Physics 1A - Lecture 3 Final

Zhiyuan Yang

TOTAL POINTS

99 / 113

QUESTION 1

1 Problem 1 24 / 24

- - 1 (a) Idea to use strategy of finding acceleration and using this to determine velocity at bottom.
- - 3 (a) FBD x-direction and resulting equation
- - 3 (a) FBD y-direction and resulting equation
- - 1 (a) Relationship between friction and normal force
- - 4 (a) Kinematics + algebra to solve for final velocity as function of angle or equivalent
- - 3 (a) Argument that statement 1 is true appealing to math.
- - 4 (b) Energy conservation + equation
- - 2 (b) Solve for final speed
- - 3 (c) Take mathematical limit and discuss
- **- 0 Point adjustment**

QUESTION 2

2 Problem 2 26 / 27

- - 3 (a) Reasonable argument that mass m_A will free fall
- - 4 (b) FBD + NSL equation mass A
- - 4 (b) FBD + NSL equation mass B
- - 4 (b) FBD + NSL equation for bottom pulley
- - 4 (b) Constraint relating accelerations of both masses (can be obtained via combination of more than one constraint as in solution)
- **- 3 (b) Algebra and solve for a_{A,y}**
- - 3 (c) Plug in and solve for tension
- - 2 (d) Take limit and compare to prediction
- **+ 2 Point adjustment**

QUESTION 3

3 Problem 3 21 / 24

- + 3 (a) Convincing argument that block on string will require more initial speed.
- **+ 2 (b) Momentum conservation**
- **+ 3 (b) Mechanical energy conservation**
- **+ 3 (b) force analysis at top with recognition of condition on tension to just make it around**
- **+ 3 (b) Solve for desired speed**
- **+ 3 (c) Angular momentum conseravation (linear momentum also works)**
- **+ 3 (c) Mechanical energy conservation with recognition that velocity zero at top to just make it around circle**
- **+ 2 (c) Solve for desired speed**
- **+ 2 (d) Determine from math with speed is greater and comment on whether agrees with prediction**
- + 1 (a) plausible but incorrect reasoning (partial credit)
- + 2 (b) speed solved but with arithmetic error
- + 1 (c) speed solved but with arithmetic error
- + 2 energy conservation but used incorrectly
- + 2 momentum conservation used incorrectly
- + 0 no credit

no justification for part a, 0 credit

QUESTION 4

4 Problem 4 14 / 24

- **+ 3 (a.1) horizontal total force is zero (no external horizontal force actually)**
- **+ 1 (a.2) vertical net forces is close to zero, as long as the system center of mass doesn't change much in the vertical direction**
- **+ 3 (b) Argue stays at rest using momentum conservation and initial condition**
- + 3 (c) Argue net external torque in parallel-direction zero since external forces are vertical, so ang. mom. in that direction conserved
- **+ 3 (d.1) angular momentum conservation equation**

+ **2** (d.2) the correct result (c+1)w

+ **3** (e.1) Compute kinetic energy change result expression correctly

+ **2** (e.2) give reasonable argument that why the energy increases

+ **4** **(f) when c goes to zero, all the changes are negligible. relate math with physics.**

+ **0** zero

QUESTION 5

5 Problem 5 **14 / 14**

+ **5** **(a) Kinematics or equivalent argument to obtain the correct range expression and simplify**

+ **3** **(b) Compute correct Taylor expansion coefficients and put together to write answer.**

+ **6** **(c)**

+ **4** (c) setup correct, but arithmetic errors

+ **4** (a) correct setup, but arithmetic errors/missing coefficients

+ **2** (b) unsimplified or arithmetic errors

+ **3** (c) half credit for giving two answers/good attempt

+ **2** (a) incorrect attempt

+ **1** (b) incorrect attempt

+ **2** (c) qualitative attempt/incorrect attempt

+ **5** (c) correct but with minor arithmetic error

+ **0** no credit

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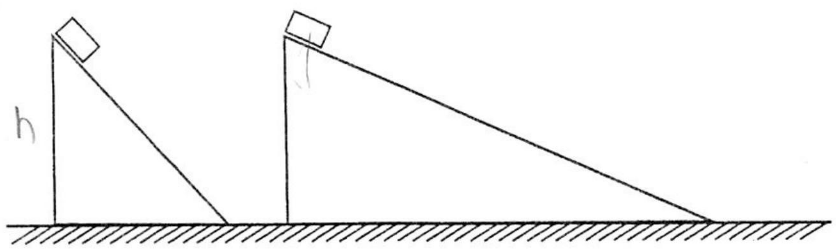
Physics 1A - Winter 2016
Lecture 3

FINAL EXAM

Problem 1.



Two identical boxes slide down different ramps pictured above having started at rest from their tops. The boxes start at the same height. The coefficient of kinetic friction μ_k between the boxes and the ramps is the same in both cases, and both ramps are fixed to the ground. The coefficient of static friction is not large enough to prevent the blocks from sliding down the ramps.

Let box A be the box on the left, and let box B be the box on the right. Consider the following statements:

I. The speed of box A is greater than the speed of box B when they reach the bottoms of their ramps.

II. The speed of box A is less than the speed of box B when they reach the bottoms of their ramps.

III. The speed of box A is the same as the speed of box B when they reach the bottoms of their ramps.

Questions.

(a) Use force methods (energy methods not allowed!) to determine which if these statements is true when $\mu_k \neq 0$. You'll find it useful to name your own relevant variables so that you can answer this question mathematically.

(b) Use energy methods to determine which if these statements is true when $\mu_k \neq 0$. You'll find it useful to name your own relevant variables so that you can answer this question mathematically.

(c) Does the answer change when $\mu_k = 0$? Justify using your answers from the previous parts.

(a) let a_A be acceleration of A, a_B be acceleration of B. They both have mass m.

Extra Space

The left ramp has angle θ_A, and the right one has θ_B.

For box A, we have $mg\sin\theta_A - \mu_k mg\cos\theta_A = ma_A$ ... ①

For B we have $mg\sin\theta_B - \mu_k mg\cos\theta_B = ma_B$ ... ②

from ①, ② we have $\begin{cases} a_A = g(\sin\theta_A - \mu_k\cos\theta_A) \\ a_B = g(\sin\theta_B - \mu_k\cos\theta_B) \end{cases}$

Also we know $0 < \theta_B < \theta_A < \frac{\pi}{2}$

when they reach the bottom,

$$V_A^2 = 2a_A x_A = 2a_A \frac{h}{\sin\theta_A} = \frac{2g(\sin\theta_A - \mu_k\cos\theta_A)h}{\sin\theta_A} = 2gh(1-\mu_k\cot\theta_A),$$

$$V_B^2 = 2a_B x_B = 2a_B \frac{h}{\sin\theta_B} = \frac{2g(\sin\theta_B - \mu_k\cos\theta_B)h}{\sin\theta_B} = 2gh(1-\mu_k\cot\theta_B)$$

$\because \cot\theta_B > \cot\theta_A \Rightarrow 1-\mu_k\cot\theta_A > 1-\mu_k\cot\theta_B$

$\therefore V_A^2 > V_B^2 \Rightarrow V_A > V_B$ when they reach the bottoms.

I is correct.

(b) Using energy methods:

for A: $mgh = \frac{1}{2}mV_A^2 + f_A x_A$

for B: $mgh = \frac{1}{2}mV_B^2 + f_B x_B$

$$\Rightarrow V_A^2 = \frac{(mgh - f_A x_A)2}{m} = 2\left(gh - \frac{\mu_k g\cos\theta_A h}{\sin\theta_A}\right)$$

$$= 2gh(1-\mu_k\cot\theta_A)$$

$$\Rightarrow V_B^2 = 2gh(1-\mu_k\cot\theta_B).$$

Again, $\cot\theta_A < \cot\theta_B$

$\therefore V_A^2 > V_B^2 \Rightarrow \underline{V_A > V_B}$ (I) is still true!

(c) if $\mu_K = 0$,

using energy methods we have

$$mgh = \frac{1}{2} m V_A^2$$

$$mgh = \frac{1}{2} m V_B^2,$$

because mechanic energy is conserved in this case.

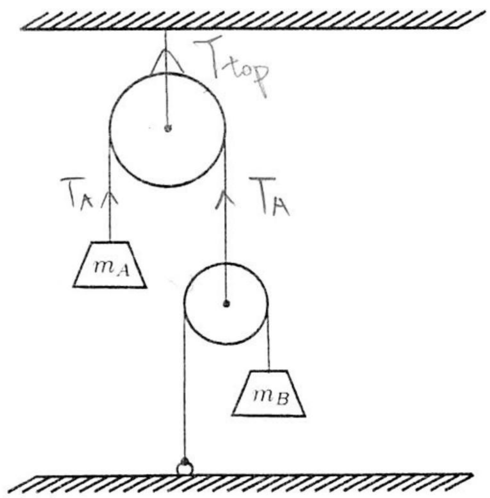
So $V_A = V_B$ when they reach the bottoms,

when $\mu_K = 0$

Extra Space

$h - x_A + h - x_B = L$

Problem 2.



Consider the apparatus above. All pulleys and ropes are massless.

(a) In limit $m_B \to 0$, would you expect the magnitude of the acceleration of mass A to be greater than, equal to, or less than g? Explain using physical reasoning.

(b) Determine an expression for the acceleration of mass A in terms of the given variables.

(c) Determine an expression for the tension in the top rope in terms of the given variables.

(d) Does your mathematical answer in part (b) agree with your answer in part (a)? Explicitly verify this mathematically. If the answers don't agree, you should consider re-evaluating either your math, or your intuition, or both.

(a) We name the tension of the rope of the upper pully to be T_A, similarly tention of the lower rope T_B.

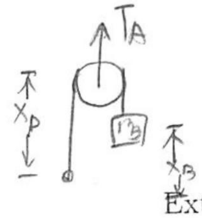
when $m_B \to 0$, $T_B = 0$ since pulleys and ropes themselves are massless.

Then $T_A = 2T_B = 0$

So for box A,

$$m_A g = m_A a_A$$

$$a_A = g$$

For the lower system, 

Extra Space

$a \quad -a_p = -\frac{1}{2}a_B$

using physical constraints we have $X_p + X_p - X_B = L_{rope}$

$$\Rightarrow 2a_p = a_B \quad \cdots ①$$

For mass B, $T_B - m_B g = m_B a_B \quad \cdots ②$

Using physical constraints of the upper pulley system,

$$a_A = -a_p \quad \cdots ③$$

$$m_A g - T_A = m_A a_A \quad \cdots ④$$

Finally for the whole system, $T_A = 2T_B \quad \cdots ⑤$

we have five equations for five unknowns, solving them we get

$$T_B = m_B(a_B + g)$$

$$m_A g - 2m_B(a_B + g) = -\frac{1}{2}m_A a_B$$

$$m_A g - 2m_B g = (2m_B - \frac{1}{2}m_A)a_B$$

$$a_B = \frac{m_A - 2m_B}{2m_B - \frac{1}{2}m_A} g$$

$$a_A = -a_p = -\frac{1}{2}a_B = \frac{m_B - \frac{1}{2}m_A}{2m_B - \frac{1}{2}m_A} g$$

We can also find

$$T_A = m_A(g - a_A) = \frac{m_A m_B}{2m_B - \frac{1}{2}m_A} g$$

(c). $T_{top} = 2T_A = \frac{2m_A m_B}{2m_B - \frac{1}{2}m_A} g$ (we found T_A in the previous question)

(d) My answer for part (b) is

$$a_A = \frac{m_B - \frac{1}{2}m_A}{2m_B - \frac{1}{2}m_A} g$$

when $m_B \to 0$,

$$a_A = \frac{-\frac{1}{2}m_A}{-\frac{1}{2}m_A} g = g,$$

which agrees with my answer in part (A).

Extra Space

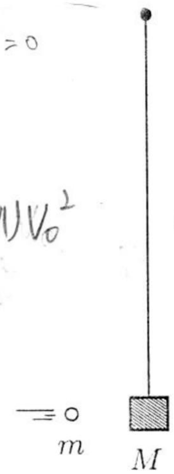
Problem 3.

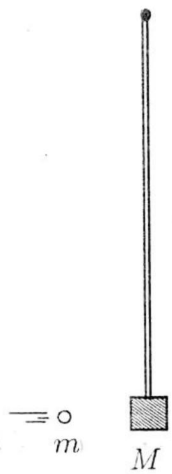
$T = 0$

$T = \frac{mv^2}{R}$,

$\frac{1}{2}mv^2 + mgR = \frac{1}{2}(m+M)V_0^2$

$(m+M)V_0^2 = mV_f$





In the diagram on the left, a small block of mass M is connected to a massless string of length ℓ that is frictionlessly pivoted at its end so it can spin around in a vertical circle. In the diagram on the right, an identical block of mass M is connected to a rigid, massless rod of length ℓ that is frictionlessly pivoted at its end so it can spin around in a vertical circle. In both cases, the block starts out hanging at rest, and a clay pellet of mass m is fired horizontally at the block and gets lodged inside.

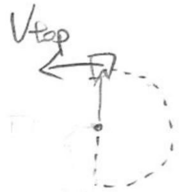
(a) In which case would you expect the pellet needs to be shot with a higher speed for the block to move all the way around in a vertical circle with radius ℓ?

(b) In the case on the left, with what speed does the pellet need to be shot at the block so the block will make it all the way around in a vertical circle of radius ℓ?

(c) In the case on the right, with what speed does the pellet need to be shot at the block so the block will make it all the way around in a vertical circle of radius ℓ?

(d) According to your answers to parts (b) and (c), in which case does the speed of the pellet need to be greater? Does your mathematics agree with your intuition from part (a)?

(a) my intuition is that the bullet on the left has to be shot with a higher speed



(b) For the case on the left, the limiting case is when the block reaches the top, it still has some velocity, V_{top}, that allows it to travel the other half of the circle. The tension of the rope would be 0 in the limiting case.

Let V_{left} denotes the initial speed of the bullet, and V_0 be the speed of the block and the bullet right after the bullet lodged in.

Using conservation of momentum and conservation of energy,

$$\begin{cases} (m+M)g = \dfrac{(m+M)V_{top}^2}{l} \\ \frac{1}{2}(m+M)V_{top}^2 + (m+M)g\cdot 2l = \frac{1}{2}(m+M)V_0^2 \\ mV_{left} = (m+M)V_0 \end{cases} \Rightarrow V_{left} = \frac{M+m}{m}\sqrt{5gl}$$

(c) for the case on the left, as long as the block is able to make it all the way to the top of the vertical circle, it is garanteed that they would make the whole circle, because the distance from the block to the center of the circle is fixed in this case.

We still have, in the limiting case,

$$\begin{cases} mV_{right} = (m+M)V_0 \\ \frac{1}{2}(m+M)V_0^2 = (m+M)g\cdot 2l + \frac{1}{2}(m+M)V_{top}^2 \\ V_{top} = 0 \end{cases}$$

$$\Rightarrow \underline{V_{right} = \frac{M+m}{m}\sqrt{4gl}}$$

(d) See NEXT PAGE

(d) according from part (b) and (c),

$V_{left} = \sqrt{5gL} \cdot \frac{M+m}{m}$,

$V_{right} = \sqrt{4gL} \frac{M+m}{m} < V_{left}$,

which agrees with my intuitive prediction in part (A)
(that the bullet on the left should be shot at a higher speed).

Extra Space

Problem 4.

A large, uniform solid disk of mass M and radius R initially spins on the surface of a flat, frictionless surface at an angular speed ω. Its center of mass is initially at rest relative to the table. Recall that the moment of inertia of a uniform solid disk for rotations about an axis passing through its center of mass and perpendicular to its face is $MR^2/2$.

Josh and Nancy are initially standing diametrically opposite one another on the edge of the disk (so the initial distance between them is $2R$). Next they walk directly toward one another along the diameter joining them until they meet at the disk's center.

They both move with the same speed as a function of time and they both have mass $(c/4)M$ where c is a unitless constant. Let $\mathbf{P}$ denote the total momentum of the Josh + Nancy + disk system. Let $\mathbf{L}_{\parallel}$ denote the angular momentum of the Josh +Nancy + disk system in the direction parallel to the axis of rotation.

(a) Is $\mathbf{P}$ conserved as Josh and Nancy walk to the center?

(b) What is the motion of the center of mass of the disk as Josh and Nancy walk to the center?

(c) Is $\mathbf{L}_{\parallel}$ conserved as Josh and Nancy walk to the center?

(d) Determine the angular speed of the disk when Josh and Nancy are at its center.

(e) Is the mechanical energy of the system conserved as Josh and Nancy walk to the center? If it is conserved, prove it. If not, compute the change in mechanical energy and show that it's nonzero. In both cases, give physical reasoning to explain why your answer makes sense as well. If it doesn't make sense, you may consider re-evaluating either your intuition about this scenario, or your math, or both.

(f) What would you expect the answer to parts (d) and (e) would be in the limit $c \to 0$? Do your mathematical answers agree with these expectations?

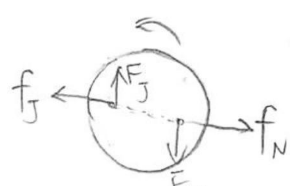
(a)

since Josh and Nancy walks in the opposite direction,

$\vec{V}_J = -\vec{V}_N$

let $\vec{V}_{disk}$ denotes the linear velocity of the center of mass of the disk,

For the disk, the only forces that are parallel to the axis of rotation are the forces from Josh and Nancy that resist the disk's rotation. let them be $\vec{F}_J$ and $\vec{F}_N$. $\vec{F}_J = -\vec{F}_N$. Also there are friction that makes Josh and Nancy moves toward the center.



So $\vec{\tau}_{net} = \vec{r}_J \times \vec{F}_J + \vec{r}_J \times \vec{f} + \vec{r}_N \times \vec{F}_N + \vec{r}_N \times \vec{f} \neq 0$

But the net force on the disk is 0 ($\vec{f}_J = -\vec{f}_N$, $\vec{F}_J = -\vec{F}_N$)

So $M\vec{a}_{disk} = 0 \Rightarrow \vec{V}_{disk} = 0$

So $\vec{P} = \frac{cM}{4}\vec{V}_J + \frac{cM}{4}\vec{V}_N + M\vec{V}_{disk} = 0$ $\vec{P}$ is conserved.

(b) As we can see from part (A), the center of mass of the disk stays at rest

(c) $\vec{L}_{\parallel} = I\omega_z^2$, where ω_z denotes the component of the angular velocity of the system that's parallel to the axis of rotation.

$I_{total} = \frac{1}{2}MR^2 + 2 \cdot \frac{CM}{4} r^2$, where r is the displacement from the center of the disk to Josh or Nancy.

As Josh and Nancy walk toward the center, r decreases, so I_{total} decreases. Also from our calculation in part (A), there is a net torque $\vec{\tau}$ that points to the direction opposite to that of the initial angular speed ω. so ω decreases over time.

So $\vec{L}_{\parallel}$ is not conserved, since its magnitude decreases over time.

(d) Using conservation of angular momentum,

$$\vec{L}_0 = \vec{L}_f$$

$$\left(\frac{1}{2}MR^2 + 2\frac{CM}{4}R^2\right)\omega^2 = \frac{1}{2}MR^2\omega_f^2$$

$$\omega_f^2 = (1+C)\omega^2$$

$$\omega_f = \sqrt{1+C}\,\omega$$

(e) In the beginning,

$$KE_0 = \frac{1}{2} I_0 \omega^2$$

$$= \frac{1}{2}\left(\frac{1}{2}MR^2 + 2 \cdot \frac{cM}{4} R^2\right)\omega^2 = \left(\frac{1}{4} + \frac{1}{4}c\right) MR^2\omega^2$$

In the end,

$$KE_f = \frac{1}{2} I_f \omega_f^2$$

$$= \frac{1}{2} \cdot \frac{1}{2} MR^2 \cdot \left(\sqrt{1+c}\,\omega\right)^2 = \frac{1}{4}(1+c) MR^2\omega^2 = KE_0$$

So mechanical energy is conserved in this case.

It makes sense because there are no non-zero, non-conservative force acting on the whole system.

(f) when $c \to 0$, both Josh and Nancy's mass would be negligible,

so $\omega_f = \omega$, since $\frac{1}{2}MR^2\omega^2 = \frac{1}{2}MR^2\omega_f^2$

when $c \to 0$, energy is conserved:

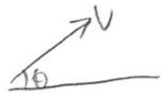
$$KE_0 = \frac{1}{2} I_{disk} \omega^2 = \frac{1}{4} MR^2\omega^2$$

$$KE_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{4} MR^2\omega^2, \quad KE_0 = KE_f$$

this still agrees from my calculation in part (e)

Extra Space

Problem 5.



(a) Consider a projectile launched at speed v angle θ relative to the horizontal on flat ground near the surface of a planet with gravitational acceleration g_P. Derive an expression for the range r_P of the projectile on this planet.

(b) If an object of mass m is a distance r away from the center of a planet of mass M, then it experiences an attractive gravitational force of magnitude

$$F = \frac{GMm}{r^2} \tag{1}$$

where G is Newton's gravitational constant. Using Newton's Second Law to set this equal to the object's mass times the magnitude a of its acceleration, we find that the object's gravitational acceleration is independent of its mass, but depends on G, M and r:

$$a = \frac{GM}{r^2} \tag{2}$$

In other words, it depends only on a fundamental physical constant, the mass of the planet, and the distance to the planet's center. If the object is at a height h above the planet's surface, and if the planet's radius is R, then the gravitational acceleration becomes

$f(0) + f'(0)x + \frac{f''(0)x^2}{2!}$

$$a = \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(1 + \frac{h}{R}\right)^{-2} \tag{3}$$

What is the Taylor expansion of the acceleration due to gravity in the variable $x = h/R$ about $x = 0$ including only the first three nonzero terms?

Useful observations. You should find that the first non-vanishing order equals GM/R^2. When you are close to the surface of the planet, only this first nonzero term is significant because $x = h/R$ will be extremely small, so this expression gives the acceleration due to gravity near the planet's surface. $x = h/R$

$$g_P = \frac{GM}{R^2}. \tag{4}$$

$\Rightarrow R = \frac{h}{x}$

In the case of Earth, one can for example show that by using this term to compute the acceleration due to gravity, the quantity GM_E/R_E^2 gives a value very close to g, where M_E is the Earth's mass, and R_E is the Earth's radius. In other words, one can predict the acceleration due to gravity near the Earth's surface using its mass and its radius!

(c) Alice is on planet A whose mass is $M_E/2$ and whose radius is R_E. Bob is on planet B whose mass is $\sqrt{3}M_E$ and whose radius is $\sqrt{2}R_E$.

Alice throws a ball at a speed v at a certain angle relative to the ground that maximizes the range of the thrown object. Bob throws a ball at speed $\sqrt{2}v$ and at an angle θ relative to the ground, and the ball ends up having the same range as Alice's ball.

At what angle θ did Bob throw the ball?

(a) the horizontal component of the velocity is never changed, so $r_p = V\cos\theta\, t$, where t denotes the time period between when the projectile is launched to when it just hits the ground.

The time it takes for the projectile to travel to the apex point. $t_{apex} = \frac{V_0 \sin\theta}{g_p}$. The time it takes for the projectile to hit the ground after it reaches the apex point is exactly the same as t_{apex}.

So the total time, $t = 2t_{apex} = \frac{2V_0 \sin\theta}{g}$

$$\therefore r_p = \frac{2V_0^2 \sin\theta\cos\theta}{g}$$

(b) $a(x) = \frac{GM}{R^2}(1+x)^{-2} \Rightarrow a(0) = \frac{GM}{R^2}(1^{-2}) = \frac{GM}{R^2}$

$a'(x) = \frac{-2GM}{R^2}(1+x)^{-3} \Rightarrow a'(0)x = \frac{-2GM}{R^2}x$

$a''(x) = \frac{6GM}{R^2}(1+x)^{-4} \Rightarrow \frac{a''(0)x^2}{2} = \frac{3GM}{R^2}x^2$

$$\therefore a(0) + a'(0)x + \frac{a''(0)x^2}{2!} = \frac{GM}{R^2} - \frac{2GM}{R^2}x + \frac{3GM}{R^2}x^2$$

$\sin\frac{\pi}{4}$

(C)

Alice: θ_A, V, $\boxed{\begin{matrix}\frac{1}{2}M_E \\ R_E\end{matrix}}$

Bob: θ, $\sqrt{2}V$, $\boxed{\begin{matrix}\sqrt{3}M_E \\ \sqrt{2}R_E\end{matrix}}$

From part (A) we know that, to maximize the range, $r_A = \dfrac{2V_0^2\sin\theta_A\cos\theta_A}{g_A}$, where g_A is the gravitational acceleration on planet A, we want to maximize $\sin\theta_A\cos\theta_A$

$\sin\theta_A\cos\theta_A = \frac{1}{2}\sin 2\theta_A \leq \frac{1}{2}$, it reaches maximum when $2\theta_A = \frac{\pi}{2}$

$$2\theta_A = \frac{\pi}{2} \Rightarrow \theta_A = \frac{\pi}{4}.$$

Also, from part (B) we know that

$$g_A(x) = \frac{G\cdot\frac{1}{2}M_E}{R_E^2} - \frac{2G\cdot\frac{1}{2}M_E}{R_E^2}x + \frac{3G\cdot\frac{1}{2}M_E}{R_E^2}x^2 = \frac{GM_E}{R_E^2}\left(\frac{1}{2} - x + \frac{3}{2}x^2\right).$$

$$g_B(x) = \frac{\sqrt{3}GM_E}{(\sqrt{2}R_E)^2} - \frac{2\sqrt{3}GM_E}{(\sqrt{2}R_E)^2}x + \frac{3\sqrt{3}GM_E}{(\sqrt{2}R_E)^2}x^2 = \frac{GM_E}{R_E^2}\left(\frac{\sqrt{3}}{2} - \sqrt{3}x + \frac{3}{2}\sqrt{3}x^2\right)$$

$$\therefore g_B(x) = \sqrt{3}g_A(x)$$

$$r_B = \frac{2V_B^2\sin\theta_B\cos\theta_B}{g_B} = \frac{4V^2\sin\theta_B\cos\theta_B}{\sqrt{3}g_A} = r_A = \frac{2V^2\sin\theta_A\cos\theta_A}{g_A}$$

$$\therefore \sin\theta_B\cos\theta_B = \frac{\sqrt{3}}{2}\sin\theta_A\cos\theta_A = \frac{\sqrt{3}}{4}$$

$$\frac{1}{2}\sin 2\theta_B = \frac{\sqrt{3}}{4}$$

$$\sin 2\theta_B = \frac{\sqrt{3}}{2}$$

$$2\theta_B = \frac{\pi}{3} \Rightarrow \theta_B = \frac{\pi}{6}$$

Extra Space

Extra Space