Physics 1A

Physics for Scientists and Engineers

Mechanics (Fall 2020)

Instructor - Nayana Rajapakse

Final Exam

Dec 14 2020 - (3hr 30min)

Rules

Now you have already viewed the exam. You MUST upload your answers to Gradescope as a single pdf file within 3hr and 30min.

You have 7hr of a submission window since you open the exam. This is to facilitate students approved for special accommodations. All others MUST submit within 3hr 30min limit. See the class email about the final for extra details.

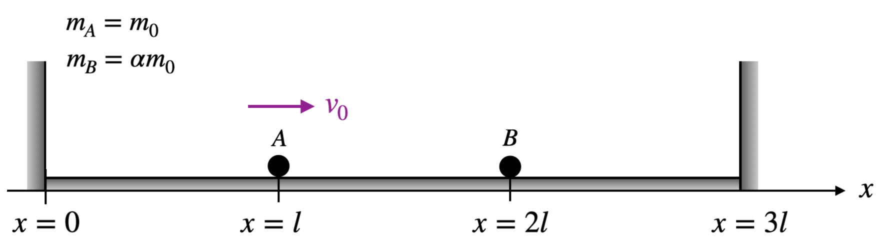
Submissions will not be accepted after 8.00 am Dec 15 2020 PT. You are prohibited to communicate about the exam until this time.

This is an open book exam. You are allowed to access everything posted on CCLE and anything that can already be found online. You are not allowed to directly or indirectly contact an actual person seeking any form of assistance or clarification. Methods of contacting include (but not limited to) meeting in person, voice call, messaging, emailing, posting on online forums, and etc. Individuals with suspicious activities will be reported to the deans office for further investigation.

If you need any clarifications while taking the exam, send a PRIVATE message in Slack to the person(s) listed below for each time period. DO NOT post any thing on the general thread.

8.00 am - 11.00 am - Nayana
12.00 pm - 3.00 pm - Bryan
4.00 pm - 7.00 pm - Mark
After 7.00pm - Nayana

Q1 (25 points)



Consider the setup above where two point masses are contained between two walls separated by a distance of $3l$. Initial positions of the point masses A and B are as shown with respective the given coordinate system. Masses are such that $m_A = m_0$ and $m_B = \alpha m_0$ where α and m_0 are positive constants. Initially (at $t = 0$), B is at rest while A is given a velocity v_0 as shown. All collisions occurring in the subsequent motion are perfectly elastic and duration of collisions are negligible compared to the other time scales of the problem. (i.e., duration of a collision is much less than $\frac{l}{v_0}$)

(a) (5pts) Find the velocities of A and B just after they collide for the first time.

Now set $\alpha < 1$

(b) (5pts) Find the time t when A and B collide for the second time.

Now set $\alpha = 1$

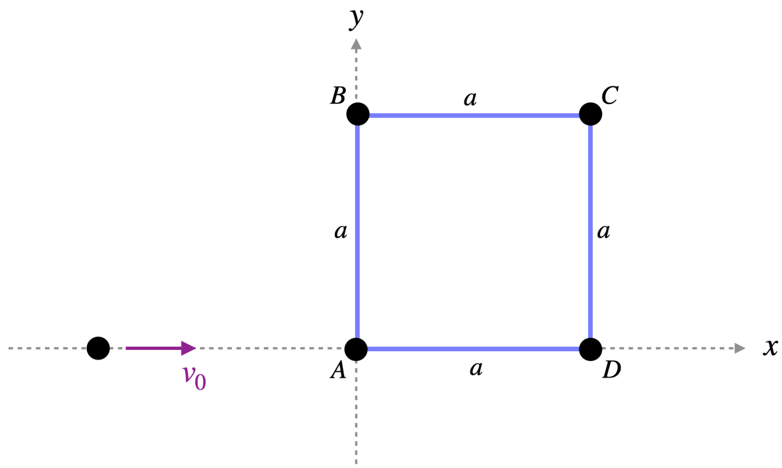
(c) (5pts) Draw the displacement vs time plot for both A and B on the same plot until $t = \frac{12l}{v_0}$.

Each curve should be correctly labeled and numerical values of special points (eg. time and displacement coordinates of specific points) should be marked.

(d) (5pts) Draw the velocity vs time plot for both A and B on the same plot until $t = \frac{12l}{v_0}$. Each curve should be correctly labeled and numerical values of special points (eg. time and velocity coordinates of specific points) should be marked.

(e) (5pts) Find the magnitude of the average force exerted on the right wall in a time scale much larger than $\frac{l}{v_0}$.

Q2 (25 points)



Four point masses A, B, C, and D are attached together with four rigid rods each with length a and are placed "on a table" as shown. Note that the shown diagram is the "top view" of the setting where all objects are on the table. For part (a) consider the coordinate system given. A fifth mass is approaching mass A along the x-axis with a velocity v_0 as shown. Upon collision this fifth mass will get "attached" to mass A for the motion to follow. All point masses have mass m each. Masses of the rods are "negligible" .

(a) (5pts) Find the location of the center of mass (CM) of the resulting object just after collision in the shown coordinate system.

(b) (5pts) Find the moment of inertia of the resulting object around the axis that is perpendicular to the table top and passes through its center of mass. For this consider a new coordinate axis system of which origin is placed at the CM yet being parallel to the old coordinate axis system.

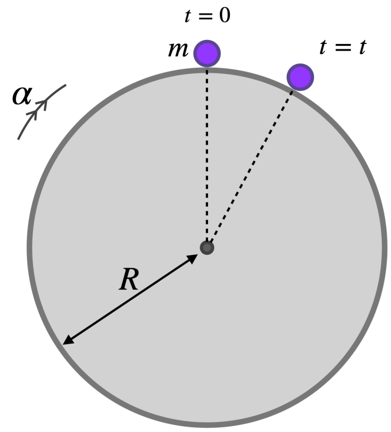
The subsequent motion can be described by a combination of a translational motion of the CM and a rotational motion about the CM. Velocity of the CM is denoted as v_{CM} while the angular velocity around the CM is denoted as ω.

(c) (5pts) Find expressions for v_{CM} and ω.

(d) (5pts) Find the fractional loss in kinetic energy during the collision.

(e) (5pts) During the subsequent motion, mind the maximum and minimum speeds of A, B, C, and D with respective to the table.

Q3 (25 points)



Consider the above setup where a "point" mass with mass m is initially sitting on the top of a disk which is connected to a motor. The plane of the disk is vertical. Note that the point mass is "not" attached to the disk but rather placed on it. The radius of the disk is R and the mass is M. At $t = 0$ the motor starts and the disk starts to spin (starting from rest) at a constant angular acceleration of α which is much less than $\frac{g}{R}$ where g is the magnitude of free fall acceleration. Coefficient of static friction between the disk and the point mass is μ_S. Note that the static friction on the point mass first points (directs) parallel to its tangential velocity but will switch direction after a certain time.

Consider the shown moment after a time t where the static friction on the point mass is still parallel to the its tangential velocity. Use an axis system where the y-axis points towards the center of the disk and the x-axis is parallel to the tangential velocity.

(a) (5pts) Find an expression for the normal force on the point mass.

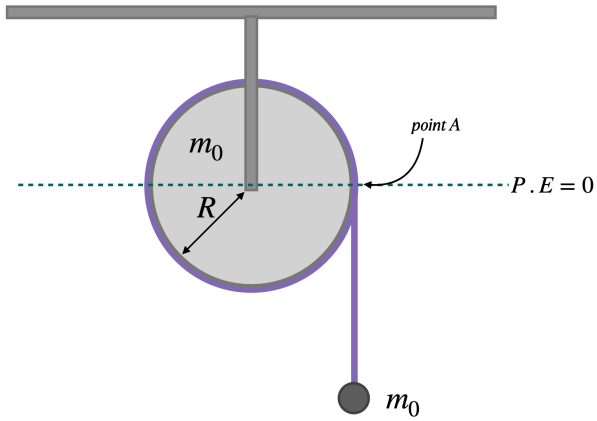
(b) (5pts) Find an expression for the friction force on the point mass.

(c) (5pts) Find the time when the instantaneous friction force drops to zero. (name this time as t_0)

(d) (5pts) Draw the free body diagram of the point mass just after $t = t_0$.

(e) (5pts) Obtain an inequality that will allow you to find the time at which the static friction between the point mass and the disk fails. (NO need to solve)

Q4 (25 points)



Consider the above setup where a rope is wrapped around a pulley that is fixed to a ceiling from its axis. The pulley is free to rotate around its axis which has negligible friction. The radius and the mass of the pulley are R and m_0 respectively. A fraction of $1/3$ of the rope is hanging vertically where a point mass (with mass m_0) is attached to the end. The total length of the rope is $3\pi R$ and the mass is m_0. Note that the rope goes around the pulley and ends at point A. This end of the rope is glued and fixed to the pulley. At $t = 0$ the system is released from rest.

(a) (5pts) For $t = 0$, draw the free body diagram for the system A defined as the pulley together with the $2/3$ of the rope wrapped around it.

(b) (5pts) For $t = 0$, draw the free body diagram for the system B defined as the $1/3$ of the vertical segment of rope together with the point mass.

(c) (5pts) Find the initial angular acceleration of the pulley.

(d) (5pts) Find the initial tension at the top of the vertical segment of the rope. (i.e., force on system B by system A)

(e) (5pts) Find the angular velocity of the pulley when the point mass drops a vertical height of $2\pi R$ relative to where it started from. For this, use conservation of energy and use the shown potential energy reference level (P.E = 0).

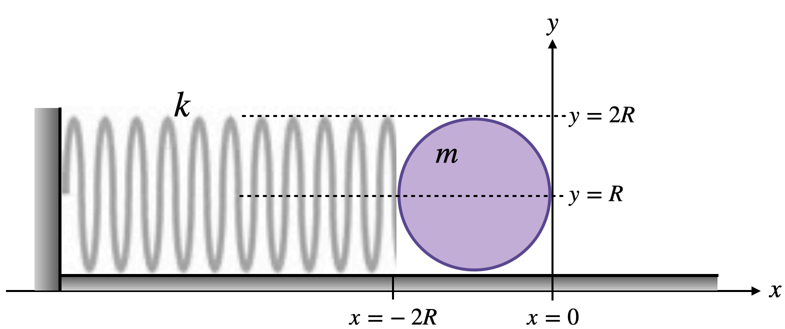
Q5 (25 points)

Consider a disk of mass m and radius R. The areal mass density of the disk is not uniform however is directly proportional to the radial distance measured from its center. That is $\sigma(r) \propto r$.

(a) (5pts) Find the proportionality constant for $\sigma(r) \propto r$ by setting up an equation to relate disk' s total mass to a proper area integral of $\sigma(r)$.

(b) (5pts) Find the moment of inertia of the disk around the axis perpendicular to its plane and passes through its center of mass (CM).

Now consider the setup shown below where the disk is compressed against a spring with spring constant k. One end of the spring is attached to a wall where the other end is in contact with the disk exerting a horizontal force on the latter that goes through its CM. Note that the disk is not attached to the spring. There is no friction force between the spring and the disk. The spring is initially compressed by an amount of $2R$. The system is released from rest and assume the static friction coefficient between the disk and the floor is high enough that the disk will start to roll "without" slipping.

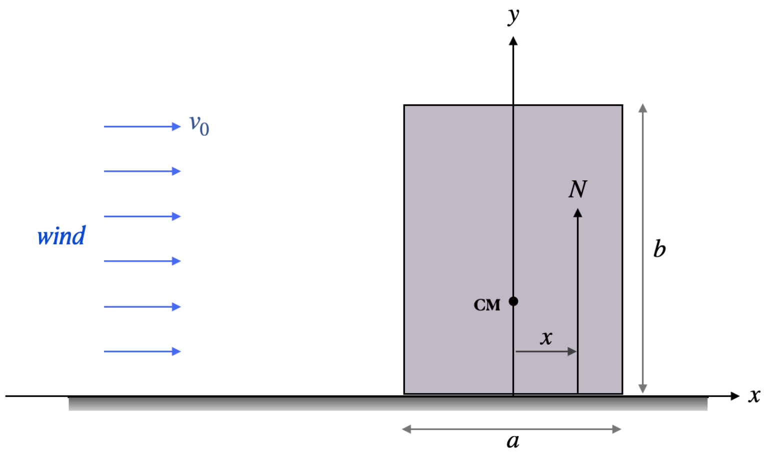


(c) (5pts) Let x be the coordinate of the point where the disk makes contact with the spring. Applying Newton' s laws, obtain a second order differential equation for x with respective to t.

(d) (5pts) Solve the equation obtained in part (c) to find an expression for the translational velocity of the disk as a function of x,

(e) (5pts) Use conservation of energy to find the translational velocity of the disk as its CM crosses the y-axis.

Q6 (25 points)



Consider the above scenario where a box of mass M is standing still on a rough horizontal floor against a wind that strikes at a speed of v_0 as shown. Consider the given coordinate system. The center of mass (CM) of the box is at $1/3$ of its height measured from the base. The dimensions of the box are as shown and the area of the face facing the wind is A. The density of air is ρ. Upon striking perpendicularly on the face of the box (area A), each air molecular bounces back (along -x direction) losing a fraction of $1/2$ of their initial kinetic energy. Assume that the friction between the box and floor is sufficiently large that as v_0 increases, box will topple before sliding. As shown, in such a scenario, the normal force from the floor on the box does not necessarily go through its center. To allow this freedom, consider the normal force as marked as in the diagram.

(a) (5pts) Find the force on the box due to wind.

(b) (5pts) Complete the free body diagram of the box and construct the force polygon for its static equilibrium.

(c) (5pts) Considering torque around the edge of the base on the opposite side to the wind, determine the location of the normal force (i.e., obtain an expression for shown x) in terms of v_0 assuming it is sufficiently small that the box is still in static equilibrium.

(d) (5pts) Find the minimum required v_0 that will lift the base of the box from the ground. (Name this v_0 as $v_{0,min}$)

(e) (5pts) Find the magnitude and the direction of the reaction force on the edge of the box (edge in contact with the floor in the marginal lift up position).