Midterm 1 Solutions

Physics 1A, Winter 2021

Full Name (Printed) ____________________________________

Full Name (Signature) ____________________________________

Student ID Number ____________________________________

There is only one correct answer for each multiple-choice question. Clearly write all answers in the boxes below each question.

On free-answer questions, it is not sufficient to just present the final answer on any given problem. **You need to show all of your work** in your solution to justify your steps so that another person can understand how you arrived at the final answer.

All work must be your own and <u>do not</u> communicate with anyone about the exam. Be prepared to defend anything you write down in a potential follow-up oral examination.

Your submitted exam on Gradescope <u>must</u> contain the same number of pages as the original exam packet and your solutions <u>must</u> be clearly written in the spaces below each question with your final answer clearly boxed or circled. (You may want to work on the problems on scratch paper, and when you are happy with your solution, write down your final solution in the space provided on the exam packet. But, do not turn in scratch paper on Gradescope – we will only grade what is written in the proper sections on the exam packet.)

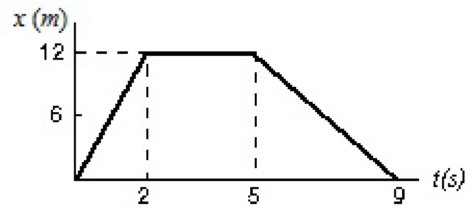
Have fun! Good luck!

<u>Multiple-choice/Short-answer Section</u> **– Each question only has one correct answer. These questions are either correct or incorrect, you do not need to show your work.**

1. During a short time interval the speed v in m/s of a car is given by $v = at^4 - bt^3$, where the time t is in seconds. The units of a and b are respectively:

Ans: $[a] = \frac{m}{s^5}$, $[b] = \frac{m}{s^4}$

2. This graph shows the position of a particle as a function of time. What is its average velocity between t = 5s and t = 9s?



Ans: -3 m/s

3. Throughout a time interval, while the speed of a particle increases as it moves along the x axis, its acceleration and velocity could be:

   A. negative and negative, respectively
   B. positive and negative, respectively
   C. negative and positive, respectively
   D. positive and zero, respectively

Ans: A

<u>**Free Answer Section**</u> **– Show all of your work for full credit. Partial credit will be awarded where warranted.**

4. The velocity as a function of time for an object moving in 1-dimensional motion along the x-axis is given by $v(t) = at^4 - bt^3$, where a and b are constants. The object starts at the origin at time $t = 0$.

- 4a) Derive an expression for the position as a function of time.

Since $x(0) = 0$, at any later time t we can write $\Delta x = x(t) - x(0) = x(t)$. Thus,

$$\Delta x = \int_{t_o=0}^{t} v(t')dt' = \int_{t_o=0}^{t} (at^4 - bt^3)dt = \boxed{\frac{a}{5}t^5 - \frac{b}{4}t^4 = x(t)}$$

- 4b) Derive an expression for the acceleration as a function of time.

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(at^4 - bt^3) = \boxed{4at^3 - 3bt^2}$$

5. A 2-dimensional vector $\vec{F}$ has a magnitude of $16\ m$ and direction 270^o. Another 2-dimensional vector $\vec{T}$ has a magnitude of $8\sqrt{2}\ m$ and direction of 225^o. All of the given angles are measured counter-clockwise from the positive x-axis.

   We define a new vector $\vec{B} = \vec{F} - \vec{T}$.

- 5a) Find the x and y components of $\vec{B}$.

First, apply trig to find the components of the given vectors. Don't forget that if the magnitude of the vectors are in meters, then the components must also have these units.

$$F_x = F\cos\theta_F = 16\cos 270^o = 16\cos 90^o = 16(0) = 0\ m$$

$$F_y = F\sin\theta_F = 16\sin 270^o = -16\sin 90^o = -16(1) = -16\ m$$

$$T_x = T\cos\theta_T = 8\sqrt{2}\cos 225^o = -8\sqrt{2}\cos 45^o = -8\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = -8\ m$$

$$T_y = T\sin\theta_T = 8\sqrt{2}\sin 225^o = -8\sqrt{2}\sin 45^o = -8\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = -8\ m$$

Then, the vector $\vec{B} = \vec{F} - \vec{T}$ has components

$$B_x = F_x - T_x = 0\ m - (-8\ m\,) = \mathbf{8\ m}$$

$$B_y = F_y - T_y = -16\ m - (-8\,) = \mathbf{-8\ m}$$

In component form, we can write in this out in several ways:

$$\boxed{\vec{B} = (8\ m, -8\ m) = (8\ m)\hat{x} + (-8\ m)\hat{y} = (8\ m)\hat{\imath} + (-8\ m)\hat{\jmath}}$$

[Finding the correct value of the components of $\vec{B}$ is enough for full credit; writing out the vector in component form is optional, but it's a nice way to display the final answer. 😊]

- 5b) Find the magnitude and direction of $\vec{B}$.

To determine the magnitude of $\vec{B}$ apply Pythagorean's theorem:

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(8)^2 + (-8)^2} = \sqrt{16\ + 16}\ = \sqrt{128} = \boxed{8\sqrt{2}\,m \approx 11.31\,m}$$

The direction of $\vec{B}$ is determined from the inverse tangent:

$$\theta_B = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{-8}{8}\right) = \tan^{-1}(-1) = -45^o$$

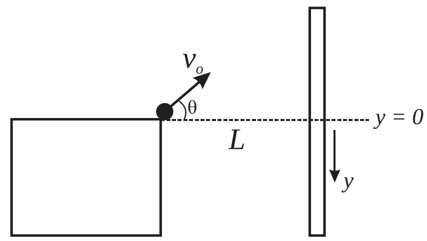
The components of $\vec{B}$ suggest the vector lies in Quadrant IV. The inverse tangent must imply an angle of <u>45^o below the x-axis</u>, or equivalently,

$\boxed{\theta_B = 360^o - 45^o = 315^o}$, measured CCW from the x-axis,

or $\boxed{\theta_B = -45^o}$, measured CCW from the x-axis (placing it below the x-axis),

or $\boxed{\theta_B = 45^o}$, measured CW from the positive x-axis.

6. A ball is thrown from the from the edge of a table toward vertical wall a distance L away. The ball is launched at angle θ above horizontal with a speed v_o. Take $y = 0$ as the top of the table and treat *downwards* as the positive y-direction.



- 6a) What is the final y-position of the ball when it strikes the opposite wall?

  (Hint: Your final answer should be a symbolic expression in terms of the known quantities L, v_o, θ, and possibly the acceleration of gravity, g.)

Take the origin to be the initial position of the ball. The ball will hit the wall when it has traveled a horizontal distance of L, so the time of flight is

$$x_f = v_{o_x} t + x_o \quad \Longrightarrow \quad L = (v_o \cos\theta)t \quad \Longrightarrow \quad t = \frac{L}{v_o \cos\theta}$$

The final height when the ball hits the wall is (notice that we take g as positive *downwards* and the initial y-velocity *upwards* is negative):

$$y_f = \frac{1}{2} g t^2 - v_{o_y} t + y_o$$

$$y_f = \frac{1}{2} g \left(\frac{L}{v_o \cos\theta}\right)^2 - v_o \sin\theta \left(\frac{L}{v_o \cos\theta}\right)$$

$$\boxed{y_f = \frac{gL^2}{2v_o^2 \cos^2\theta} - L\tan\theta}$$

- 6b) Evaluate your answer to part a) in the case that the ball is launched horizontally to the right, towards the wall.

If $\theta = 0$, then $\cos\theta = 1$ and $\tan\theta = 0$. The final height becomes

$$y_f = \frac{gL^2}{2v_o^2 \cos^2\theta} - L\tan\theta \quad \longrightarrow \quad \boxed{y_f = \frac{gL^2}{2v_o^2}}$$

- 6c) Use your answer in part b) to interpret whether the ball hits the wall above or below its initial starting height? Briefly explain your reasoning.

Since we defined y as *positive downwards*, notice that the final height will be *lower* than the initial height if $y_f > 0$ and *higher* than the initial height if $y_f < 0$.

The answer to part b) is positive, so the ball hits the wall **below** its starting height. Of course, conceptually this *has* to the case – if we launch the ball straight forward, gravity can only make the ball drop as it travels towards the wall!

- 6d) How does the final height of the ball in part b) change if the initial speed is doubled?

  Derive an expression for the "new height" when $v_o \to 2v_o$ and comment on whether it will hit the wall *above* or *below* the "original height".

In part b) we have a final height $y_1 = \frac{gL^2}{2v_o^2}$. The new height when $v_o \to 2v_o$ is

$$y_2 = \frac{gL^2}{2(2v_o)^2} = \frac{1}{4}\left(\frac{gL^2}{2v_o^2}\right) = \frac{1}{4}y_1 \quad \Longrightarrow \quad y_1 = 4y_2$$

When the initial speed is doubled, the final height is ¼ as low, or equivalently, the original height is 4 times lower than the new height.

With greater initial speed, the ball reaches the wall sooner and experiences less drop due to gravity – the new height is <u>above</u> the original height.