

# Physics 1A - Lecture 2 final

TOTAL POINTS

QUESTION 1

## 1 Problem 113

- 1 Recognize that rotational analog of Newton's Second Law (or equivalent) is useful, and write it down.
- 1 Notice that if the submarine has maximum acceleration, then the propeller will spin with some constant, maximum angular velocity.
- 1 Use the fact that the problem stated max acceleration is  $g/4$  and the acceleration expression given in the question to write an equation that can be solved for  $\omega_{\max}$ .
- 2 Notice that when the propeller reaches max angular velocity, its angular acceleration will be zero, and therefore the net torque will vanish by the rotational analog of NSL.
- 2 Notice that there are two sources of torque: the shaft and the water, and that these must sum to zero by the last observation.
- 3 Use the formula for  $f$  given in the question to determine an expression for the torque of the water on the shaft at maximum angular velocity
- 3 Combine various results above and solve for  $N$ .
- 3 Small math error in result
- 3 Wrong Torque
- 2 Small math error
- 10 Unclear, and does not look correct
- 0 Correct
- 13 Totally wrong

QUESTION 2

## 2 Problem 2 25

- +2 a-i
- +2 (a-ii) Recognize this is like free fall, so length is just natural length.

+2 (a-iii) Recognize that this is just like gravity being twice as strong as if stationary from the perspective of someone inside the elevator, so same as (i) with  $g$  replaced by  $2g$ .

+2 (b) Give some reasonably convincing argument that the tension will be less than the total weight.

+2 (c) NSL mass A

+2 (c) NSL mass B

+2 (c) NSL Pulley

+3 (c) Torque equation (rotational analog of NSL)

+2 (c) Constraint  $a_A, a_B$ , pulley

+2 Constraint relating acceleration and angular acceleration (tricky!)

+4 (c) Algebra to solve for length of spring.

+2 EXTRA CREDIT (d-i)

+2 EXTRA CREDIT (d-ii)

+2 EXTRA CREDIT (d-iii)

+2 EXTRA CREDIT (e)

+0 no points

QUESTION 3

## 3 Problem 3 12

- 3 (a) Recognize that gravity causes the net nonzero force to be zero in the  $z$ -direction during the flight of the bottle and then reason that this leads to non-conservation
- 2 (b) Recognize that there are no external forces in the  $x$ - $y$  plane and conservation follows
- 4 (c) Recognize that normal and gravity in  $z$ -direction but argue that torques only in  $x$ - $y$  plane, so conservation follows
- 3 (d) Recognize that gravity causes torque in  $z$ -direction while bottle flying so non-conservation follows
- 0 Correct

QUESTION 4

4 Problem 4 17

**+1 (a) recognize that the force from the falling water contributes to the measured weight**

**+1 (a) recognize that " $N > (M+m)g$ "**

**+8 (b) full credit: correctly recognize each term in the mass flow equation or equivalent procedure leading to correct equation**

**+3 (b) partial credit:** recognize that scale reads normal force and try solving for it.

**-1 (b) adjustment:** minor error (correct answer:  $N = (M + mt/T)g + m \sqrt{2gh}/T$ )

**+5 (b) partial credit:** attempt made at recognizing each term in the mass flow equation or equivalent procedure leading to the correct equation (correct answer:  $N = (M + mt/T)g + m \sqrt{2gh}/T$ )

**+3 (c) full credit: Take  $t \rightarrow T$  limit and interpret correctly.**

**+2 (c) partial credit:** attempt made at taking  $t \rightarrow T$  limit and a meaningful interpretation

**+4 (d) full credit: Notice that time  $t$  where reading is just weight exists and solve for the time by setting  $N = (m+M)g$**

**+2 (d) partial credit:** Notice that time  $t$  where reading is just weight exists between  $t=0$  and  $t=T$ .

**-1 (d) adjustment:** minor error

**+1 (d) partial credit:** gave an answer of  $t > T$  with realistic physical interpretation that the scale will read  $(M+m)g$  with no more falling water contributing to the weight.

**and y-directions, notice that x and y take special values at corner, solve for  $v_0$**

**+5 (b) Partial Credit:** Use kinematics equations for x- and y-directions, notice that x and y take special values at corner, solve for  $v_0$  with minor math error

**+3 (b) Partial Credit:** Use kinematics equations for x- and y-directions, notice that x and y take special values at corner, solve for  $v_0$  with significant error and/or missing steps

**+1 (b) Partial Credit:** Attempt made

**+6 (c) Full Credit: correct reasoning on what happens and if it makes sense at both limits**

**+4 (c) Partial Credit:** partially correct reasoning on what happens and if it makes sense at both limits

**+1 (c) Partial Credit:** attempt made

QUESTION 5

5 Problem 5 17

**+5 (a) Full Credit: Convincing reasoning for the possible launch angle range to hit corner, ( $\pi/2, \pi/4$ )**

**+3 (a) Partial Credit:** Convincing reasoning for the possible launch angle range to hit corner, missed the range ( $\pi/2, \pi/4$ ) or, got the range without enough reasoning

**+1 (a) Partial Credit:** attempt made at reasoning, missed the range

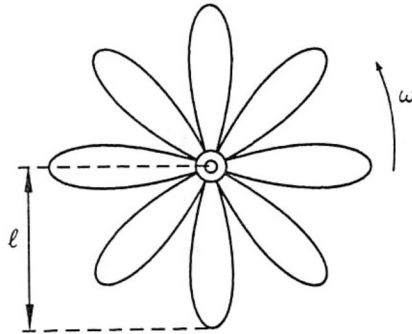
**+6 (b) Full Credit: Use kinematics equations for x-**

Physics 1A - Winter 2016  
Lecture 2

---

FINAL EXAM

Problem 1.



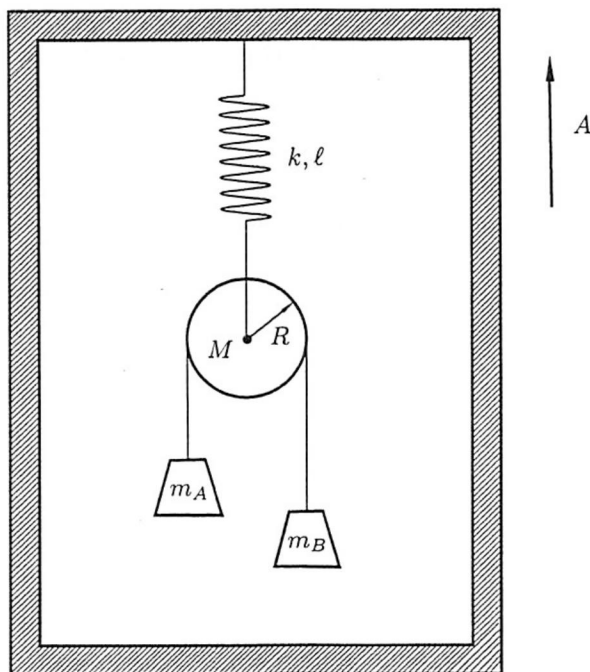
A submarine moves forward because it spins a propeller which displaces water backward. The propeller is spun by a crankshaft that is connected to the submarine's engine. The propeller has  $N$  blades (for example in the diagram  $N = 8$ ) of length  $\ell$ . Because of fluid resistance, the water exerts an angular speed-dependent force  $f = c\omega^2$  on each blade, tangent to the circle in which the blade is rotating. You can treat this force as though it's acting at the center of each blade. When the propeller spins, the resulting magnitude  $a$  of the ship's acceleration is related to the angular speed  $\omega$  of the propeller as  $a = gN\omega/\omega_0$  where  $\omega_0$  is a given constant.

If  $\tau$  is the magnitude of the maximum torque the engine can exert on the crankshaft, and if this torque gives the submarine a maximum acceleration of  $g/4$ , how many blades does its propeller have?

Extra Space

Extra Space

Problem 2.



An Atwood's machine with a pulley in the shape of a uniform disk of mass  $M$  and radius  $R$  is in an elevator having vertical acceleration  $A$  relative to the ground. Let positive  $A$  correspond to the elevator accelerating upward. Mass  $m_A$  hangs on the left side of the pulley while mass  $m_B$  hangs on the right. The rope connecting masses  $m_A$  and  $m_B$  is massless. The pulley is suspended from the ceiling of the elevator by a spring of spring constant  $k$  and natural length  $\ell$ .

- (a) Consider the special case  $m_A = m_B$ . What would you expect the length of the spring to be in the following limits:
- i.  $A \rightarrow 0$
  - ii.  $A \rightarrow -g$
  - iii.  $A \rightarrow g$

Justify your answers using physical reasoning and minimal math if possible.

- (b) When  $m_A \neq m_B$  but  $A = 0$ , would you expect the tension in the spring to be less than, equal to, or greater than the weight of the pulley plus the weight of the masses hanging from the pulley? Justify your answers using physical reasoning and minimal math if possible.

- (c) Determine a general expression for the length of the spring in terms of the given variables.
- (d) Does your mathematical answer from part (c) agree with your answers from part (a) in each special case? If so, demonstrate this mathematically for each case. If not, you may want to re-evaluate your intuition, or your mathematical answer, or both.
- (e) Does your mathematical answer from part (c) agree with your answer from part (b)? If so, demonstrate this mathematically for each case. If not, you may want to re-evaluate your intuition, or your mathematical answer, or both.



Extra Space



### Problem 3.

Nancy is spinning on an ice rink (effectively frictionless) at a certain angular speed  $\omega$ . Josh throws her a water bottle, and she catches it. Let the word "system" refer to the bottle + Nancy. Let the  $z$ -direction point vertically, away from the ice and perpendicular to it.

For all of the following questions, consider the time interval from the moment just after Josh lets the water bottle go, to the moment just after Nancy catches it.

- (a) Is the total linear momentum of the system conserved in the  $z$ -direction? Justify mathematically, and explain the math in words.
- (b) Is the total linear momentum of the system conserved in the  $x$ - $y$ -direction? Justify mathematically, and explain the math in words.
- (c) Is the total angular momentum of the system conserved in the  $z$ -direction? Justify mathematically, and explain the math in words.
- (d) Is the total angular momentum of the system conserved in the  $x$ - $y$ -direction? Justify mathematically, and explain the math in words.

Extra Space

Extra Space

**Problem 4.**

A large-diameter bowl of mass  $M$  sits on a cooking scale. A small amount of water of mass  $m$  is slowly poured into the bowl out of a cup from a height  $h$  above the bottom of the bowl. It takes a time  $T$  from the moment when the water first hits the bottom of the bowl to the moment when all of the water has flowed in. The water flows in at a constant mass per unit time. The bowl starts empty. Let  $t = 0$  be the moment at which the water first strikes the bowl.

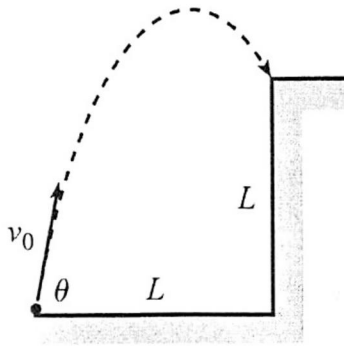
- (a) As  $t$  approaches  $T$  from values less than  $T$ , will the scale show a weight less than, equal to, or greater than  $(M + m)g$ ? Use physical reasoning and minimal math if possible.
- (b) Determine an expression for the weight as a function of time that the scale reads from  $t = 0$  until the moment when the last of the water strikes the bowl?
- (c) Does your answer in part (b) agree with your answer in part (a)? Explain.
- (d) Is there a time when the scale reads a weight  $(M + m)g$ ? If not, explain why not. If so, determine an expression for this time.

Extra Space

Extra Space



Problem 5.



A small ball is launched from the ground onto the corner of a cliff as shown.

- (a) For which launch angles  $\theta$  in the range  $[0, \pi]$  is it possible for the ball to hit the corner? Explain in the most convincing way you can.
- (b) For a given angle  $\theta$  that does allow the ball to hit the corner, what launch speed  $v_0$  is necessary for the ball to precisely hit the corner?
- (c) Examine the limiting cases  $\theta \rightarrow \pi/2$  and  $\theta \rightarrow \pi/4$ . What happens to the required launch speed in each of these cases according to the formula you derived from part (b)? Does the behavior of your formula make sense in these limiting cases? Explain with physical reasoning.

Extra Space

$$L(\tan\theta - 1) = \frac{gL^2}{2v_0^2 \cos^2\theta}$$

$$(\tan\theta - 1)\cos^2\theta = \frac{gL}{2v_0^2}$$

$$v_0^2 = \frac{gL}{2(\tan\theta - 1)\cos^2\theta}$$

$$v_0 = \sqrt{\frac{gL}{2(\tan\theta - 1)\cos^2\theta}}$$

$$a) \lim_{\theta \rightarrow \frac{\pi}{2}} v_0 = \lim_{\theta \rightarrow \frac{\pi}{2}} \sqrt{\frac{gL}{2(\tan\theta - 1)\cos^2\theta}} = \lim_{\theta \rightarrow \frac{\pi}{2}} \sqrt{\frac{gL}{2(\cos\theta \sin\theta - \cos^2\theta)}} = \infty$$

$$\lim_{\theta \rightarrow \frac{\pi}{4}} v_0 = \lim_{\theta \rightarrow \frac{\pi}{4}} \sqrt{\frac{gL}{2(\tan\theta - 1)\cos^2\theta}} = \infty$$

These behaviors make sense, since the ball would need an infinitely large velocity to hit the corner from  $\frac{\pi}{4}$  without dropping, and would need an infinitely high velocity to not drop below a height of  $L$  before hitting the corner when  $\theta = \frac{\pi}{2}$ .

Extra Space

