$

$N_2 + F\sin\theta - m_2 g = 0$

$$\text{solve for } a_2 = \frac{F\cos\theta - \mu_k(m_2 + 2m_1)g + \mu_k F\sin\theta}{m_2 + 4m_1}$$

$$= \boxed{0.643 \ \frac{m}{s^2}}$$

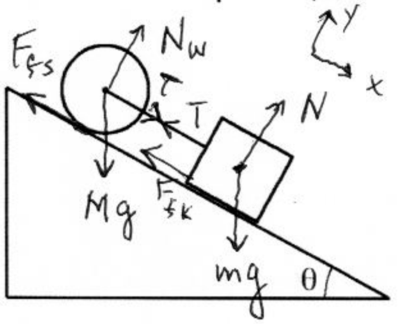
$$a_1 = 2a_2 = \boxed{1.29 \ \frac{m}{s^2}}$$

force on the floor is $-N_2 = F\sin\theta - m_2 g = -12.5$ N (downward)

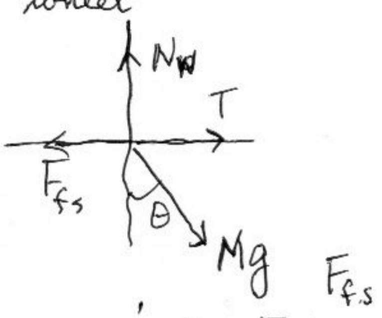
2. A solid cylindrical wheel of mass M = 1.0 kg and radius R = 0.5 m rolls down a 30° incline without slipping. The wheel is attached with a massless string at its axle to a block whose mass m = M = 1.0 kg. The coefficient of kinetic friction between the block and the incline is $\mu_k = 0.1$.

a) Find the acceleration of the system when it is released from rest, and the tension in the string.

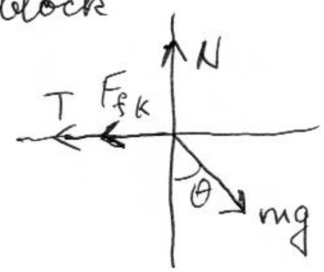
b) What is the angular velocity of the wheel after it has moved a distance x = 1.0 m down the plane, if it started from rest? (35 pts.)



a) wheel



block



if block + wheel moves x down incline,

$$x = R\theta$$

$$\ddot{x} = a = R\ddot{\theta} = R\alpha$$

$$T + Mg\sin\theta - F_{fs} = Ma$$

$$\Sigma\tau = F_{fs}R = I\alpha = \tfrac{1}{2}MR^2\left(\tfrac{a}{R}\right)$$

$$mg\sin\theta - \mu_k N - T = ma$$

$$N - mg\cos\theta = 0$$

solve for $a = \dfrac{(m+M)\,g\sin\theta - \mu_k mg\cos\theta}{m + \frac{3}{2}M}$

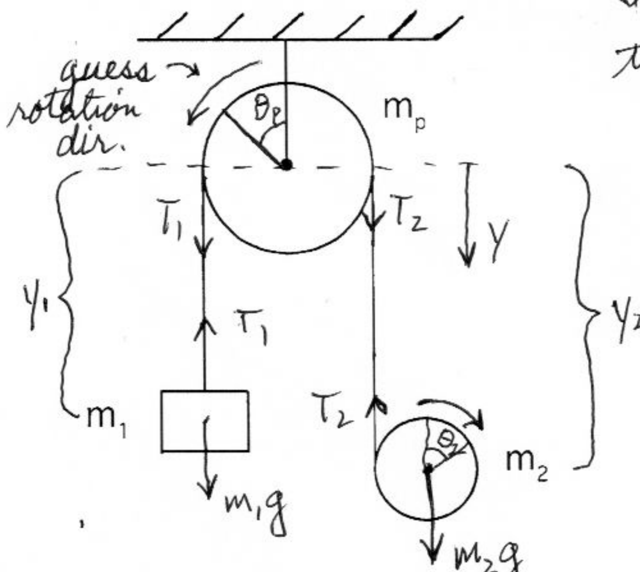
$$= \frac{4}{5}g\sin\theta - \frac{2}{5}\mu_k g\cos\theta = \boxed{3.58\ \frac{m}{s^2}}$$

$$T = \frac{3}{2}Ma - Mg\sin\theta = \boxed{0.471\ N}$$

b) $v^2 = v_0^2 + 2ax$

$$\omega = \frac{v}{R} = \sqrt{\frac{2ax}{R^2}} = \boxed{5.35\ \frac{rad}{s}}$$

3. An Atwood machine has $m_1 = 1.50$ kg and $m_2 = 1.00$ kg, attached with a rope over a cylindrical pulley of mass $m_p = 1.00$ kg and radius $r_p = 0.5$ m, which rotates without slipping by the rope. The rope is wound many turns around the cylindrical mass m_2 of radius $r_2 = 0.4$ m, and begins to unwind as the system is released. Find the acceleration of the center of mass of m_2.(40 pts.)



If l_0 is the initial length of rope, then the length l at any later time is

$$l = l_0 + r_2\theta_2 = y_1 + y_2$$

$$\ddot{l} = r_2\ddot{\theta}_2 = \ddot{y}_1 + \ddot{y}_2 \Rightarrow r_2\alpha_2 = a_1 + a_2$$

If the pulley turns through θ_p, then m_1 drops by $\Delta y_1 = r_p\theta_p$,

$$\Delta\ddot{y}_1 = a_1 = r_p\ddot{\theta}_p = r_p\alpha_p$$

2^{nd} law equations (y positive down)

$$m_1 g - T_1 = m_1 a_1$$

$$m_2 g - T_2 = m_2 a_{2CM}$$

$$\Sigma\tau_{pulley} = T_1 r_p - T_2 r_p = I_p\alpha_p = \tfrac{1}{2}m_p r_p^2\left(\frac{a_1}{r_p}\right)$$

$$\Sigma\tau_{2CM} = T_2 r_2 = I_2\alpha_2 = \tfrac{1}{2}m_2 r_2^2\left(\frac{a_1+a_2}{r_2}\right)$$

solve these 4 eqns in 4 unknowns T_1, T_2, a_1, a_2

$$\Rightarrow a_2 = \frac{2}{3}g - \frac{1}{3}\left(\frac{m_1 g - \frac{1}{3}m_2 g}{m_1 + \frac{1}{3}m_2 + \frac{1}{2}m_p}\right) = \boxed{+4.90\ \frac{m}{s^2}}$$

(down)

$$a_1 = \frac{m_1 g - \frac{1}{3}m_2 g}{m_1 + \frac{1}{3}m_2 + \frac{1}{2}m_p} = +4.90\ \frac{m}{s^2}$$ (both go down)

note for $m_p \to \infty$ (effectively a stationary pulley and rope)

$a_1 = 0$, $a_2 = \frac{2}{3}g \leftarrow$ value found for a yo-yo in lecture

4. A satellite is launched radially outward from the surface of the earth (R_E = 6400 km, $M_E = 5.98 \times 10^{24}$ kg, $G = 6.67 \times 10^{-11}$ N m^2/kg^2) with an initial velocity v_i = 12.0 km/s. (Neglect rotation of the earth and the gravitational pull of the sun, moon, and other planets)

a) What is the speed of the satellite when it gets to the orbital radius of the moon, $r_m = 3.84 \times 10^5$ km?

b) What is its speed when it completely leaves the solar system ($r \to \infty$) ?

c) Find how far it goes from the center of the earth if instead it is launched at an initial velocity of 10.0 km/s.

d) The rocket is now fired at an angle to the earth's surface at 10.0 km/s, and is found to go into a circular orbit about the earth. What is the radius of the orbit, and how long does one orbit take? (30 pts.)

a) $$E_i = \frac{1}{2}mv_i^2 - G\frac{M_E m}{R_E} = E_f = \frac{1}{2}mv^2 - G\frac{M_E m}{r_m}$$

$$\Rightarrow v = \sqrt{v_i^2 - 2GM_E\left(\frac{1}{R_E} - \frac{1}{r_m}\right)} = \boxed{4.63 \ \frac{km}{s}}$$

b) $$E_i = \frac{1}{2}mv_i^2 - G\frac{M_E m}{R_E} = E_f = \frac{1}{2}mv_f^2 \quad (r \to \infty)$$

$$\Rightarrow v_f = \sqrt{v_i^2 - 2GM_E\left(\frac{1}{R_E}\right)} = \boxed{4.40 \ \frac{km}{s}}$$

c) r_{max} where $v = 0$

$$E_i = \frac{1}{2}mv_i^2 - G\frac{M_E m}{R_E} = E_f = -G\frac{M_E m}{r_{max}}$$

$$\Rightarrow r_{max} = \frac{1}{\frac{1}{R_E} - \frac{v_i^2}{2GM_E}} = \boxed{3.24 \times 10^4 \text{ km}}$$

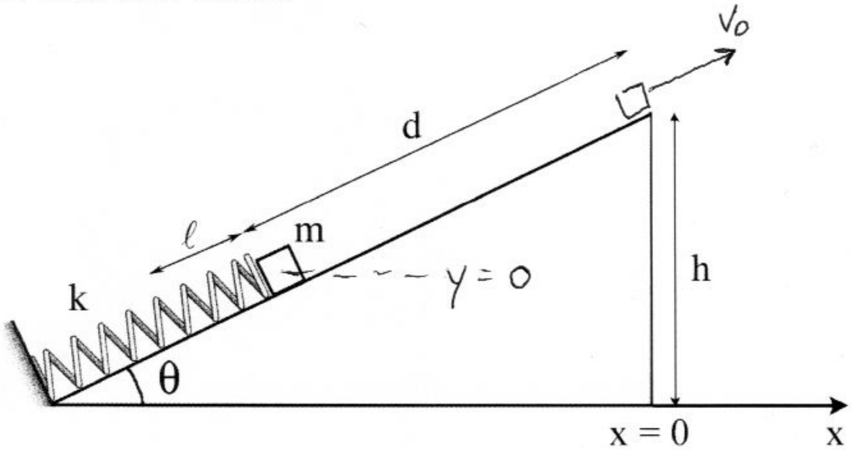
d) $$E_i = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = E_{orbit} = \frac{1}{2}mv_o^2 - \frac{GM_E m}{r_o}$$

radial force $F_g = G\frac{M_E m}{r_o^2} = ma_c = m\frac{v_o^2}{r_o} \Rightarrow v_o^2 = \frac{GM_E}{r_o}$

solve for $r_o = \dfrac{1}{\frac{2}{R_E} - \frac{v_i^2}{GM_E}} = \boxed{1.62 \times 10^4 \text{ km}}$

$$T = \frac{2\pi r_o}{v_o} = \boxed{2.05 \times 10^4 \frac{s}{rev} = 5.70 \frac{hrs}{rev}}$$

5. A spring with k = 100 N/m along a 30° incline has its end a distance d = 3.00 m from the top of the incline. A mass m = 1.00 kg is pushed against the spring, compressing it a distance of ℓ = 1.00 m. It is released from rest and travels up the incline before leaving the top of the incline, which is h = 3.00 m from the floor. The coefficient of kinetic friction with the incline is μ_k = 0.2. What is the distance x the mass has traveled from the base of the incline when it hits the floor? (30 pts)



$$E_i = \tfrac{1}{2}k\ell^2 - mg\ell\sin\theta$$

$$E_f = \tfrac{1}{2}mv_0^2 + mgd\sin\theta$$

$$E_f - E_i = W_{NC} = -\mu_K N(\ell+d) = -\mu_K mg\cos\theta(\ell+d)$$

$$\Rightarrow V_0 = \sqrt{\tfrac{k}{m}\ell^2 - 2g(\ell+d)(\sin\theta + \mu_K\cos\theta)} = 6.87\ \tfrac{m}{s}$$

Time to get from $y_0 = h$ to 0

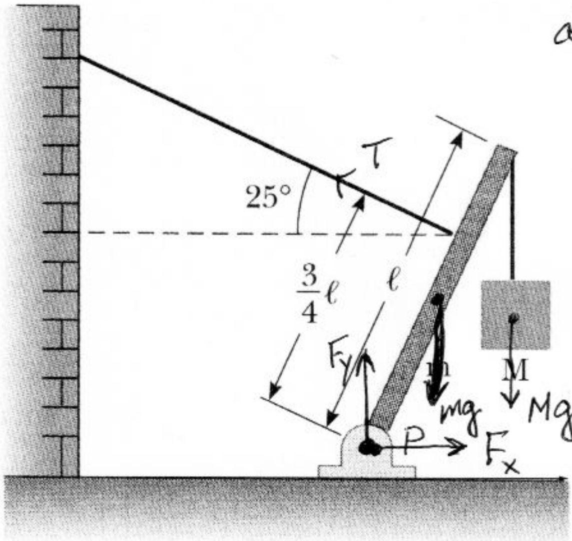
$$0 = h + V_0\sin\theta - \tfrac{1}{2}gt^2 \quad \Rightarrow t = 1.208\ s$$

$$x = V_0\cos\theta\, t = \boxed{7.19\ m}$$

6. A beam of mass m = 100 kg has a block of mass M = 200 kg hanging from its top end. The beam is supported at its bottom by a pivot, and by a wire at 3/4 the length of the beam, that makes an angle θ = 25° with the horizontal and a right angle with the beam.

a) What is the tension in the wire attached to the wall

b) What is the (vector) force exerted by the pivot on the beam?(30 pts.)



a) $\Sigma \tau_P = T\left(\frac{3}{4}\ell\right) - mg\frac{\ell}{2}\sin(180° - \theta)$

$- Mg\,\ell \sin(180° - \theta) = 0$

$\sin(180° - \theta) = \sin\theta$

$\Rightarrow T = \frac{4}{3}\left(\frac{mg}{2} + Mg\right)\sin\theta$

$= \boxed{1380\ N}$

b) $\Sigma F_x = F_x - T\cos\theta = 0$

$\Sigma F_y = F_y + T\sin\theta - (m+M)g = 0$

$\Rightarrow F_x = T\cos\theta = \boxed{1250\ N}$

$F_y = (m+M)g - T\sin\theta$

$= \boxed{2360\ N}$