Catherine Hu - Physics 1A Final

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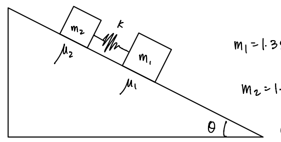
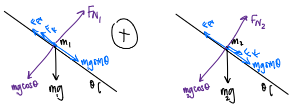
Physics 1A Final — Question 1



$m_1 = 1.38\ kg$ $\mu_1 = 0.144$

$m_2 = 1.35\ kg$ $\mu_2 = 0.340$

$\theta = 45.2^\circ$ $k = 247\ \frac{N}{m}$



$$\sum F_{1x} = mg\sin\theta - F_{fr} - F_k = m_1 a_x$$

$$mg\sin\theta - \mu_{k1} m_1 g\cos\theta - F_k = m_1 a_x$$

$$a_x = g\sin\theta - \mu_{k1} g\cos\theta - \frac{F_k}{m_1}$$

$$\sum F_{2x} = m_2 g\sin\theta - F_{fr} + F_k = m_2 a_x$$

$$m_2 g\sin\theta - \mu_{k2} m_2 g\cos\theta + F_k = m_2 a_x$$

$$a_x = g\sin\theta - \mu_{k2} g\cos\theta + \frac{F_k}{m_2}$$

$$\cancel{g\sin\theta} - \mu_{k1} g\cos\theta - \frac{F_k}{m_1} = \cancel{g\sin\theta} - \mu_{k2} g\cos\theta + \frac{F_k}{m_2}$$

$$(\mu_{k2} - \mu_{k1}) g\cos\theta = \frac{F_k}{m_1} + \frac{F_k}{m_2} = \frac{(m_1 + m_2) F_k}{m_1 m_2}$$

$$\frac{(m_1 m_2)(\mu_{k2} - \mu_{k1}) g\cos\theta}{(m_1 + m_2)} = F_k$$

$$F_k = \frac{(1.38\ kg)(1.35\ kg)(0.340 - 0.144)(9.8\ \frac{m}{s^2})\cos 45.2^\circ}{1.38\ kg + 1.35\ kg} = 0.9236\ N$$

$$a_x = g\sin\theta - \mu_{k1} g\cos\theta - \frac{F_k}{m_1}$$

$$a_x = \left(9.8\ \frac{m}{s^2}\right)\sin 45.2 - 0.144\left(9.8\ \frac{m}{s^2}\right)\cos 45.2 - \frac{F_k}{1.35kg}$$

$$a_x = 5.2901193 \rightarrow \boxed{5.29\ \frac{m}{s^2}}$$

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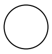
Physics 1A Final — Question 2



sphere $m_{ball} = 2.86\,kg$ $r = 1.03\,m$

$\omega = 3.97 \frac{rev}{s} \times \frac{2\pi\,rad}{1\,rev} = 24.944 \frac{rad}{s}$

$K = 1.94 \times 10^3 \frac{N}{m}$ how much is the spring compressed by

$E_1 = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$ ball rolling

$E_2 = \frac{1}{2} kx^2$ spring compressed / ball stops

$\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = \frac{1}{2} kx^2$

$I\omega^2 + mv^2 = kx^2$ $\omega = \frac{v}{r}$ $I = \frac{2}{5} mr^2$

$\frac{2}{5} m\cancel{r^2}\left(\frac{v^2}{\cancel{r^2}}\right) + mv^2 = kx^2$

$\frac{7}{5} mv^2 = kx^2$

$\sqrt{\frac{7mv^2}{5k}} = x = \sqrt{\frac{7m}{5k}} \times v = \sqrt{\frac{7m}{5k}} \times \omega r$

$x = \sqrt{\frac{7(2.86\,kg)}{5(1.94 \times 10^3 \frac{N}{m})}} \times 24.944 \frac{rad}{s} (0.103\,m)$

$x = 0.116722 \rightarrow \boxed{0.117\ m}$

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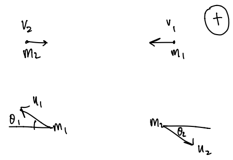
Physics 1A Final — Question 3



$m_1 = 1\text{kg} \quad v_1 = 2.74 \frac{m}{s}$

$m_2 = 1.47\text{kg} \quad v_2 = 6.79 \frac{m}{s}$

$|\vec{u_1}| = 1.39 \frac{m}{s} \quad \theta_1 = 33.7°$

a) $p_{ix} = p_{fi} \qquad p_{iy} = p_{fy}$

$-m_1v_1 + m_2v_2 = -m_1u_1\cos\theta_1 + m_2u_2\cos\theta_2 \qquad 0 = m_1u_1\sin\theta_1 - m_2u_2\sin\theta_2$

$m_1u_1\cos\theta_1 - m_1v_1 + m_2v_2 = m_2u_2\cos\theta_2 \qquad m_1u_1\sin\theta_1 = m_2u_2\sin\theta_2$

$$\frac{m_1u_1\cos\theta_1 - m_1v_1 + m_2v_2}{\cos\theta_2} = \frac{m_1u_1\sin\theta_1}{\sin\theta_2}$$

$$\tan\theta_2 = \frac{m_1u_1\sin\theta_1}{m_1u_1\cos\theta_1 - m_1v_1 + m_2v_2}$$

$$\theta_2 = \arctan\left(\frac{(1\text{kg})(1.39\frac{m}{s})\sin 33.7}{(1\text{kg})(1.39\frac{m}{s})\cos 33.7 - (1\text{kg})(2.74\frac{m}{s}) + (1.47\text{kg})(6.79\frac{m}{s})}\right)$$

$$\theta_2 = 5.2472398 \rightarrow \boxed{5.25°}$$

b) $KE_1 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad KE_2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

solve for u_2

$p_{iy} = p_{fy}$

$0 = m_1u_1\sin\theta_1 - m_2u_2\sin\theta_2$

$$u_2 = \frac{m_1u_1\sin\theta_1}{m_2\sin\theta_2} = \frac{(1\text{kg})(1.39\frac{m}{s})\sin 33.7}{1.47\text{kg}\ \sin 5.25}$$

$$u_2 = 5.736773 \frac{m}{s}$$

ratio of $\frac{K_{after}}{K_{before}} = \frac{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2}{\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2} = \frac{m_1u_1^2 + m_2u_2^2}{m_1v_1^2 + m_2v_2^2} = \frac{(1\text{kg})(1.39\frac{m}{s})^2 + (1.47\text{kg})(5.73\frac{m}{s})^2}{(1\text{kg})(2.74\frac{m}{s})^2 + (1.47\text{kg})(6.79\frac{m}{s})^2} = 0.6683077 \rightarrow \boxed{0.668}$

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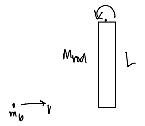
Physics 1A Final — Question 4



$L = 2.37\,m$

$m_{rod} = 3.44\,kg = M$

$m_b = 1.37\,kg$

a) $v_1 = 12.6\,\frac{m}{s}$

conservation of angular momentum

$$L_1 = L_2$$

$$mv_1r = I\omega - mv_2r$$

$$\frac{mr(v_1+v_2)}{I} = \omega$$

conservation of kinetic energy (elastic collision)

$$E_1 = E_2$$

$$\tfrac{1}{2}mv_1^2 = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}mv_2^2$$

$$\frac{m(v_1^2 - v_2^2)}{I} = \omega^2$$

$$\omega^2 = \frac{m^2r^2(v_1+v_2)^2}{I^2}$$

$$\frac{m^2r^2(v_1+v_2)^2}{I^2} = \frac{m(v_1^2-v_2^2)}{I}$$

I of rod $= \frac{1}{3}ML^2$

$r = L$

$$\frac{mr^2(v_1+v_2)}{I} = v_1 - v_2$$

$$\frac{3m\cancel{L^2}(v_1+v_2)}{M\cancel{L^2}} = v_1 - v_2$$

$$3mv_1 + 3mv_2 = Mv_1 - Mv_2$$

$$(3m+M)v_2 = (M-3m)v_1$$

$$v_2 = \frac{(M-3m)v_1}{3m+M} = \frac{3.44\,kg - 3(1.37\,kg)}{3(1.37\,kg) + 3.44\,kg}\left(12.6\,\frac{m}{s}\right) = -1.118145695\,\frac{m}{s}$$

$$\frac{mr(v_1+v_2)}{I} = \omega$$

$$\frac{1.37\,kg\,(2.37\,m)\left(12.6\,\frac{m}{s} - 1.118145\,\frac{m}{s}\right)}{\frac{1}{3}(3.44\,kg)(2.37\,m)^2} = \omega$$

$$\omega = 5.788242 \longrightarrow \boxed{5.79\,\frac{rad}{s}}$$

b) $v_1 = 13.9\,\frac{m}{s}$ $\quad m_{rod} \Rightarrow \infty$

If the rod is significantly heavier, then v_2 of the pointlike object is equal to $v_1 = \boxed{13.9\,\frac{m}{s}}$

This is because kinetic energy is conserved in this system, and if the rod is very heavy, it won't move.

Therefore, the speed of the point mass must be equal to its initial value (its velocity will change due to a change in direction)

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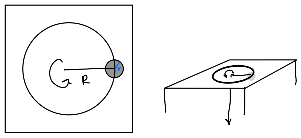
Physics 1A Final — Question 5



$m = 1.74 kg \quad r = 0.170 m$

$R = 1.78 m \quad v = 6.97 \frac{m}{s}$

a) magnitude of angular momentum

parallel axis theorem

$I_{total} = \frac{1}{2} m r^2 + M R^2$

$I_{total} = \frac{1}{2}(1.74kg)(0.170m)^2 + (1.74kg)(1.78m)^2$

$I_{total} = 5.538159$

$L = I\omega = I\frac{v}{R} = \frac{I(6.97\frac{m}{s})}{1.78m} = 21.685937 \rightarrow \boxed{21.7\ kg\frac{m^2}{s^2}}$

b) force acts on string

new $R = 0.674$ m

work done on cylinder $\longrightarrow$ $W = \Delta K$

$L_1 = L_2$

$= 21.685937\ kg\frac{m^2}{s} = I_2\omega_2$

$W = \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}I_1\omega_1^2$

$I_1 = \frac{1}{2}mr^2 + MR_1^2 = \frac{1}{2}(1.74kg)(0.17m)^2 + (1.74kg)(1.78m)^2 = 5.538159$

$I_2 = \frac{1}{2}mr^2 + MR_2^2 = \frac{1}{2}(1.74kg)(0.17m)^2 + (1.74kg)(0.674m)^2 = 0.81558324$

$\omega_2 = \frac{L}{I_2} = 26.58948\ \frac{rad}{s}$

$\omega_1 = \frac{v_1}{R} = \frac{6.97\frac{m}{s}}{1.78m} = 3.9157\ \frac{rad}{s}$

$W = \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}I_1\omega_1^2$

$W = \frac{1}{2}(0.81558)(26.58948)^2 - \frac{1}{2}(5.538159)(3.9157)^2$

$W = 245.85079 \rightarrow \boxed{246\ J}$

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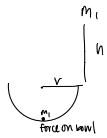
Physics 1A Final — Question 6

$m_1 = 1.72kg$ $h = 4.11m$ $r = 1.37m$

a)



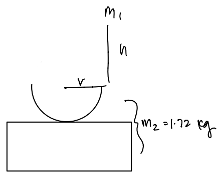
$$mgH = \tfrac{1}{2}mv^2$$
$$mgH = \tfrac{1}{2}mv^2$$
$$v^2 = 2gH$$

$$\sum F_c = F_N - mg = \frac{mv^2}{r}$$
$$F_N = mg + \frac{mv^2}{r} = mg + \frac{2mgH}{r} \quad \text{where } H = h+r$$

$$F_N = mg\left(1 + \frac{2(h+r)}{r}\right) = (1.72kg)(9.8\tfrac{m}{s^2})\left(1 + \frac{2(4.11m + 1.37m)}{1.37m}\right)$$

$$= 151.704 \rightarrow \boxed{152\ N}$$

b)



when m_1 reaches bottom of bowl:

$m_1v_1 - m_2v_2 = 0$ due to conservation of momentum

$m_1v_1 = m_2v_2$

so $v_1 = v_2$ $\Rightarrow$ m_1 moves in one direction, m_2 will move in the opposite

$$E_1 = E_2$$
$$mgH = \tfrac{1}{2}(m_1)v_1^2 + \tfrac{1}{2}(m_2)v_2^2 \quad \text{where } m_1 = m_2$$
$$\text{so} \quad mgH = \tfrac{1}{2}(2m)v^2 \quad \text{and} \quad H = h+r$$
$$v^2 = gH$$
$$v = \sqrt{gH}$$

$v_1 = \sqrt{gH}$ $v_2 = -\sqrt{gH}$

relative velocities of v_1 and v_2:

$$V_{rel} = \sqrt{gH} - (-\sqrt{gH})$$
$$V_{rel} = 2\sqrt{gH}$$

$$\sum F_c = F_N - mg = ma_c = \frac{mv_{rel}^2}{r}$$

$$F_N = \frac{mv_{rel}^2}{r} + mg = \frac{m(4gH)}{r} + mg$$

$$F_N = mg\left(\frac{4(h+r)}{r} + 1\right) = (1.72kg)(9.8\tfrac{m}{s^2})\left(\frac{4(4.11m + 1.37m)}{1.37m} + 1\right)$$

$$= 286.552 \rightarrow \boxed{287\ N}$$

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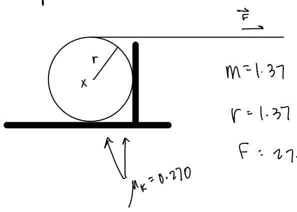
Physics 1A Final — Question 7

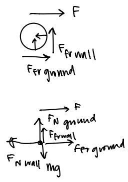


$m = 1.37$ kg

$r = 1.37$ m

$F = 27.2$ N

a)



$\sum \tau = Fr - F_{fr\,wall}r - F_{fr\,ground}r = I\alpha$

$Fr - \mu_k F_N r - \mu_k m g r = I\alpha = \frac{1}{2} m R^2 \alpha$

$\sum F_x = 0 = F_{frg} + F - F_{NW}$

$0 = \mu_k F_{Ng} + F - F_{NW}$

$F_{NW} = \mu_k F_{Ng} + F$

$\sum F_y = 0 = F_{Ng} + F_{fr\,w} - mg$

$0 = F_{Ng} + \mu_k F_{NW} - mg$

$0 = F_{Ng} + \mu_k(\mu_k F_{Ng} + F) - mg$

$0 = F_{Ng} + \mu_k^2 F_{Ng} + \mu_k F - mg$

$\dfrac{mg - \mu_k F}{\mu_k^2 + 1} = F_{Ng}$

$F_{NW} = \mu_k F_{Ng} + F$

$F_{NW} = \mu_k\left(\dfrac{mg - \mu_k F}{\mu_k^2 + 1}\right) + F$

$= 0.270\left(\dfrac{(1.37\text{kg})(9.8\frac{m}{s^2}) - 0.270(27.2\text{N})}{0.270^2 + 1}\right) + 27.2$

$= 28.73056 \rightarrow \boxed{28.7\ \text{N}}$

b) $\sum \tau = I\alpha \qquad I = \frac{1}{2} m r^2$

$Fr - F_{frw}r - F_{frg}r = I\alpha = \frac{1}{2} m r^2 \alpha$

$F - F_{frw} - F_{frg} = \frac{1}{2} m r \alpha$

$\dfrac{2(F - F_{frw} - F_{frg})}{mr} = \alpha$

$F = 27.2$N

$F_{frw} = \mu_k F_{NW} \qquad F_{NW} = 28.7$

$\hookrightarrow 7.757251$ N

$F_{frg} = \mu_k F_{Ng} \qquad F_{Ng} = \dfrac{mg - \mu_k F}{\mu_k^2 + 1} = \dfrac{(1.37\text{kg})9.8\frac{m}{s^2} - 0.270(27.2\text{N})}{0.27^2 + 1} = 5.6687$ N

$\hookrightarrow 1.53056$ N

$\alpha = \dfrac{2(27.2 - 7.757 - 1.5306\text{ N})}{(1.37\text{kg})(1.37\text{m})} = 19.0869 \rightarrow \boxed{19.1\ \frac{\text{rad}}{s^2}}$