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Dis 2A

Physics 1A - Winter 2016

Lecture 2
Midterm Exam #1

Advice. Your answers will be graded to a large extent on how convincing your reasoning is. A correct answer without good reasoning won't get much credit. Often convincing reasoning is a mixture of mathematics, explanations, and diagrams.

Problem 1.

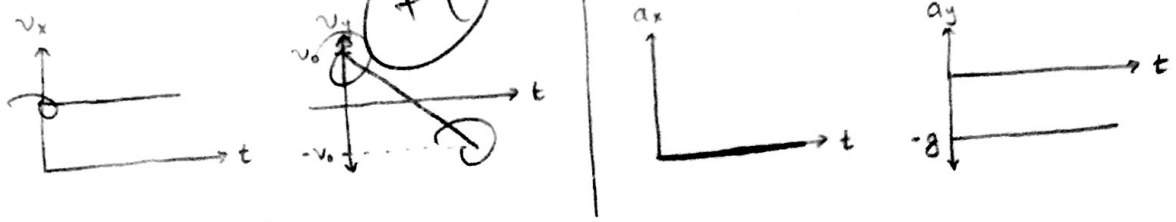
A ball is thrown into the air at a speed v and an angle θ above the horizontal direction. Air resistance may be neglected.

(a) Sketch graphs of the x- and y-components of the velocity and acceleration of the ball as functions of time while it's in the air.

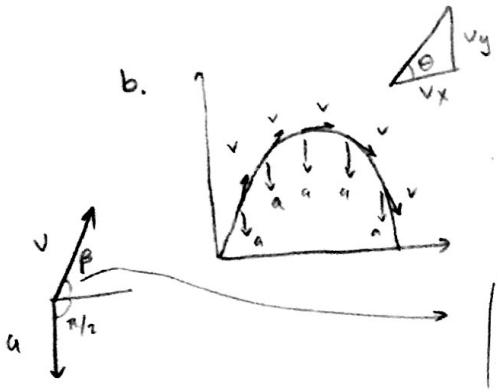
(b) Determine an expression for the angle between the velocity and acceleration vectors of the ball as a function of time while it's in the air.

(c) Use your expression from the last part to determine the angle between the acceleration and velocity vector at the top of its trajectory. Comment on whether your result makes physical sense.

a.



b.



$\frac{y\text{-dir}}{v_f^2 = v_i^2 + 2a\Delta y}$

$v_{fy} = \sqrt{v_y^2 - 2g\Delta y}$

$v_\theta = \tan^{-1}\left(\frac{\sqrt{v_y^2 - 2g\Delta y}}{v\cos\theta}\right) = \tan^{-1}\left(\frac{\sqrt{v^2\sin^2\theta - 2g\Delta y}}{v\cos\theta}\right)$

$a_\theta = -\frac{\pi}{2}$

$\Delta y = v_0\sin\theta t - \frac{1}{2}gt^2$

$\tan^{-1}\left(\frac{\sqrt{v^2\sin^2\theta - 2g(\quad)}}{v\cos\theta}\right) + \frac{\pi}{2}$

$\tan^{-1}\left(\frac{\sqrt{v^2\sin^2\theta - 2g\left(v\sin\theta t - \frac{1}{2}gt^2\right)}}{v\cos\theta}\right) + \frac{\pi}{2}$

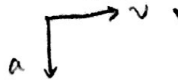
$\tan^{-1}\left(\frac{\sqrt{v^2\sin^2\theta - 2gv\sin\theta t + g^2t^2}}{v\cos\theta}\right) + \frac{\pi}{2}$

+8

c.

angle will be $\frac{\pi}{2}$

solve / show



$t = \frac{v\sin\theta}{2g}$

$\frac{v^2\sin^2\theta}{4g^2}$

$4g^2 \cdot v\sin\theta$

Problem 2.

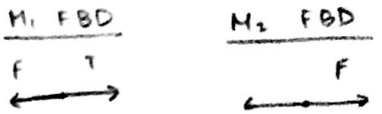
Two identically constructed spacecraft in outer space are trying to fly in opposite directions, but they're prevented from doing so by a strong cable that connects them. When they turn on their engines, they are each pulling on opposite ends of the cable with a force of 5×10^6 N.

(a) If the cable is massless, what is the tension in the middle of the rope?

(b) If the cable has nonzero mass M and length ℓ, then does the answer change? If so, explain why. If not, explain why not.

a.

Tension = $F = 5 \times 10^6$ N ✓

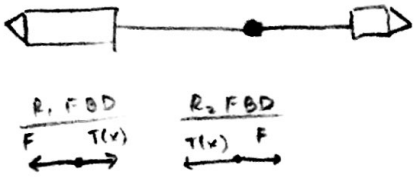


$F - T = M_1 a_1$ $\quad F - T = M_2 a_2$

$a_1 = a_2 = 0$

$F - T = 0 \rightarrow F = T$

b.



✗

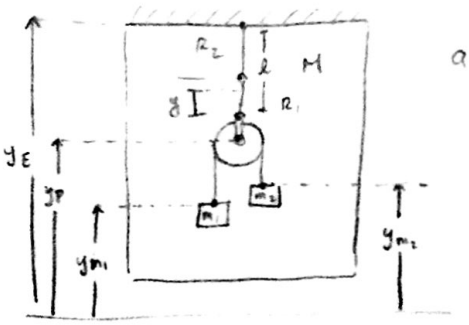
The answer does change because the tension in the middle will need to pull both the spaceships' mass AND the mass of the rope on each side ($M\ell/2$).

Problem 3.

A massless pulley hangs from the ceiling of an elevator by a uniform chain of mass M and length ℓ connected to the pulley's center. Masses m_1 and m_2 are connected by a massless rope and hang on either side of the pulley. The elevator has acceleration A relative to the ground. Let $T(y)$ denote the tension in the chain a distance y above the point at which it is connected to the pulley.

(a) What would you predict $T(0)$ would be if $m_1 = m$, $m_2 = m$ and $A = 0$? Explain using physical reasoning.

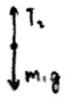
(b) Determine the general expression for $T(y)$ in terms of the given variables.



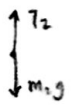
a. $m_1 = m_2 = m$
$A = 0$
$T(0)$?

The tension would be $2mg$ because the section of rope at $y=0$ is only responsible for holding up two masses of mass m each. Furthermore, there are no external forces besides gravity since $A=0$. Thus, the elevator is an inertial reference frame. +5

M_1 FBD

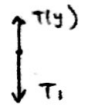


M_2 FBD



Pulley FBD

R_1 FBD



R_2 FBD

NTL

$T_3 =$

$T_2 - m_1 g = m_1 a_1$

$T_2 - m_2 g = m_2 a_2$

$T_1 = 2T_2$

$T(y) - T = m_{R1} a_{R1}$

$T_3 - T(y) = m_{R2} a_{R2}$

$y_p - y_{m_1} + y_p - y_{m_2} + \pi R = \ell$

$2y_p = y_{m_1} + y_{m_2} - \pi R - \ell$

$2a_p = a_{m_1} + a_{m_2} \rightarrow 2A = a_{m_1} + a_{m_2}$

Constraints

$a_{R1} = a_{R2} = a = A$

$T_2 - m_1 g = m_1 a_1$

$T_2 - m_2 g = m_2 a_2$

$T_1 = 2T_2$

$T(y) - T_1 = m_{R1} a$

$T(y) = T_1 + m_{R1} a = 2T_2 + m_{R1} a = \frac{2(2A+2g)\, m_1 m_2}{m_1 + m_2} m_{R1} A$, $m_{R1} = \frac{y}{\ell} M$

$$\boxed{T(y) = \frac{4(A+g)\, m_1 m_2 M y}{\ell(m_1+m_2)}}$$ +17

$2A = a_1 + a_2$

5 eq's, 5 unk's

$2A = \frac{T_2 - m_1 g}{m_1} + \frac{T_2 - m_2 g}{m_2}$

$2A = \frac{T_2}{m_1} + \frac{T_2}{m_2} - g - g$

$2A = \frac{m_2 T_2 + m_1 T_2}{m_1 m_2} - 2g$

$2A = T_2 \left(\frac{m_1+m_2}{m_1 m_2}\right) - 2g$

$T_2 = (2A+2g) \cdot \frac{m_1 m_2}{m_1+m_2}$

Problem	Score
1	12
2	5
3	22
Total	39