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MIDTERM 2 – PHYSICS 1A

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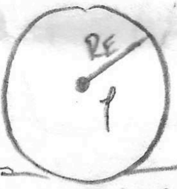
Specify here if you want your exam returned privately:________

Each question is worth 25 points. The exam is closed book; no notes or calculators. If necessary, use the back of the page. If you do use the back of the page, write OVER on the page whose backside you are using. Please make the organization of your answer as clear as possible.

1. Assume that the Earth has radius R_E and density ρ. (1) Expressed in terms of these parameters, what is the mass of the Earth? (2 pts.) (2) If $R > R_E$, what is the force of gravity on a test mass, m_{test}? (5 pts.). (3) If $R < R_E$, what is the force of gravity on a test mass? (5 pts.) (4) Define the potential energy at infinity to equal 0. What is the potential energy $R = R_E$? (5 pts.) (5) What is the potential energy at $R = 0$? (8 pts.)



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1) $\boxed{M_E = \frac{4}{3}\pi R_E^3 \rho \;\; \text{~~kg~~} \; \text{kg}}$

2) $R > R_E$:

$$\boxed{F = \frac{GM_E M_{test}}{R^2}} \quad N$$

3) $R < R_E$:

$$F = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right) m_{test}}{R^2}$$

$$\boxed{F = G\left(\frac{4}{3}\pi R \rho\right) m_{test}}$$

-4

4) $U = \int_{\infty}^{R_E} \frac{GM_E M_{test}}{R^2} dR$

$$= GM_E M_{test} \int_{\infty}^{R_E} \frac{1}{R^2} dr$$

$$= GM_E M_{test} \left(-\frac{1}{R}\Big|_{\infty}^{R_E}\right)$$

$$= GM_E M_{test} \left(-\frac{1}{R_E} - 0\right)$$

$$= -\frac{GM_E M_{test}}{R_E}$$

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5) $U = m_{test} \int_{R_E}^{0} \frac{4}{3} G\pi R \rho \, dR - \frac{GM_E M_{test}}{R_E}$

$$= m_{test} \frac{4}{3} G\pi\rho \left(\frac{1}{2}R^2 \Big|_{R_E}^{0}\right) - \frac{GM_E M_{test}}{R_E}$$

$$= m_{test} \frac{3}{2} G\pi\rho \left(-R_E^2\right) - \frac{GM_E M_{test}}{R_E}$$

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$$-\frac{3}{2}G\pi\rho\left(R_E^2\right)^{m_{test}} - \frac{GM_E M_{test}}{R_E} \qquad M_e = \frac{4}{3}\pi R_e^3\rho$$

$$= -\frac{3}{2}G\pi\rho R_E^2 - \frac{G\left(\frac{4}{3}\pi R_E^{\cancel{3}\,2}\rho\right)}{\cancel{R_E}}\left[m_{test}\right]$$

$$= -m_{test}\,G\pi\rho R_E^2\left(\frac{3}{2}+\frac{4}{3}\right) \qquad \frac{4}{6}+\frac{8}{6} = \frac{12}{6} = 2$$

$$= \boxed{-2m_{test}G\pi\rho R_E^2}$$

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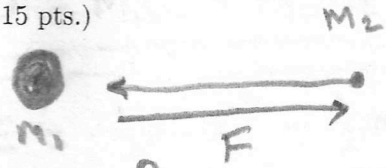
2. Assume that there is a repulsive force between m_1 which is at rest and an incident particle of mass m_2, incoming on a trajectory along the X axis from x = $+\infty$. Assume that $m_1 >> m_2$. Assume the potential energy is of the form:

$$U(x) = K\left(e^{x_0/x} - 1\right) \tag{1}$$

(1) What is the vector force associated with this potential energy? (10 pts.) (2) Very far from mass #1, the incoming particle has speed v_0, how close can it approach mass #1? (15 pts.)



1) $U = -\int F dx$

$$F = \frac{d}{dt}(U(x)) = \frac{d}{dt}\left(ke^{x_0/x} - k\right) \qquad \frac{d}{dt}\left(e^{x_0/x}\right)$$

$$F = \left(\frac{-x_0}{x^2} e^{x_0/x}\right)k \qquad = -\frac{x_0}{x^2} e^{x_0/x}$$

$$\boxed{\vec{F} = \left(\frac{x_0}{x^2} e^{x_0/x}\right)k} \rightarrow F \text{ is in } (+)\,x \text{ direction}$$

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2) KE = PE

$$\frac{1}{2} m_2 v_0^2 = k\left(e^{x_0/x}\right) - k$$

$$\frac{m_2 v_0^2}{2} + k = k\left(e^{x_0/x}\right)$$

$$\frac{m_2 v_0^2}{2k} + 1 = e^{x_0/x}$$

$$\ln\left(\frac{m_2 v_0^2}{2k} + 1\right) = \frac{x_0}{x}$$

$$\boxed{x = x_0 \Big/ \left[\ln\left(\frac{m_2 v_0^2}{2k} + 1\right)\right]}$$

15

3. Consider a radioactive decay where a particle at rest decays into two particles of mass m_1 and m_2. Let E denote the energy released during this radioactive decay and let $\vec{u}_1$ and $\vec{u}_2$ denote the velocities of particles 1 and 2 after the decay. What is the expression for conservation of momentum after the decay? (5 pts.) What is the expression for conservation of energy after the decay? (5 pts.) Compute the kinetic energy of particle #1 after the decay in terms of E, m_1 and m_2 only (15 pts.).

a) $\boxed{0 = m_1\vec{u}_1 + m_2\vec{u}_2}$ 5

b) $\boxed{E = \frac{1}{2}m_1\vec{u}_1^{\,2} + \frac{1}{2}m_2\vec{u}_2^{\,2}}$ 5

c) $u_2 = \dfrac{-m_1 u_1}{m_2}$

$E = \frac{1}{2}m_1 u_1^2 + \frac{1}{2}\cancel{m_2}\left(\dfrac{m_1^2 u_1^2}{m_2^{\cancel{2}}}\right)$

$E = \frac{1}{2}m_1 u_1^2 + \frac{1}{2}\dfrac{m_1^2 u_1^2}{m_2}$

$E = \frac{1}{2}m_1 u_1^2\left(1 + \dfrac{m_1}{m_2}\right)$

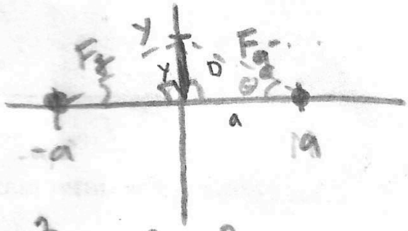
$E = \cancel{\tfrac{1}{2}}\cancel{m_1} u_1^2\left(\dfrac{m_2+m_1}{\cancel{m_2}}\right)$

$\dfrac{2Em_2}{m_1(m_2+m_1)} = u_1^2$

$KE_1 = \cancel{\tfrac{1}{2}}\cancel{m_1}\left(\dfrac{\cancel{2}Em_2}{\cancel{m_1}(m_2+m_1)}\right)$ 15

$\boxed{KE_1 = \dfrac{Em_2}{m_2+m_1}}$ J

4. Consider two objects of equal mass, M, located on the X-axis at $(a, 0)$ and $(-a, 0)$. Find the total gravitational force on a test mass, m, that lies on the Y axis at position $(0, y)$. Note that at $y = 0$ and $y = \infty$ your expression should indicate no net force. Also, recognize that the force will be aligned along the Y axis so compute the component of the total force along this axis (10 pts.) Find the positive value of y where $|\vec{F}|$ is a maximum. (15 pts.).



a) $F_g = \dfrac{GMM}{D^2}$ $\qquad D^2 = y^2 + a^2$

$F_g = \dfrac{GMm}{y^2+a^2}$

$F_y = 2F_g \sin\theta = \boxed{\dfrac{2GMm}{y^2+a^2}\sin\theta}$

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b) $\dfrac{dF_y}{dy} = 0 \rightarrow$ max value of Force $\qquad u = y^2 + a^2$, $du = 2y\,dy$

$= 2GMm\sin\theta \left(\dfrac{d}{dt}\left(\dfrac{1}{y^2+a^2}\right)\right)$

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$= 2GMm\sin\theta \left(\dfrac{0 - 2y}{(y^2+a^2)^2}\right)$ $\qquad$ min value @ $y = 0$; $y = \infty$

$= \dfrac{-4GMm\sin\theta\, y}{y^4 + 2a^2y^2 + a^4}$ $\rightarrow$ $\dfrac{y}{(y^2+a^2)^2}$

$\boxed{F \text{ is max when } y = a}$

$y = y^4 + 2a^2y^2 + a^4$

$a = a^4 + 2a^6 + a^4$

$2a^4 + 2a^6 = a$