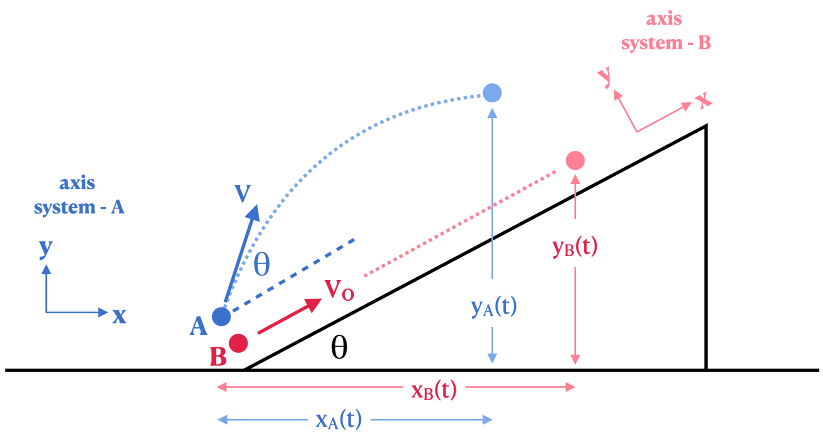
Q1 (35 points)



Two balls (A and B) are at the base of a ramp that makes an angle θ with the horizontal as shown in the figure above. At $t = 0$, B is given an initial velocity v_0 along the ramp while A is given an initial velocity v that makes an angle θ with the ramp (see diagram). In the subsequent questions, use axis systems A and B (shown in the diagram) for balls A and B respectively. Take g to be the "magnitude" of the free fall acceleration. As shown in the figure $x_A(t)$, $y_A(t)$, $x_B(t)$, and $y_B(t)$ define the location of the balls at a general time t.

(a) (2pts) Find acceleration components of ball A with respective to axis system-A.

(b) (3pts) Find acceleration components of ball B with respective to axis system-B.

(c) (7pts) Obtain expressions for $x_A(t)$ and $y_A(t)$.

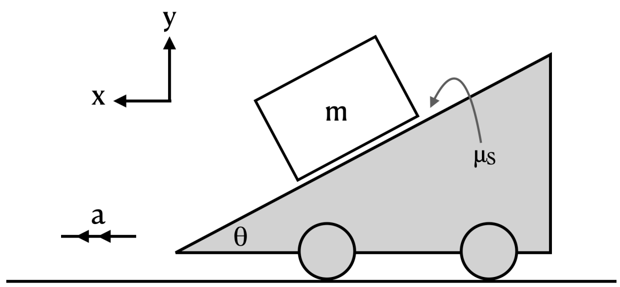
(d) (7pts) Obtain expressions for $x_B(t)$ and $y_B(t)$. (You may first work in axis system-B and then transform the results to find $x_B(t)$ and $y_B(t)$)

Now consider v is adjusted so that ball A will strike back on ball B moving on the ramp at some time $t = t_0$. Using results obtained in parts (c) and (d),

(e) (8pts) Show that $v = \dfrac{v_0}{\cos\theta}$

(f) (8pts) Show that $t_0 = \dfrac{2v_0}{g}\dfrac{\tan\theta}{\cos\theta}$

Q2 (35 points)



As shown in the figure, a box with mass m is sitting on a ramp that makes an angle θ with the horizontal. The ramp is capable of accelerating at a along the positive x direction as shown. Static friction coefficient between the ramp floor and the box is μ_S. Use the axis system shown in the diagram. Take g to be the "magnitude" of the free fall acceleration. Here $\theta > \tan^{-1}(\mu_S)$

(a) (5pts) Find the ideal value for a (call it a_{ideal}) such that the box can remain still with respective to the ramp without any assistance from the static friction.

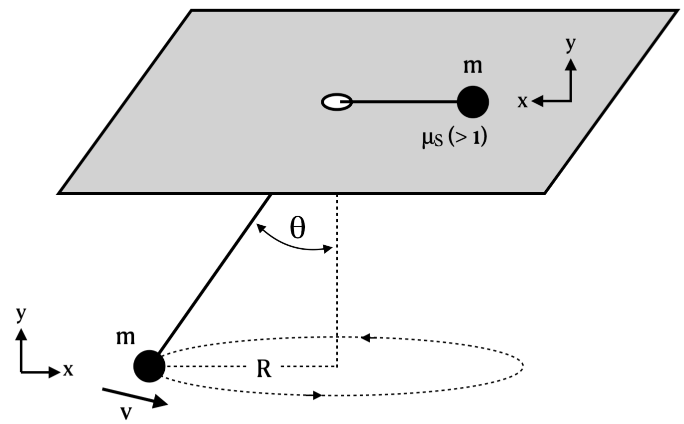
Now consider the ramp is accelerating at a which is slightly smaller than a_{ideal} yet large enough that the box will not slide.

(b) (3pts) Draw the free body diagram for the box.
(c) (6pts) Show that the magnitude of the static friction force is given by $F_S = m\left(g\sin\theta - a\cos\theta\right)$
(d) (6pts) Show that the magnitude of the normal force between the box and the ramp is give by $N = m\left(g\cos\theta + a\sin\theta\right)$
(e) (8pts) Show that the minimum allowed value for a (a_{min}) such that the box will not slide on the ramp is given by $a_{\min} = \left(\frac{1-\mu_S\cot\theta}{1+\mu_S\tan\theta}\right)a_{ideal}$. Here a_{ideal} is the result found in part (a).

Now consider the ramp is accelerating at a which is slightly greater than a_{ideal} yet small enough that the box will not slide.

(f) (3pts) Draw the free body diagram for the box.
(g) (4pts) Without rewriting the equations, using the results obtained in part (e), deduce an expression for a_{max} which is the maximum allowed value for a such that the box will not slide on the ramp. Make sure to clearly articulate your argument in relevance to the free body diagram.

Q3 (15 points)

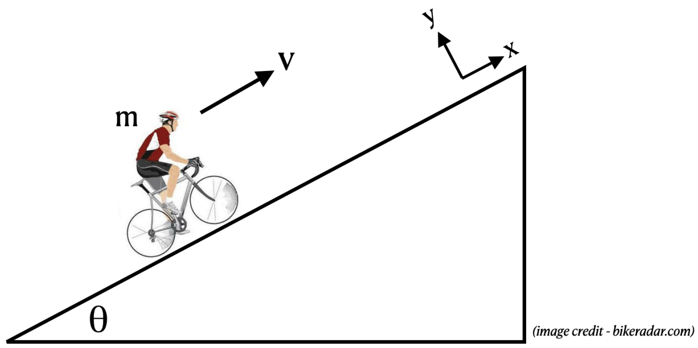


As shown in the figure, two equal balls each with mass m are connected with a massless string that passes through a small hole on a table. "Ball on the top of the table will remain stationary throughout the problem." Static friction between the table and this ball is μ_S which is greater than one. Ball below the table is undergoing uniform circular motion with a radius of R and a constant tangential speed of v. This v is small enough so that the friction can support the ball on the table to remain stationary. Take g to be the "magnitude" of the free fall acceleration. For each ball, use the provided axis system next to them.

(a) (8pts) Show that the tension of the string is given by $T = mg\left[1 + \frac{v^4}{R^2 g^2}\right]^{\frac{1}{2}}$

(b) (7pts) In order for the ball on the table to remain in equilibrium, show that v is bound by $v \leq \left[g^2 R^2 \left(\mu_s^2 - 1\right)\right]^{\frac{1}{4}}$

Q4 (15 points)



As shown in the figure, a cyclist with mass m is climbing up a hill which can be modeled as a ramp with constant slope. Here θ is the angle that the ramp makes with the horizontal and v is the instantaneous velocity of the cyclist moving up the ramp. The cyclist is exerting a velocity dependent drive force given by $F_{drive} = \frac{P_0}{v}$ to propel the bike up hill (for $v > 0$) while experiencing a drag force from air that has a magnitude of $F_D = \beta v^2$. Here P_0 and β are constants. Neglect the static friction force on the front wheel. Take g to be the "magnitude" of the free fall acceleration. Use the axis system shown in the diagram and in your answers you may use only the symbols given.

(a) (4pts) Draw the free body diagram for the cyclist.

(b) (5pts) Applying Newton's second law, obtain a differential equation for the velocity v with respective to time. (no need to solve it)

(c) (6pts) By using the expression obtained in part (b) find the terminal velocity (v_t) of the cyclist if he was riding on a flat road (i.e., use $\theta = 0$). Here terminal velocity is the maximum velocity he can attain due to air drag.

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