# Physics 1A

# Physics for Scientists and Engineers

# Mechanics (Fall 2020)

## Instructor - Nayana Rajapakse

# Midterm 2

Nov 20 2020 - (2hr 15min)

### Rules

Now you have already viewed the exam. You MUST upload your answers to Gradescope as a single pdf file within 2hr and 15min.

You have 4hr of a submission window since you open the exam. This is to facilitate students approved for special accommodations. All others MUST submit within 2hr 15min limit. See the class email about midterm for extra details.

Submissions will not be accepted after 10.30am Nov 21 2020 PT. You are prohibited to communicate about the exam until this time.

This is an open book exam. You are allowed to access everything posted on CCLE and anything that can already be found online. You are not allowed to directly or indirectly contact an actual person seeking any form of assistance or clarification. Methods of contacting include (but not limited to) meeting in person, voice call, messaging, emailing, posting on online forums, and etc. Individuals with suspicious activities will be reported to the deans office for further investigation.

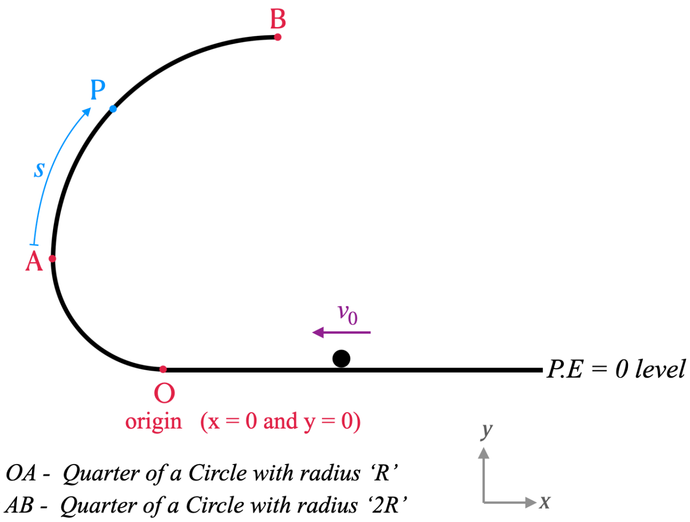
Even though you are allowed to access resources available online, you are not allowed to quote pre-derived results for specific examples. All reasoning and steps on the road to the final answer should be clearly provided to receive full credit.

If you need any clarifications while taking the exam, send a PRIVATE message in Slack to the person(s) listed below for each time period. DO NOT post any thing on the general thread.

10.15 am - 1.30 pm - Nayana
1.30 pm - 3.30 pm - Bryan
3.30 pm - 5.30 pm - Mark
After 5.30pm - BOTH Nayana and Bryan

slack group - https://join.slack.com/t/ucla-ksp9681/shared_invite/zt-hvwk6586-2bFe0eRKvrB9eQB1QmQnpA

## Q1 (30 points)




*Segment OA - Quarter of a Circle with radius 'R'*
*Segment AB - Quarter of a Circle with radius '2R'*

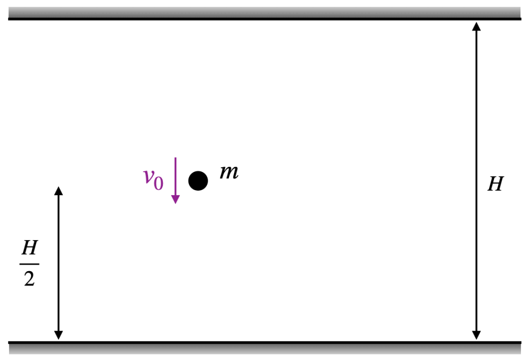
Consider the setup above where a ball moves on a frictionless track. Coming from the horizontal segment at a velocity of $v_0$ the ball first enters segment OA which is one quarter of a circle with radius $R$. The ball then enters segment AB which is one quarter of a circle with radius $2R$. All segments are connected smoothly. (i.e., if the shown track was a curve on a graph it is differentiable at points O and A). Point P is a variable point on the segment AB. Location of P is defined with the distance $s$ measured along the circumference from point A (See the diagram). Axis system in grey is for part (c) below. For the purpose of writing energy conservation equations, use the potential energy reference (P.E = 0) shown in the diagram.

(a) (10pts) Find the magnitude of the normal force on the ball from the track at point P. Assume that $v_0$ is sufficient for the ball to move pass P in circular motion.

(b) (10pts) Find the minimum required value for $v_0$ so that the ball will indeed reach point B in circular motion. Call this value $v_{min}$ for part (c).

(c) (10pts) Assume that $v_0$ is set to $v_{min}$ found in part (b). Find the $x$ coordinate of the location where the ball would land on the horizontal segment after leaving the track. As shown in the diagram, $x = 0$ is at point O. For this part, your answer must not contain $v_0$ for full credit.

## Q2 (30 points)




A ball with mass $m$ is set to bounce back and forth (vertically) between a roof and the floor. Height of the roof is $H$. The ball is first shot straight down from a height of $\dfrac{H}{2}$ with an initial speed of $v_0$. At each bouncing event, the ball will lose a fraction of $\epsilon$ out of its "kinetic energy" as heat. Kinetic energy of the ball right before and right after a bouncing event $n$ ($n = 1,2,3,...$) are defined as follows.

$K_{n,i} =$ kinetic energy just before $n^{th}$ bouncing
$K_{n,f} =$ kinetic energy just after $n^{th}$ bouncing

Note that odd-n correspond to ball bouncing off the floor while even-n correspond to ball bouncing off the roof. Here $v_0$ is sufficiently large such that, $v_0^2 > gH\left(\dfrac{1+\varepsilon}{1-\varepsilon}\right)$

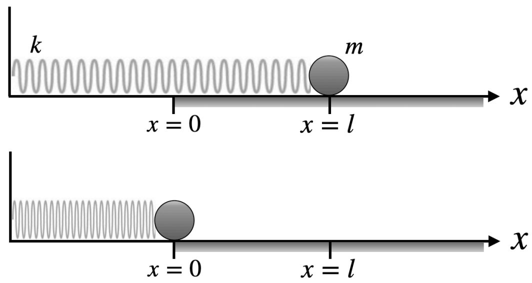
(a) (10pts) Show that the ball will in fact reach the roof at least once.

(b) (10pts) Find expressions for $K_{2,f}$ and $K_{3,f}$.

(c) (10pts) Find the general expression for $K_{n,f}$. You may have to write a couple of more terms to identify the pattern. Leave the answer as a summation series. NO need to use any identity to simplify.

Note - This expression is only valid while the ball has sufficient energy to reach the roof. NO need to find the number of bouncing events after which ball does not reach the roof anymore.

Q3 (20 points)




Consider a horizontal spring mass setup as shown above along with the defined axis system. Mass of the object and the spring constant of the spring are $m$ and $k$ respectively. Right end of the spring is attached to the mass while its left end is attached to a wall. $x < 0$ region of the floor is frictionless. However, $x > 0$ region (shaded) has space dependent friction given as follows.

kinetic friction coefficient, $\mu_k(x) = \mu_{k0}\left(\frac{x}{l}\right)$

static friction coefficient, $\mu_s(x) = \mu_{s0}\left(\frac{x}{l}\right)$

here $\mu_{k0}$ and $\mu_{s0}$ are dimensionless constants while $l$ is as same as marked in the diagram. Initially, the mass is at $x = l$ where the spring is "relaxed". Then the mass is compressed against the spring to $x = 0$ point before it is released.

(a) (10pts) Using work-energy theorem with pre-derived spring potential energy term, find the $x$ coordinate of the furthest location the mass will reach once it is released.

(b) (10pts) Find the condition on $\mu_{s0}$ such that the mass will turn back after reaching the location found in part (a).

Q4 (20 points)


A cyclist is set to sprint a distance $d$ as shown above. Use the shown axis system for convenience. $m$ is the combined mass of the cyclist and the bike. Cyclist starts off from rest at $x = 0$ and maintains a constant acceleration of $a$ until the end of the sprint (i.e., $x = d$). There is no wind but however, the cyclist is under the usual drag force that has a magnitude in the form $F_D = \beta v^2$. Here $\beta$ is a constant and $v$ is the instantaneous speed of the cyclist with respective to the surrounding air. The drive force is applied via the static friction between the ground and the rear wheel. For simplicity, neglect any static friction between the front wheel and the ground.

(a) (10pts) Find an expression for the instantaneous power $P(x)$ generated by the cyclist when he is at a general location between start and end points. Note that this is a function of $x$.

(b) (10pts) Find the average power $P_{avg}$ generated by the cyclist during the sprint. This should not contain $x$.

END