

LECTURE 1  
MIDTERM EXAM #2

Guidelines/Advice

- On multiple choice and true/false questions, **the correct answer is worth 1 point, and your reasoning is worth 4 points** for a total of 5 points.
- On true/false questions, if you select the answer “false,” it’s often best to provide a physically realizable counterexample demonstrating that the statement is false.
- Your answers will be graded to a large extent on how convincing your reasoning is. It is recommended that you explain your logic in complete sentences whenever possible.
- Convincing reasoning is often a mixture of mathematics, explanations, and diagrams.
- If time allows, check units and limiting cases of symbolic answers! If a limiting case analysis demonstrates that your answer isn’t sensible, then explaining this in some detail can gain you more credit, even if the answer is incorrect.
- You may write on the back of each sheet on the exam paper. Any work on the back of any sheet will be considered as part of your graded work.

Name. JOSHUA SAMANI

ID#. \_\_\_\_\_



Question 1. (5 points)

**Reminder:** A correct answer is worth only 1/5 points - you must give convincing reasoning for your choice. See instructions on front page.

If the net external force on a system is zero, then the mechanical energy of the system is conserved.

(A) True

(B) False

As a counterexample, consider two equal-mass pieces of clay colliding in deep space and sticking together:



For simplicity, let their initial speeds be the same, then momentum conservation shows that after they stick together, their combined velocity is zero.

$$mv + m(-v) = mV_{\text{combined}}$$

$$\Rightarrow V_{\text{combined}} = 0$$

So all of the initial kinetic energy was lost.



Question 2. (5 points)

Reminder: A correct answer is worth only 1/5 points – you must give convincing reasoning for your choice. See instructions on front page.

Consider the following two scenarios:

**Scenario A.** A rocket is towing 1000 kg of cargo with a very light cable of uniform mass density. The rocket exerts a constant pulling force of  $10^6$  N on the cable. The cargo is pulled in a straight line without changing direction for 10 km

**Scenario B.** The same as scenario A except the cable is quite heavy.

Let  $W_A$  denote the work performed by the cable on the cargo in scenario A, and let  $W_B$  denote the work performed by the cable on the cargo in scenario B. Which of the following is true?

(A)  $W_A > W_B$

(B)  $W_A = W_B$

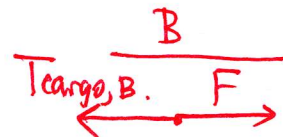
(C)  ~~$W_A > W_B$~~   $W_A < W_B$ .

FBD for cable:



$$F - T_{\text{cargo,A}} = ma_x$$

$$T_{\text{cargo,A}} = F$$



$$F - T_{\text{cargo,B}} = ma_x$$

$$T_{\text{cargo,B}} = F - ma_x$$

$$\Rightarrow T_{\text{cargo,A}} > T_{\text{cargo,B}}$$

$$W_A = T_{\text{cargo,A}} d$$

$$W_B = T_{\text{cargo,B}} d$$

$$W_A > W_B$$



Question 3. (5 points)

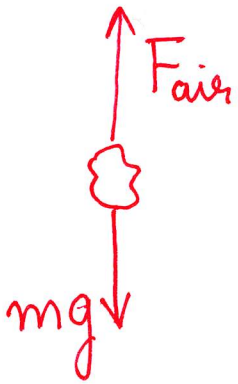
**Reminder:** A correct answer is worth only 1/5 points – you must give convincing reasoning for your choice. See instructions on front page.

If the net non-conservative work on an object is negative, then its kinetic energy cannot increase.

(A) True

(B) False

Counterexample: ball falling through the air under the influence of air ~~resistance~~ resistance / gravity.



~~Answer~~ Air resistance is the only/net non-conservative force and performs negative work since the force is opposite the velocity, but the speed and therefore the kinetic energy increases during the motion (at least until terminal velocity is reached).





Question 4. (5 points)

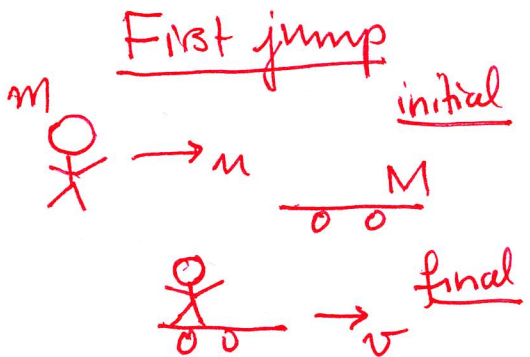
**Reminder:** A correct answer is worth only 1/5 points – you must give convincing reasoning for your choice. See instructions on front page.

Nancy jumps onto a skateboard having effectively frictionless axles. The skateboard starts at rest on the ground, and Nancy jumps on with a horizontal velocity  $u$  relative to the ground. A short time later, she jumps off of the skateboard with horizontal velocity  $u$  relative to the skateboard.

Just before she impacts the ground after jumping off the skateboard, Nancy has zero horizontal velocity relative to the ground.

(A) True

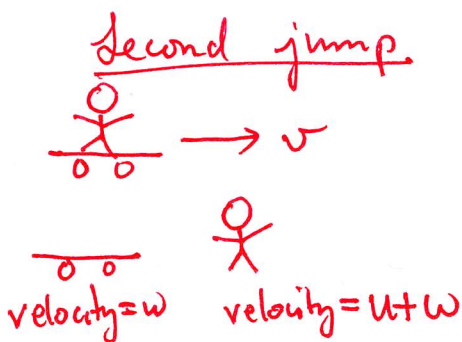
(B) False



First jump momentum conservation

$$mu \cancel{=} = (m+M)v$$

$$\Rightarrow v = \frac{m}{m+M}u$$



Second jump momentum conservation.

$$(m+M)v = Mw + (m)(w+u)$$

$$w = v - \frac{m}{m+M}u$$

$$= \frac{m}{m+M}u - \frac{m}{m+M}u$$

$$= 0$$

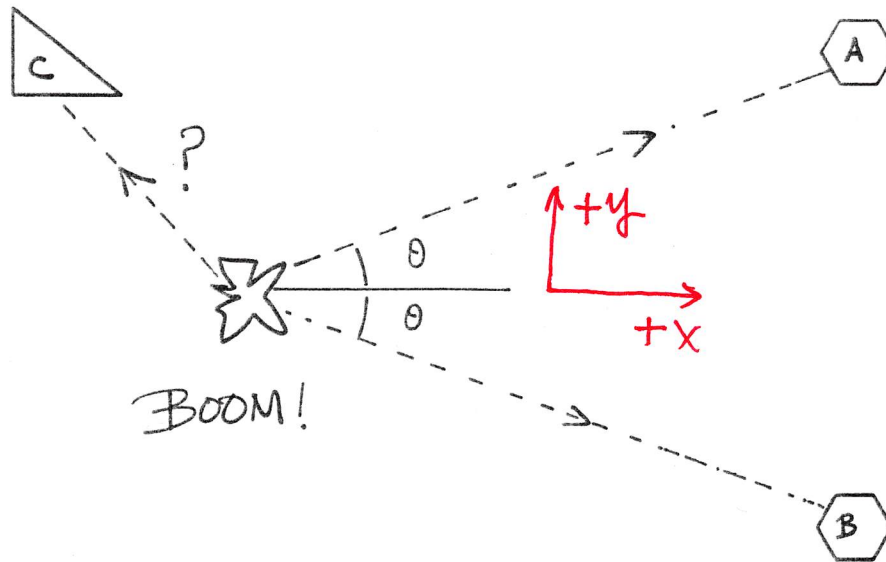
$$\Rightarrow \text{nancy velocity after second jump} = \vec{v} + u = u$$



Question 5. (5 points)

**Reminder:** A correct answer is worth only 1/5 points – you must give convincing reasoning for your choice. See instructions on front page.

An imperial starship is flying through space on its way to assault a rebel fleet in the Dagobah star system. Unfortunately for the empire, the starship's circuitry was designed by a USC engineering student, and a malfunction causes the starship to explode into three pieces A, B, and C, all lying in a plane. Pieces A and B have the same mass and fly away at the same speed and at the same angle  $\theta$  relative to the original line of flight of the starship.



What are the possible directions of the velocity of piece C?

Notice:

- total initial momentum in  $x$ -direction (no  $y$ -momentum)
- $y$ -components of momentum of A and B cancel.

(OVER)



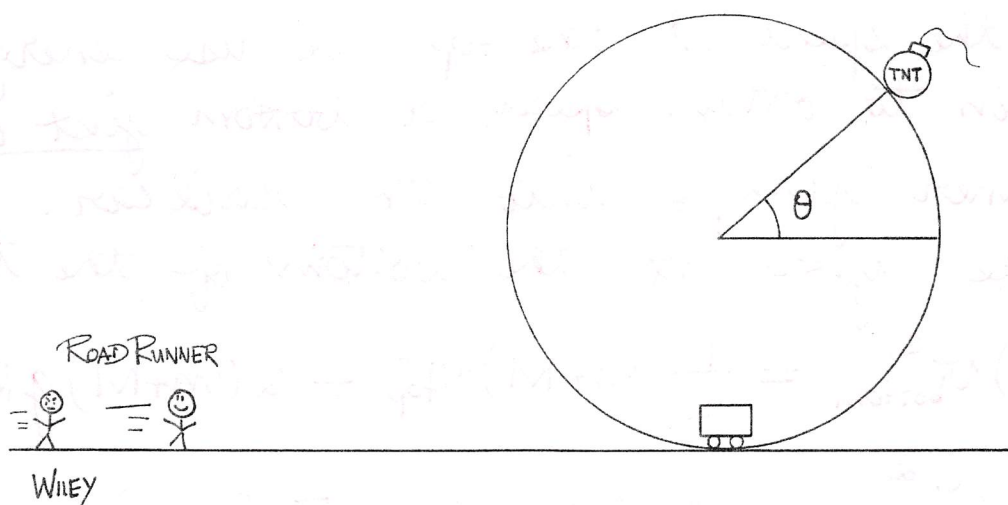
combining the facts above implies that piece C ~~has~~ has no momentum in the  $y$ -direction.

It follows that after the explosion, piece C either moves ~~along~~<sup>in</sup> the original ~~direction~~ of flight, or opposite the original direction of flight.



Problem 1. (15 points)

Wile E. Coyote is chasing Road Runner on a track. Road Runner has mass  $m$ . Far down the track is a circular loop of radius  $R$ , and at the bottom of the loop is a small railcar of mass  $M$ . Road Runner plans to jump onto the railcar to escape from Wiley, but Wiley knows this, and he has rigged a bomb to go off at some angle  $\theta$  above a horizontal diameter as pictured. The bomb mechanism assumes (i.e. the bomb will only detonate under this assumption) that Road Runner will run fast enough so that when he jumps onto the railcar, he'll make it all the way around the loop without falling off. The track-railcar interface is frictionless.

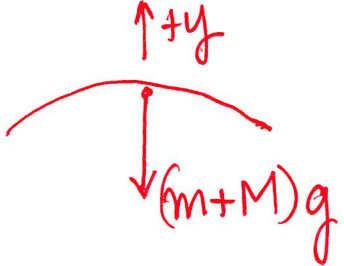


- (A) If the bomb goes off, with what minimum initial speed was the road runner running toward the railcar?
- (B) The bomb mechanism works by sensing the normal force between the railcar and the track at the location of the bomb when roadrunner + the railcar pass by on the track. If the bomb goes off, then what is the minimum normal force the sensor has to be able to measure? Note that your answer should be written in terms of  $\theta$  and the other variables given.

(OVER)



(A) If the bomb goes off, roadrunner + railcar barely made it around the track so normal force at the top is zero. Drawing an FBD at the top gives



$$-(m+M)g = \cancel{(m+M)} \left( -\frac{v_{\text{top}}^2}{R} \right)$$

$$\Rightarrow v_{\text{top}}^2 = gR.$$

Knowing the speed at the top, we use energy conservation to obtain speed at bottom just after Road Runner jumps into the railcar.

We take  $y=0$  at the bottom of the track:

$$\frac{1}{2} \cancel{(m+M)} v_{\text{bottom}}^2 = \frac{1}{2} \cancel{(m+M)} v_{\text{top}}^2 + 2 \cancel{(m+M)} gR$$

$$v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gR = 5gR.$$

$$\Rightarrow v_{\text{bottom}} = \sqrt{5gR}$$

Finally, we use momentum conservation to obtain Road Runner's running speed  $v_{RR}$ :

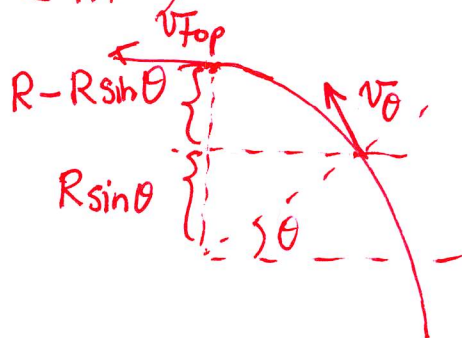
$$m v_{RR} = (m+M) v_{\text{bottom}}$$

$$v_{RR} = \frac{m+M}{m} \sqrt{5gR}$$

Note: this is the desired minimum speed necessary - any higher speed would have resulted in a nonzero normal force at the top.



(B) The sensor has to be able to measure at least the normal force between the railcar and track ~~and~~ at the angle  $\theta$  when  $v_{RR}$  is  $\sqrt{5gR} \left( \frac{m+M}{m} \right)$  as determined in part (A).

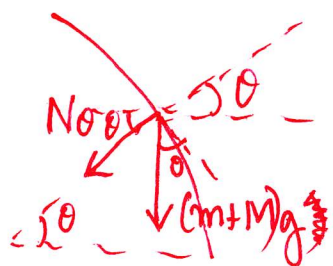


Applying energy conservation between position  $\theta$  and the top gives

$$\frac{1}{2} \frac{(m+M)}{m} v_{\theta}^2 = \frac{1}{2} \frac{(m+M)}{m} v_{top}^2 + \frac{(m+M)}{m} g (R - R \sin \theta)$$

$$\frac{v_{\theta}^2}{R} = \frac{v_{top}^2}{R} + 2g(1 - \sin \theta) = g + 2g(1 - \sin \theta)$$

Using Newton's 2nd Law at position  $\theta$  allows us to now determine the normal force there



NSL in radial direction:  
(radially inward chosen positive)

$$N_{\theta} + (m+M)g \sin \theta = (m+M) \left( \frac{v_{\theta}^2}{R} \right)$$

$$N_{\theta} = (m+M) (g \sin \theta + g + 2g(1 - \sin \theta))$$

$$N_{\theta} = (m+M) (3g) (1 - \sin \theta)$$

