Physics 1A - Lecture 2 final

Karen Li

TOTAL POINTS

77 / 84

QUESTION 1

1 Problem 1 13 / 13

- 1 Recognize that rotational analog of Newton's Second Law (or equivalent) is useful, and write it down.
- 1 Notice that if the submarine has maximum acceleration, then the propeller will spin with some constant, maximum angular velocity.
- 1 Use the fact that the problem stated max acceleration is g/4 and the acceleration expression given in the question to write an equation that can be solved for omega_max.
- 2 Notice that when the propeller reaches max angular velocity, its angular acceleration will be zero, and therefore the net torque will vanish by the rotational analog of NSL.
- 2 Notice that there are two sources of torque: the shaft and the water, and that these must sum to zero by the last observation.
- 3 Use the formula for f given in the question to determine an expression for the torque of the water on the shaft at maximum angular velocity
- 3 Combine various results above and solve for N.
- 3 Small math error in result
- 3 Wrong Torque
- 2 Small math error
- 10 Unclear, and does not look correct
- 0 Correct
- 13 Totally wrong
- **0 Point adjustment**

Correct

QUESTION 2

2 Problem 2 25 / 25

+ 2 a-i

+ 2 (a-ii) Recognize this is like free fall, so length is just natural length.

+ 2 (a-iii) Recognize that this is just like gravity being twice as strong as if stationary from the perspective of someone inside the elevator, so same as (i) with g replaced by 2g.

+ 2 (b) GIve some reasonably convincing argument that the tension will be less than the total weight.

+ 2 (c) NSL mass A

+ 2 (c) NSL mass B

+ 2 (c) NSL Pulley

+ 3 (c) Torque equation (rotational analog of NSL)

+ 2 (c) Constraint a_A, a_B, pulley

+ 2 Constraint relating acceleration and angular acceleration (tricky!)

+ 4 (c) Algebra to solve for length of spring.

+ 2 EXTRA CREDIT (d-i)

+ 2 EXTRA CREDIT (d-ii)

+ 2 EXTRA CREDIT (d-iii)

+ 2 EXTRA CREDIT (e)

+ 0 no points

QUESTION 3

3 Problem 3 12 / 12

- 3 (a) Recognize that gravity causes the net nonzero force to be zero in the z-direction during the flight of the bottle and then reason that this leads to non-conservation
- 2 (b) Recognize that there are no external forces in the x-y plane and conservation follows
- 4 (c) Recognize that normal and gravity in z-direction but argue that torques only in x-y plane, so conservation follows
- 3 (d) Recognize that gravity causes torque in z-direction while bottle flying so non-conservation follows

- **0** Correct

QUESTION 4

4 Problem 4 15 / 17

+ **1** **(a) recognize that the force from the falling water contributes to the measured weight**
+ **1** (a) recognize that "N > (M+m)g"
+ **8** **(b) full credit: correctly recognize each term in the mass flow equation or equivalent procedure leading to correct equation**
+ **3** (b) partial credit: recognize that scale reads normal force and try solving for it.
- **1** (b) adjustment: minor error (correct answer: N = (M + mt/T)g + m sqrt(2gh)/T)
+ **5** (b) partial credit: attempt made at recognizing each term in the mass flow equation or equivalent procedure leading to the correct equation (correct answer: N = (M + mt/T)g + m sqrt(2gh)/T)
+ **3** (c) full credit: Take t->T limit and interpret correctly.
+ **2** **(c) partial credit: attempt made at taking t->T limit and a meaningful interpretation**
+ **4** **(d) full credit: Notice that time t where reading is just weight exists and solve for the time by setting N = (m+M)g**
+ **2** (d) partial credit: Notice that time t where reading is just weight exists between t=0 and t=T.
- **1** (d) adjustment: minor error
+ **1** (d) partial credit: gave an answer of t>T with realistic physical interpretation that the scale will read (M+m)g with no more falling water contributing to the weight.

QUESTION 5

5 Problem 5 12 / 17

+ **5** (a) Full Credit: Convincing reasoning for the possible launch angle range to hit corner, (pi/2, pi/4)
+ **3** **(a) Partial Credit: Convincing reasoning for the possible launch angle range to hit corner, missed the range (pi/2, pi/4) or, got the range without enough reasoning**
+ **1** (a) Partial Credit: attempt made at reasoning, missed the range
+ **6** (b) Full Credit: Use kinematics equations for x- and y-directions, notice that x and y take special values at corner, solve for v_0
+ **5** (b) Partial Credit: Use kinematics equations for x- and y-directions, notice that x and y take special values at corner, solve for v_0 with minor math error
+ **3** **(b) Partial Credit: Use kinematics equations for x- and y-directions, notice that x and y take special values at corner, solve for v_0 with significant error and/or missing steps**
+ **1** (b) Partial Credit: Attempt made
+ **6** **(c) Full Credit: correct reasoning on what happens and if it makes sense at both limits**
+ **4** (c) Partial Credit: partially correct reasoning on what happens and if it makes sense at both limits
+ **1** (c) Partial Credit: attempt made

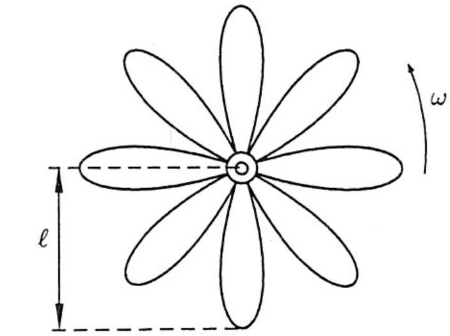
Physics 1A - Winter 2016
Lecture 2

Karen Li

FINAL EXAM

* neglect mass of the propellor blades

Problem 1.

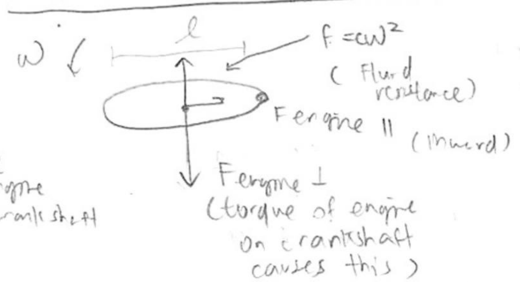


A submarine moves forward because it spins a propeller which displaces water backward. The propeller is spun by a crankshaft that is connected to the submarine's engine. The propeller has N blades (for example in the diagram $N = 8$) of length ℓ. Because of fluid resistance, the water exerts an angular speed-dependent force $f = c\omega^2$ on each blade, tangent to the circle in which the blade is rotating. You can treat this force as though it's acting at the center of each blade. When the propeller spins, the resulting magnitude a of the ship's acceleration is related to the angular speed ω of the propeller as $a = gN\omega/\omega_0$ where ω_0 is a given constant.

If τ is the magnitude of the maximum torque the engine can exert on the crankshaft, and if this torque gives the submarine a maximum acceleration of $g/4$, how many blades does its propeller have?

$+z = \odot$ FBD left horizontal blade

τ = max torque of engine on crankshaft



① $\tau_{net_z} = I\alpha_z$

For one blade, the torque that the engine causes on it is $\frac{\tau}{N}$.

$$\frac{\tau}{N} - f\left(\frac{\ell}{2}\right) = I\alpha_z$$

$$\frac{\tau}{N} - \frac{c\omega^2\ell}{2} = I\alpha_z$$

Since we can neglect the mass of the propellor blades, then $I = 0$

② $$\frac{\tau}{N} - \frac{c\omega^2\ell}{2} = (0)\alpha_z$$

$$\frac{\tau}{N} = \frac{c\omega^2\ell}{2}$$

$$\omega^2 = \frac{2\tau}{Nc\ell} \rightarrow \omega = \sqrt{\frac{2\tau}{Nc\ell}}$$

③ $$a = gN\omega/\omega_0$$

$$\frac{g}{4} = \frac{gN\left(\sqrt{\frac{2\tau}{Nc\ell}}\right)}{\omega_0}$$

$$\frac{\omega_0}{4} = N\sqrt{\frac{2\tau}{Nc\ell}}$$

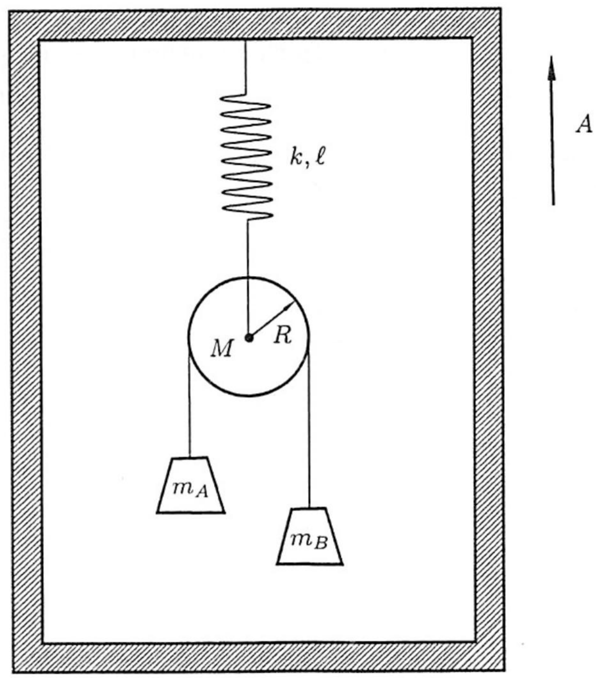
$$\frac{\omega_0}{4} = \sqrt{N}\cdot\sqrt{\frac{2\tau}{c\ell}}$$

$$\sqrt{N} = \frac{\omega_0\sqrt{c\ell}}{4\sqrt{2\tau}} \rightarrow N = \frac{\omega_0^2 c\ell}{16(2\tau)} = \frac{\omega_0^2 c\ell}{32\tau}$$

Extra Space

Extra Space

Problem 2.



An Atwood's machine with a pulley in the shape of a uniform disk of mass M and radius R is in an elevator having vertical acceleration A relative to the ground. Let positive A correspond to the elevator accelerating upward. Mass m_A hangs on the left side of the pulley while mass m_B hangs on the right. The rope connecting masses m_A and m_B is massless. The pulley is suspended from the ceiling of the elevator by a spring of spring constant k and natural length ℓ.

(a) Consider the special case $m_A = m_B$. What would you expect the length of the spring to be in the following limits:

i. $A \to 0$

ii. $A \to -g$

iii. $A \to g$

Justify your answers using physical reasoning and minimal math if possible.

(b) When $m_A \neq m_B$ but $A = 0$, would you expect the tension in the spring to be less than, equal to, or greater than the weight of the pulley plus the weight of the masses hanging from the pulley? Justify your answers using physical reasoning and minimal math if possible.

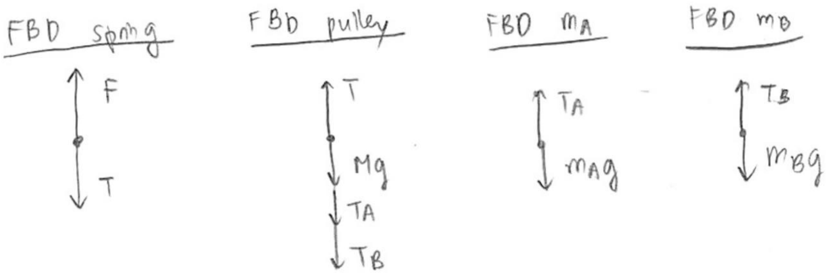
$mgd = \frac{1}{2}kd^2$

$2mg = kd$ $\quad d = \frac{2mg}{k}$

(c) Determine a general expression for the length of the spring in terms of the given variables.

(d) Does your mathematical answer from part (c) agree with your answers from part (a) in each special case? If so, demonstrate this mathematically for each case. If not, you may want to re-evaluate your intuition, or your mathematical answer, or both.

(e) Does your mathematical answer from part (c) agree with your answer from part (b)? If so, demonstrate this mathematically for each case. If not, you may want to re-evaluate your intuition, or your mathematical answer, or both.



a) i) If $m_A = m_B$ and $A \to 0$, this means that the elevator is not accelerating and neither are the masses because $m_A = m_B$. Thus, the spring only has to support the weights of the two masses and the pulley. So, since

$F_{spring} = -kx$ and we let $m = m_A = m_B$, then ...

$\rightarrow 2(2m + M)g = -kx$ $\quad$ ($+x$ is upward)

spring pulled from both ends by equal force $\quad x = \frac{2(2m+M)g}{-k}$

so the length of the spring is $\ell + (-x) \Rightarrow L = \ell + \frac{2(2m+M)g}{k}$

ii) If $m_A = m_B = m$ and $A \to -g$, then the elevator is accelerating downward at the rate of g so it's like the whole system is in free fall, and so, the spring does not have to exert any force on the pulley and it will have its natural length. $L = \ell$

iii) If $m_A = m_B = m$ and $A \to g$, then the spring will be responsible for providing a force to accelerate the pulley and the masses at a rate of g.

Thus, $F_{spring} = \underbrace{2(2(m+M)g}_{\text{the weight of the pulley}} + \underbrace{2(m+M)(g))}_{\text{force to accelerate @ rate of } g} \rightarrow -kx = 8(m+M)g$

$x = \frac{8(m+M)g}{-k}$

So the length of the spring is

$L = \ell + \frac{8(m+M)g}{k}$

greater?

b) When $m_A \neq m_B$ but $A = 0$, the tension in the spring would be ___ than the weight of the pulley plus the weight of the masses hanging from the pulley because

c) <u>Newton's Second Law</u>

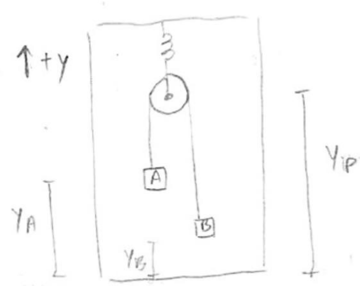
① $F - T = m_{spring}\, a_{spring}$

② $T - Mg - T_A - T_B = M a_{pulley}$

③ $T_A - m_A g = m_A a_{A,y}$

④ $T_B - m_B g = m_B a_{B,y}$

<u>Constraints</u>



Length of the rope is constant, so ...

$$(y_P - y_A) + (y_P - y_B) = \text{const.}$$

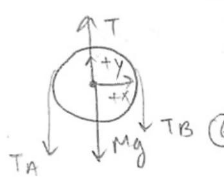
$$\ddot{y}_P - \ddot{y}_A + \ddot{y}_P - \ddot{y}_B = 0$$

$$2\ddot{y}_P = \ddot{y}_A + \ddot{y}_B$$

⑤ $2\, a_{pulley} = a_{A,y} + a_{B,y}$

<u>Torques</u>

Pulley:



Let $+z = \odot$ (out of page)

$\tau_{net,z} = I\alpha_z$

⑥ $T_A R - T_B R = I\alpha_z$

$\hookrightarrow I = \frac{1}{2}MR^2$

<u>Torque Constraint</u>

$a_T = r\alpha$

⑦ $a_{B,y} = R\alpha_z$

$\alpha_z = \dfrac{a_{B,y}}{R}$

$\rightarrow \quad T_A R - T_B R = \left(\frac{1}{2}MR^2\right)\left(\frac{a_{B,y}}{R}\right)$

$T_A R - T_B R = \dfrac{MR\,a_{B,y}}{2}$

<u>Clean Up the Equations</u>

$F - T = m_{spring}\, a_{spring} \rightarrow F - T = (0)\, a_{spring}$

① $F = T$

② $T - Mg - T_A - T_B = Ma_P$

③ $T_A - m_A g = m_A a_{A,y}$

④ $T_B - m_B g = m_B a_{B,y}$

⑤ $T_A R - T_B R = \frac{1}{2}MR\,a_{B,y}$

⑥ $2a_P = a_{A,y} + a_{B,y}$

⑦ $a_P = A$

7 unknowns, 6 eq's so need one more eq.

If the elevator is accelerating upward at rate of A, then the acceleration of the pulley must also be A since once the spring stretches, its length becomes constant as the elevator moves up.

Necessary Equations

1. $T - Mg - T_A - T_B = Ma_p$
2. $T_A - m_A g = m_A a_{A,y}$
3. $T_B - m_B g = m_B a_{B,y}$
4. $T_A R - T_B R = \frac{1}{2} M R a_{B,y}$
5. $2a_p = a_{A,y} + a_{B,y}$
6. $a_p = A$

$\boxed{\text{solve for } T}$

$T - Mg - T_A - T_B = MA \rightarrow T = Mg + T_A + T_B + MA$

$T_A - m_A g = m_A a_{A,y} \rightarrow T_A = m_A a_{A,y} + m_A g$

$T_B - m_B g = m_B a_{B,y} \rightarrow T_B = m_B a_{B,y} + m_B g$ } subtract

$T_A \cancel{R} - T_B \cancel{R} = \frac{1}{2} M \cancel{R} a_{B,y} \rightarrow T_A - T_B = \frac{1}{2} M a_{B,y}$

$2A = a_{A,y} + a_{B,y}$

$\downarrow$

$a_{A,y} = 2A - a_{B,y}$

equal { ① $T_A - T_B = \frac{1}{2} M a_{B,y}$

② $T_A - T_B = m_A a_{A,y} + m_A g - m_B a_{B,y} - m_B g$

$\frac{1}{2} M a_{B,y} = m_A a_{A,y} + m_A g - m_B a_{B,y} - m_B g$

$\frac{1}{2} M a_{B,y} + m_B a_{B,y} = m_A a_{A,y} + m_A g - m_B g$

$\frac{1}{2} M a_{B,y} + m_B a_{B,y} = m_A (2A - a_{B,y}) + m_A g - m_B g$

$\frac{1}{2} M a_{B,y} + m_B a_{B,y} + m_A a_{B,y} = 2 m_A A + m_A g - m_B g$

$$a_{B,y} = \frac{2 m_A A + m_A g - m_B g}{(\frac{1}{2} M + m_B + m_A)}$$

$$a_{A,y} = 2A - \frac{2 m_A A + m_A g - m_B g}{(\frac{1}{2} M + m_B + m_A)}$$

$$T_A = m_A \left(2A - \frac{2 m_A A + m_A g - m_B g}{\frac{1}{2} M + m_B + m_A} \right) + m_A g$$

$$T_A = m_A \left(2A - \frac{2 m_A A + m_A g - \cancel{m_B g} + \frac{1}{2} M g + \cancel{m_B g} + m_A g}{\frac{1}{2} M + m_B + m_A} \right)$$

$$T_A = m_A \left(2A - \frac{2 m_A A + 2 m_A g + \frac{1}{2} M g}{\frac{1}{2} M + m_B + m_A} \right)$$

$+ \cancel{m_B g} + m_A g)$ ✓

$$T_B = m_B \left(\frac{2 m_A A + m_A g - m_B g}{\frac{1}{2} M + m_B + m_A} + g \right) = m_B \left(\frac{2 m_A A + m_A g - \cancel{m_B g} + \frac{1}{2} M g}{\frac{1}{2} M + m_B + m_A} \right)$$

$$T_B = m_B \left(\frac{2 m_A A + 2 m_A g + \frac{1}{2} M g}{\frac{1}{2} M + m_B + m_A} \right)$$

$$T = Mg + m_A \left(2A - \frac{2 m_A A + 2 m_A g + \frac{1}{2} M g}{\frac{1}{2} M + m_B + m_A} \right) + m_B \left(\frac{2 m_A A + m_A g + \frac{1}{2} M g}{\frac{1}{2} M + m_B + m_A} \right) + MA$$

★ CONTINUED ON VERY BACK OF TEST ★

Problem 3.

Nancy is spinning on an ice rink (effectively frictionless) at a certain angular speed ω. Josh throws her a water bottle, and she catches it. Let the word "system" refer to the bottle + Nancy. Let the z-direction point vertically, away from the ice and perpendicular to it.

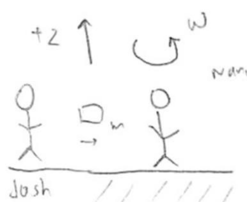
For all of the following questions, consider the time interval from the moment just after Josh lets the water bottle go, to the moment just after Nancy catches it.

(a) Is the total linear momentum of the system conserved in the z-direction? Justify mathematically, and explain the math in words.

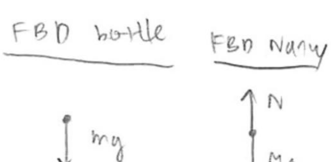
(b) Is the total linear momentum of the system conserved in the x-y-direction? Justify mathematically, and explain the math in words.

(c) Is the total angular momentum of the system conserved in the z-direction? Justify mathematically, and explain the math in words.

(d) Is the total angular momentum of the system conserved in the x-y-direction? Justify mathematically, and explain the math in words.



System = bottle + Nancy

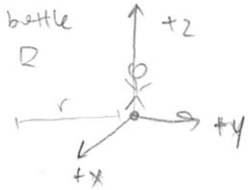


a) Total linear momentum is conserved if the net external force is 0. In the z-direction, there is gravity (an external force) and normal force on Nancy and just gravity on the bottle. For Nancy, the normal force and gravity cancel out because $N - Mg = Ma_y \rightarrow N - Mg = M(0) \rightarrow N = Mg$ but there is the net force due to gravity on the bottle $= mg = ma_{y,\text{bottle}}$ so no, the total linear momentum of the bottle + Nancy system is not conserved in the z-direction since $\vec{F}_{\text{net external},z} = \frac{dP_z}{dt}$.

b) The only external forces acting on the bottle and Nancy during the specified time interval are the normal force from the ice on Nancy and the force of gravity on both Nancy and the bottle. Since there are no external forces in the x-y direction (Nancy grabbing the bottle is internal force), then the total linear momentum is conserved in the x-y direction, since $\vec{F}_{\text{net},xy\text{ external}} = \frac{d\vec{P}_{xy}}{dt}$ and $\vec{F}_{\text{net},xy\text{ external}} = 0$ so $\frac{d\vec{P}_{xy}}{dt} = 0$.

c) Total angular momentum is conserved if the net external torque on a system is conserved. Again, since the only external forces (normal force and gravity) are in the z-direction, then since $\vec{\tau} = \vec{r} \times \vec{F}$, based on the properties of the cross product, there can be no extegnal torque in the z-direction because $\vec{F}$ is already in the z-direction. Thus, total angular momentum is conserved in the z-direction since $\tau_{\text{net},z} = \frac{d\vec{L}}{dz} = 0$.

d) Nancy + bottle



If we choose our origin to be at the point where Nancy touches the ice, then we see that the force of gravity and the normal force both act in the z-direction. The radius is in the y-direction. Thus, if you take the cross product of the radius and force of gravity on the water bottle,

$$\vec{\tau}_{net,x} = \vec{r} \times \vec{F} = (-r\hat{j}) \times (-mg\hat{k}) = mgr\hat{i}$$

Thus, since when the bottle is flying towards Nancy there is a net external force due to gravity in the z direction and radius is in the $\hat{j}$ direction, then there is a net external torque in the x direction. Thus, the total angular momentum is not conserved in the xy-direction because $\vec{\tau}_{net, external, xy} = \frac{d\vec{L}_{xy}}{dt} \neq 0$.

Extra Space

Problem 4.

A large-diameter bowl of mass M sits on a cooking scale. A small amount of water of mass m is slowly poured into the bowl out of a cup from a height h above the bottom of the bowl. It takes a time T from the moment when the water first hits the bottom of the bowl to the moment when all of the water has flowed in. The water flows in at a constant mass per unit time. The bowl starts empty. Let $t = 0$ be the moment at which the water first strikes the bowl.

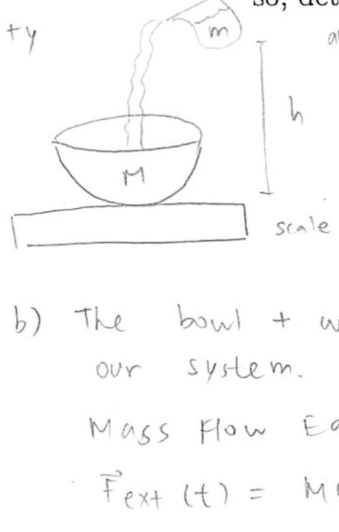
(a) As t approaches T from values less than T, will the scale show a weight less than, equal to, or greater than $(M + m)g$? Use physical reasoning and minimal math if possible.

(b) Determine an expression for the weight as a function of time that the scale reads from $t = 0$ until the moment when the last of the water strikes the bowl?

(c) Does your answer in part (b) agree with your answer in part (a)? Explain.

(d) Is there a time when the scale reads a weight $(M + m)g$? If not, explain why not. If so, determine an expression for this time.

$+y$



all water done pouring @ $t = T$

Let rate of water flowing into bowl be f (units of mass over time) $f = \frac{m}{T}$

a) As t approaches T from values less than T, the scale will show a weight less than $(M+m)g$ because $(M+m)g$ is the total weight of the bowl and all of the water, and when $t < T$, not all of the water is in the bowl so the scale will show a weight less than $(M+m)g$.

b) The bowl + water already inside is our system.

Mass Flow Equation:

$$\vec{F}_{ext}(t) = M(t)\frac{d\vec{v}}{dt} - \frac{dM(t)}{dt}\cdot\vec{u}(t)$$

Consider only forces / motion in the y-dir. The net external force is just the F_{scale} and the force of gravity (on bowl & water already inside).

$\frac{d\vec{v}(t)}{dt}$ = acceleration $= 0$ because bowl is not moving in the y-direction.

FBD bowl

N_{scale}

Mg

$m_{water}g$

FBD water

mg

$$N_{scale} - Mg - \underbrace{\left(\frac{m}{T}\right)(t)}_{\text{mass of water @ time } t \text{ in bowl}}g = M(t)(0) - \underbrace{\left(\frac{m}{T}\right)}_{+ \text{ b/c mass flowing IN}}\cdot\vec{u}(t)$$

speed of water relative to the system.

Solve for $\vec{u}(t)$ using kinematics.

$$v_f^2 = \underbrace{v_0^2}_{0} + 2ad \rightarrow v_f^2 = 2ad \rightarrow v_f = \sqrt{2(-g)(-h)} = \sqrt{2gh}$$

since falling down, $\vec{u}(t) = -\sqrt{2gh}$

$d = v_0 t + \frac{1}{2} a t^2$

$h = \frac{1}{2} a t^2$

$v_f = v_0 + at$

$v_f = -gt$

Extra Space

$$N_{scale} - Mg - \left(\frac{m}{T}\right)(t)\, g = -\left(\frac{m}{T}\right)(-\sqrt{2gh})$$

$$N_{scale} = Mg + \frac{mt}{T} g + \frac{m}{T}\sqrt{2gh}$$

c) Yes, my answer from part b) agrees with my answer from part c) because when $t < T$ (eg) when $\frac{t}{T} \to 0$, the scale shows a weight less than $(M+m)g$.

(eg) $N_{scale} = Mg + m(0)g + \frac{m}{T}\sqrt{2gh}$

$$N_{scale} = Mg + \underbrace{\frac{m}{T}\sqrt{2gh}}$$

$$\frac{\sqrt{2gh}}{T} < g \qquad \text{because} \quad d = \overset{0}{v_0 t} + \frac{1}{2} a t^2$$

$$h = \frac{1}{2} g t^2$$

$$\frac{\sqrt{2h}}{T\sqrt{g}}\, g < g$$

$$\frac{\sqrt{2h}}{T\sqrt{g}} < 1 \rightarrow \frac{\sqrt{2\left(\frac{1}{2} g t_{fall}^2\right)}}{T\sqrt{g}} = \frac{\sqrt{g} \cdot t_{fall}}{T\sqrt{g}}$$

$$\therefore \underbrace{\frac{t_{fall}}{T} < 1}_{\text{true}}$$

d) $N_{scale} = Mg + \frac{mt}{T} g + \frac{m}{T}\sqrt{2gh}$

$$(M+m)g = Mg + \frac{mt}{T} g + \frac{m}{T}\sqrt{2gh}$$

$$\cancel{Mg} + mg = \cancel{Mg} + \frac{mt}{T} g + \frac{m}{T}\sqrt{2gh}$$

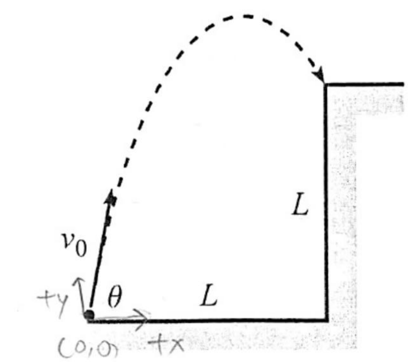
$$\frac{\cancel{m}t}{T} g = \cancel{m}g - \frac{\cancel{m}}{T}\sqrt{2gh}$$

$$t = \frac{\left(g - \frac{\sqrt{2gh}}{T}\right)T}{g} = \frac{(Tg - \sqrt{2gh})}{g} = \left(T - \sqrt{\frac{2h}{g}}\right)$$

$$\therefore t = T - \sqrt{\frac{2h}{g}}$$

Extra Space

Problem 5.



Identity

$\sin 2\theta = 2\sin\theta\cos\theta$

A small ball is launched from the ground onto the corner of a cliff as shown.

(a) For which launch angles θ in the range $[0, \pi]$ is it possible for the ball to hit the corner? Explain in the most convincing way you can.

(b) For a given angle θ that does allow the ball to hit the corner, what launch speed v_0 is necessary for the ball to precisely hit the corner?

(c) Examine the limiting cases $\theta \to \pi/2$ and $\theta \to \pi/4$. What happens to the required launch speed in each of these cases according to the formula you derived from part (b)? Does the behavior of your formula make sense in these limiting cases? Explain with physical reasoning.

a) If $\theta \geq \frac{\pi}{2}$, then the ball will definitely not hit the corner because the ball will go to the left if $\theta > \frac{\pi}{2}$ or it will go straight up for $\theta = \frac{\pi}{2}$.

part a) continued above

b) x-dir.: no acceleration so $x(t) = v_0 \cos\theta\, t$

y-dir: $y(t) = v_0 \sin\theta\,(t) - \frac{1}{2} g t^2$

we want $x(t) = L$ and $y(t) = L$. Thus, $L = v_0 \cos\theta\, t \rightarrow t = \frac{L}{v_0 \cos\theta}$

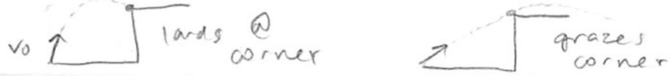
$$L = v_0 \sin\theta \left(\frac{L}{v_0\cos\theta}\right) - \frac{1}{2} g \left(\frac{L}{v_0 \cos\theta}\right)^2$$

$$L = L\tan\theta - \frac{gL^2}{2v_0^2\cos^2\theta} \rightarrow 1 = \tan\theta - \frac{gL}{2v_0^2\cos^2\theta} = 1 = \frac{2v_0^2\cos^2\theta\tan\theta - gL}{2v_0^2\cos^2\theta}$$

$$1 = \frac{2v_0^2\cos^2\theta\left(\frac{\sin\theta}{\cos\theta}\right) - gL}{2v_0^2\cos^2\theta} \Rightarrow 1 = \frac{2v_0^2\sin\theta\cos\theta - gL}{2v_0^2\cos^2\theta} = \frac{v_0^2\sin 2\theta - gL}{2v_0^2\cos^2\theta}$$

$$\therefore\ 2v_0^2\cos^2\theta = v_0^2\sin 2\theta - gL$$

It's possible for the ball to hit the corner if the value of θ satisfies the equation $2v_0^2\cos^2\theta = v_0^2\sin 2\theta - gL$. There are multiple values for θ because the ball could land @ the corner or just graze it. As well, θ changes because v_0 can change.



b) Using math from part a) ...

$$2v_0^2\cos^2\theta = v_0^2\sin 2\theta - gL$$

$$2v_0^2\cos^2\theta - v_0^2\sin 2\theta = -gL$$

$$v_0^2(2\cos^2\theta - \sin 2\theta) = -gL$$

$$v_0^2 = \frac{-gL}{2\cos^2\theta - \sin 2\theta} \rightarrow v_0 = \sqrt{\frac{gL}{\sin 2\theta - 2\cos^2\theta}}$$

c) when $\theta \rightarrow \frac{\pi}{2}$, the ball is travelling almost directly vertically upward.

$$\lim_{\theta \to \frac{\pi}{2}} v_0(\theta) = \sqrt{\frac{gL}{\sin 2(\frac{\pi}{2}) - 2\cos^2(\frac{\pi}{2})}} = \sqrt{\frac{gL}{\sin(\pi) - 2\cos^2(\frac{\pi}{2})}} = \sqrt{\frac{gL}{(0) - 2(0)^2}}$$

$$\lim_{\theta \to \frac{\pi}{2}} v_0(\theta) = \sqrt{\frac{gL}{0}} \rightarrow +\infty$$

This makes sense because if the ball were travelling almost directly upward, it would need to have ∞ large velocity so it would have ∞ time in the air to compensate for the fact that v_0 in the x-direction is so small since $v_{0,x} = v_0\cos\theta$, and $\cos\theta \rightarrow 0$ if $\theta \rightarrow \frac{\pi}{2}$.

When $\theta \rightarrow \frac{\pi}{4}$, the ball is travelling directly at the corner but is being pulled down due to gravity.

$$\lim_{\theta \to \frac{\pi}{4}} v_0(\theta) = \sqrt{\frac{gL}{\sin(\frac{\pi}{2}) - 2\cos^2(\frac{\pi}{4})}} = \sqrt{\frac{gL}{(1) - 2(\frac{1}{\sqrt{2}})^2}} = \sqrt{\frac{gL}{1 - 2(\frac{1}{2})}} = \sqrt{\frac{gL}{0}}$$

$$\lim_{\theta \to \frac{\pi}{4}} v_0(\theta) \Rightarrow +\infty$$

This also makes sense because since the trajectory of the ball if there ~~were~~ no gravity would directly hit the corner, since there is gravity, it is impossible for the ball to hit the corner since gravity is pulling it down UNLESS the initial velocity is so large (eg. approaches ∞) such that the time that the ball is in the air approaches 0 so gravity does not have enough time to pull the ball down from its gravityless trajectory.

Extra Space

Since by Newton's 3rd law, $F_{spring\ on\ pulley} = F_{pulley\ on\ spring}$

Then $2T = F_{spring}$ since spring being pulled from both ends (top and bottom)

$$kx = 2T \rightarrow x = \frac{2T}{k}$$

$\therefore$ Length $= l + \frac{2T}{k}$ } plug in funky expression for T

d) $\underline{A=0}$ $\quad \underline{m_A = m_B = m}$

$$T = Mg + m\left(0 - \frac{2mg + \frac{1}{2}Mg}{\frac{1}{2}M + 2m}\right) + m\left(\frac{0 + mg + \frac{1}{2}Mg}{\frac{1}{2}M + 2m}\right)$$

$$T = Mg + m(-g) + m(g) = Mg$$

* messed up algebra somewhere