Physics 1A - Lecture 2 Midterm 2

Karen Li

TOTAL POINTS

22 / 36

QUESTION 1

1 Problem 1 6 / 6

- **+ 1 Recognize that there is no net external force in the x-direction.**
- **+ 2 Recognize that no external force in the x-direction implies that the acceleration of the center of mass in that direction is zero.**
- **+ 2 Reason that since the center of mass acceleration in the x-direction is zero, and since it falls from rest, it falls straight down**
- **+ 1 Diagram of the trajectory**
- + 0 no points

QUESTION 2

2 Problem 2 6 / 15

- + 3 (a) Correct use of energy conservation or kinematics to get equation relating initial velocity and height of monkey
- **+ 1 (a) Partial Credit: energy conservation to find vb: 1/2 M v0^2**
- + 1 (a) Partial Credit: energy conservation to find vb: 1/2 M vb^2
- + 1 (a) Partial Credit: energy conservation to find vb: Mgh
- + 1 (a) Partial Credit: use kinematics to find vb: Correct velocity equation with given variables.
- + 1 (a) Partial Credit: use kinematics to find vb: Correct position equation with given variables
- + 1 (a) Partial Credit: use kinematics to find vb: Correct time equation with given variables.
- + 3 (a) Correct use of momentum conservation when acrobat grabs monkey
- + 1 (a) Partial Credit: Initial momentum term with calculated initial velocity.
- + 1 (a) Partial Credit: Final momentum term.
- + 3 (a) Correct use of energy conservation or kinematics to get equation relating velocity right after the grab to final height reached by acrobat and monkey
- **+ 1 (a) Partial Credit: use of energy conservation to get equation for h_max: Correct final energy term (M+m)gh_max**
- + 1 (a) Partial Credit: use of energy conservation to get equation for h_max: Correct initial potential energy term (M+m)gh
- + 1 (a) Partial Credit: use of energy conservation to get equation for h_max: Correct kinetic energy term.
- + 1 (a) Partial Credit: use of kinematics to find h_max: Correct equation for h_max.
- + 1 (a) Partial Credit: use of kinematics to find h_max: Correct equation for time (from velocity).
- + 2 (a) Solve for maximum height correctly
- + 4 (b) Recognize that mechanical energy is not conserved during the grab and compute the change in mechanical energy by finding the difference in kinetic energies before and after the grab.
- + 3 (b) All steps correct, with carry-over error from part (a)
- + 1 (b) Partial Credit: Correct initial energy
- + 1 (b) Partial Credit: Correct final energy
- **+ 2 (c) Using physical reasoning and no math, correctly state the limiting behavior of the answer to part (a) in the given limits.**
- + 1 (c) Partial Credit: Using physical reasoning and no math, correctly reason that as M/m->0, h_max=h.
- + 1 (c) Partial Credit: Using physical reasoning and no math, correctly reason that as m/M->0, h_max = (v_0)^2/(2g).
- **+ 2 (c) Show that the mathematical answer agrees with the expected limiting behavior.**
- + 1 (c) Show that the mathematical answer agrees with

the expected limiting behavior. Correct use of M/m limit with answer from part (b).

+ 1 (c) Show that the mathematical answer agrees with the expected limiting behavior. Correct use of m/M limit with answer from part (b).

QUESTION 3

3 Problem 3 10 / 15

+ 3 **(a) Correct momentum conservation equation for the first burst.**

+ 3 **(a) Correct momentum conservation equation for the second burst.**

+ 2 **(a) Correct solution for the velocity after both bursts by simultaneously solving momentum conservation equations.**

+ 2 **(b) State or prove the correct expression for the velocity as a function of time for the rocket using the given speed variable.**

+ 1 (b) Correctly identify the initial and final mass of the rocket + fuel to solve for the final velocity of the rocket.

+ 2 (c) Correct calculation of final velocity in the case of two, consecutive mass bursts in the given limit.

+ 2 **(c) Correct calculation of the final velocity in the case of continuous mass flow in the given limits.**

+ 4 (d) Correct computation of Taylor expansion to first, non-vanishing order and correct statement of the relationship between the velocities in this limit.

- 2 **Point adjustment**

1/3 and 2/3 mass division, not half-half

Physics 1A - Winter 2016 Karen Li

Midterm Exam #2

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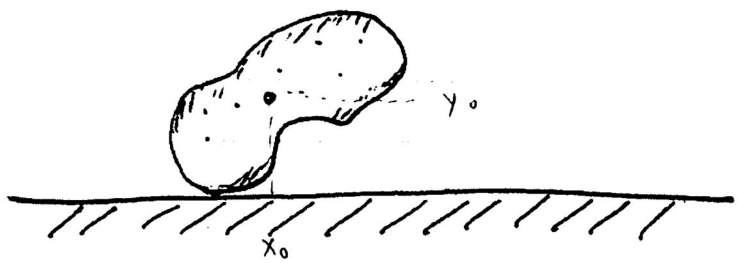
Advice. Your answers will be graded to a large extent on how convincing your reasoning is. A correct answer without good reasoning won't get much credit. Often convincing reasoning is a mixture of mathematics, explanations, and diagrams.

Problem 1. (6 points)

A perfectly smooth but irregularly-shaped rock is placed on a perfectly smooth, planar table on Earth as shown. The coefficient of friction between the rock and the surface is zero. For times $t < 0$, the rock is at rest in an unbalanced orientation, but at $t = 0$ it is released.

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} = M\vec{a}_{cm}$$



Describe the motion of the center of mass of the rock as a function of time for $t > 0$. Justify your description mathematically, with a diagram of the trajectory, and in words.

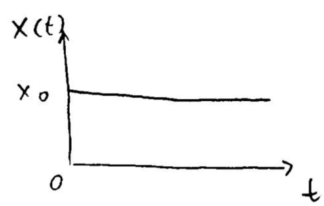
Since there are no external forces acting on the rock in the x-direction, the net, external force acting on the rock in the y-direction is the force of gravity minus normal force, which provides an acceleration downward with magnitude close to g. Thus, the x-position of the center of mass of the rock does not change and the y-position of the center of mass of the rock behaves similar to if the rock was in free fall.

x-direction motion: $x(t) = x_0$.

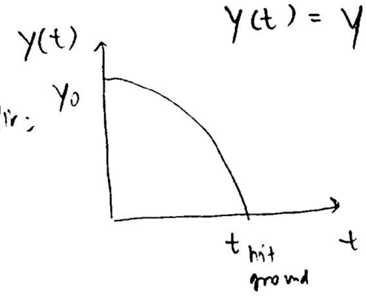
y-direction motion: $F_{net,y} = mg - F_N = ma \Rightarrow a = \frac{mg - F_N}{m}$

$$y(t) = y_0 - \frac{1}{2}\left(\frac{mg - F_N}{m}\right)t^2$$

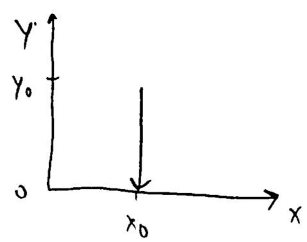
path in x-dir.



path in y-dir.



Trajectory



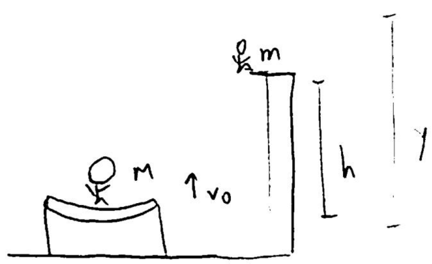
Problem 2. (15 points)

A circuis acrobat of mass M leaps straight up with initial velocity v_0 from a trampoline. As he rises up, he quickly takes a trained monkey of mass m off a perch at height h above the trampoline.

(a) What is the maximum height attained by the pair?

(b) Is mechanical energy of the acrobat-monkey system conserved in this process? If so, prove it. If not, compute the mechanical energy lost.

(c) **Extra Credit. (4 extra points possible)** What would you expect the answer to be in the limits $M/m \to 0$ and $m/M \to 0$? Does you mathematical answer agree with your expectations?



a) Since the work done by non-conservative forces is 0, total mechanical energy is conserved.

$E_0 = E_f \rightarrow U_1 + K_1 = U_2 + K_2$ ← v is 0 at top

$mgh + \frac{1}{2} M v_0^2 = (m+M) g y$

$y = \dfrac{mgh + \frac{1}{2} M v_0^2}{(m+M) g}$ (above the trampoline)

b) Mechanical energy is conserved in the acrobat-monkey system because the work done by non-conservative forces is zero. In this situation, the only force that does work is gravity when the acrobat and monkey are in the air and the spring force from the trampoline, and those are both conservative forces.

c) In the limit $M/m \to 0$, this means that the monkey is much heavier than the acrobat, so I would expect the max height to just be the height of the monkey's perch, h. In the limit $m/M \to 0$, this means that the mass of the acrobat is much larger so the mass of the monkey is insignificant so the max height would be $\frac{1}{2} M v_0^2 = Mgh$,

$h = \dfrac{\frac{1}{2} v_0^2}{g} = \dfrac{v_0^2}{2g}$. My mathematical answer agrees with my expectations because for $\lim M/m \to 0$, if I set $M = 0$, then $y = \dfrac{mgh}{mg} = h$.

In $\lim m/M \to 0$, if I set $m = 0$, then $y = \dfrac{\frac{1}{2} M v_0^2}{Mg} = \dfrac{v_0^2}{2g}$.

Extra Space

Problem 3. (15 points) rocket is in space, no gravity

Lonestar is a space rocket engineer attempting to design a rocket that can break the Andromeda rocket speed record. The rocket she designs has mass m_R and can hold a mass of fuel m_F. To break the record, Lonestar needs to launch her rocket from rest. She is considering the following propulsion strategies:

Strategy 1. The fuel is expelled in two mass bursts with the first burst expelling half as much mass as the second and with both bursts having speed u relative to the rocket.

Strategy 2. The fuel is expelled continuously at speed u relative to the rocket.

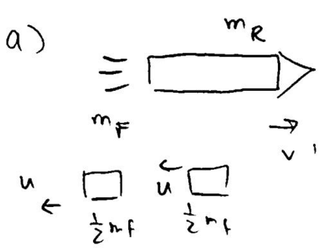
(a) What will be the final speed of the rocket for strategy 1?

(b) What will be the final speed of the rocket for strategy 2?

(c) If the mass of fuel m_F is much larger than the mass of the rocket m_R, then which strategy will yield a larger final rocket speed?

(d) **Extra Credit. (4 extra points possible)** Use Taylor expansions in the variable $x = m_F/m_R$ to determine the final rocket speed for each strategy when the mass of fuel m_R is much smaller than the mass of the rocket m_R. Which strategy yields a larger final rocket speed in this case?

a)



$F_{ext} = M(t)\frac{d\vec{v}(t)}{dt} - \frac{dM(t)}{dt}\vec{u}(t)$

0

Since net external force = 0, use conservation of momentum in the x-direction.

Speed of mass relative to inertial frame : $V = \vec{u} + \vec{V}_{after}$

① First mass burst

$\vec{P}_0 = \vec{P}_f \rightarrow \quad 0 = (\frac{m_f}{2})(-u + v_1) + (m_R + \frac{1}{2}m_f)v_1$

$0 = -\frac{1}{2}m_f u + \frac{1}{2}m_f v_1 + m_R v_1 + \frac{1}{2}m_f v_1$

$0 = -\frac{1}{2}m_f u + m_f v_1 + m_R v_1$

$v_1 = \dfrac{(\frac{1}{2}m_f u)}{m_f + m_R}$

② Second mass burst

$\vec{P}_0 = \vec{P}_f$

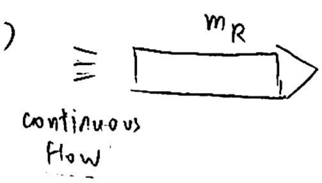
$(m_R + \frac{1}{2}m_f)v_1 = (\frac{1}{2}m_f)(-u + v_2) + (m_R)v_2$

$(m_R + \frac{1}{2}m_f)v_1 = -\frac{1}{2}m_f u + \frac{1}{2}m_f v_2 + m_R v_2$

$(m_R + \frac{1}{2}m_f)v_1 = -\frac{1}{2}m_f u + v_2(\frac{1}{2}m_f + m_R)$

$v_2 = \dfrac{(m_R + \frac{1}{2}m_f)v_1 + \frac{1}{2}m_f u}{(\frac{1}{2}m_f + m_R)} = v_1 + \dfrac{(\frac{1}{2}m_f u)}{(\frac{1}{2}m_f + M_R)} = \left(\dfrac{(\frac{1}{2}m_f u)}{m_f + m_R}\right) + \dfrac{(\frac{1}{2}m_f u)}{(\frac{1}{2}m_f + m_R)}$

$v_2 = \frac{1}{2}m_f u\left(\dfrac{1}{m_f + m_R} + \dfrac{1}{\frac{1}{2}m_f + m_R}\right)$

b) 

$$F_{ext} = M(t)\frac{d\vec{v}}{dt}(t) - \frac{dM}{dt}(t)\,\vec{u}(t)$$

$$0 = M(t)\frac{d\vec{v}}{dt}(t) - \frac{dM}{dt}(t)\,\vec{v}(t)$$

$$\frac{dM}{dt}(t)\cdot u = M(t)\frac{d\vec{v}}{dt}(t)$$

$$\frac{\left(\frac{dM}{dt}(t)\cdot u\right)}{M(t)} = \frac{d\vec{v}}{dt}(t)$$

$$\int\left(\frac{\frac{dM}{dt}(t)\cdot u}{M(t)}\right)dt = \int \frac{d\vec{v}}{dt}(t)\,dt$$

$$u\int_0^t \frac{d}{dt}(\ln(M(t)))\,dt = \int_0^t \frac{d\vec{v}(t)}{dt}\,dt$$

$M(t) = m_R$ at the end when all fuel gone

$$[u\ln(M(t))]_0^t = \vec{v}(t) - \cancel{\vec{v}(0)} \quad 0$$

$$\vec{v}(t) = u\ln(M(t)) - u\ln(M(0)) = u\ln\left(\frac{M(t)}{M(0)}\right)$$

$$\therefore V_f = u\ln\left(\frac{m_R}{m_R+m_f}\right)$$

c) For the strategy where you use two bursts of fuel, the final speed is

$$v_2 = \frac{\left(\frac{1}{2}m_f u\right)}{(m_f + m_r)} + \frac{\left(\frac{1}{2}m_f u\right)}{\left(\frac{1}{2}m_f + m_r\right)}$$

and if you take the limit $m_R/m_f \to 0$, then you get

$$v_2 = \frac{1}{2}u + u = \frac{3}{2}u$$

For the strategy where you have fuel continuously being expelled, when you take the limit $m_R/m_f \to 0$, then $V_f = u\ln\left(\frac{m_R}{m_R+m_f}\right)$ becomes $v_f = u\ln(0)$, so $v_f \to -\infty$. thus, strategy 2 will yield a larger final rocket speed.

d) Taylor expansion of a function:

$$T_n(c) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots \frac{f^n(c)}{n!}(x-c)^n$$

center

center $= m_F/m_R$

$\therefore$ Strategy 1:

Problem	Score
1	
2	
3	
Total	
Extra Credit 2	
Extra Credit 3	