Physics 1A
Spring 2015

Midterm Exam 2

May 18, 2015
4:00 PM - 4:50 PM
PAB 1-425

READ THIS BEFORE YOU BEGIN

- You are allowed to use only yourself and a writing instrument on the exam.
- Print your name on the top right of your exam.
- If you finish more than 5 minutes before the end of the exam period, then please raise your hand and a proctor will collect your exam. Otherwise, please stay in your seat until the end of time is called.
- When the exam is finished, please remain in your seat, and the proctor(s) will come around and collect your exam. Once your exam is collected, you may leave the room.
- **Show all work.** The purpose of this exam is primarily to test how you think; you will get more partial credit for a logical, well-thought-out, descriptive response.
- Please **box all of your final answers** to computational problems.

ADVICE

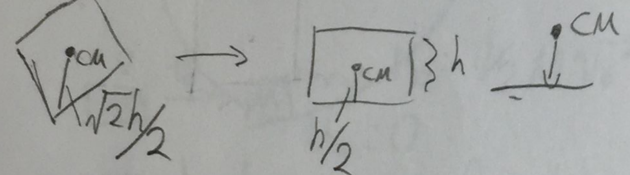
- Do not just attempt to blindly calculate answers to computational questions.
- **Use your physical and geometric intuition** first to try and determine as much about the answer as you can before you launch into computation!
- **Check units and limits** of your answers whenever possible!

Problem 1. (5 points)

Suppose you take a rigid cube (like a die), and attempt to balance it on its corner on a frictionless surface. It is extremely difficult to do so, and it is overwhelmingly likely that if you let go of the cube, it will fall down.

Suppose you let go of the unbalanced cube, and suppose that it is completely stationary just before you let go. As the cube slides and falls, describe and draw the path followed by its center of mass from the time it's released, to the time when one of its edges or faces first hits the surface.

The center of mass behaves like a point particle and travels in a smooth curve.



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In this case, the center of mass would simply behave as a point mass falling straight down a short distance due to gravity because the die's corners are slightly far away from CM than the edges and the die will slide underneathe CM on the frictionless surface.

Problem 2. (10 points)

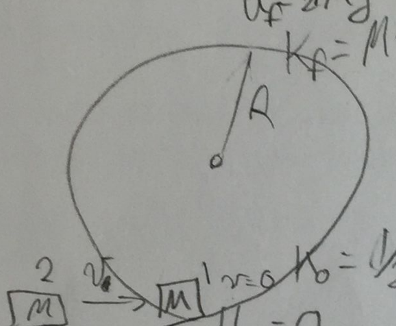
A roller coaster car (car 1) of mass M is free to slide on a frictionless track and initially sits at rest at the bottom of a circular loop of radius R. At some time later, another roller coaster car (car 2) of the same mass crashes into it, and they both begin to move up the loop such that they remain in contact with one another.

What is the minimum speed car 2 needs to have before the collision so that both cars make it to the top of the loop without ever losing contact with the track?

Hint: What is the normal force of the track on the cars if they lose contact with the track? 0

$U_f = \cancel{4}MgR$

$K_f = Mv_f^2$



$K_0 = \frac{1}{2}Mv_0^2$

$U_0 = 0$

$\frac{1}{2}Mv_0^2 \; \cancel{=} \; Mv_f^2 + 4MgR$

$\frac{1}{2}Mv_0^2 = M v_0^2/4 + 4MgR$ +2

Momentum conserved at moment of crash $v_0^2\left(M/2 - M/4\right) = 4MgR$

$2m v_f = \cancel{m v_0}$

$v_0 = 2v_f$ ✓

+2

$v_0^2 = 4MgR/(M/4) = 16gR$

$\boxed{v_0 = 4\sqrt{gR}}$

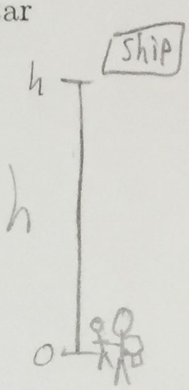
Problem 3. (20 points)

Lonestar is a galactic bounty hunter in search of a fugitive. The fugitive's last known location is the planet Alzar, a planet whose entire surface is perpetually covered by clouds. To hide her spaceship while she's looking for the fugitive, and to make a quiet descent to the surface, Lonestar plans to hover her ship above the clouds, parachute to the surface of the planet, and then use a jetpack to get her and the fugitive back to her ship.

$m + (e-1)m = em$

The combined mass of Lonestar, her empty jetpack, and the fugitive is m. Lonestar fills her jetpack with fuel having a total mass $(e-1)m$ (here $e = 2.718..$ is Euler's number). The acceleration due to gravity on Alzar is one third of g, and Lonestar's jetpack is programmed to transport her and the fugitive back to the spaceship with an acceleration of two thirds of g in the vertical direction. The jetpack fuel has an exhaust velocity u relative to the jetpack.

Neglecting air resistance, at what maximum height above Alzar's surface can Lonestar park her spacecraft and still make it back once she has apprehended the fugitive?



$F_{ext} = M(t)\, dv/dt - dM(t)/dt\, \vec{u}$

$-M(t)\, \tfrac{1}{3}g = M(t)\, \tfrac{2}{3}g - dM(t)/dt\, \vec{u}$

$\cancel{-M(t)g = -dM/dt\, \vec{u}}$

$\cancel{M(t)g = l\vec{u}}$

$u\, dM(t)/dt - M(t)\tfrac{1}{3}g = M(t)\tfrac{2}{3}g$

$M(t) = \cancel{em - lt}$

$dM/dt = \cancel{l}$ not true

$M(0) = em$

$M(t_f) = m$

$lt_f = (e-1)m$

$t_f = (e-1)m/l$

$\vec{v}(t) = 0 + \tfrac{2}{3}gt$

$l = 1.35m$

V_0, not $V(t)$

$h = 0 + \cancel{(\tfrac{2}{3}g\, t_f)} + \tfrac{1}{3}g\, t_f^2$

$= \tfrac{2}{3}g\, \frac{(e-1)}{1.35} + \tfrac{1}{3}g\, \frac{(e-1)^2}{(1.35)^2}$

$\left(\tfrac{2}{3}g\, \frac{(e-1)}{1.35}\right)^2 = 0 + 2\,\tfrac{2}{3}g\,(h)$

$h\, \tfrac{4}{3}g = \frac{4g^2(e-1)^2}{9(1.35)^2}$

$\boxed{h = \frac{g(e-1)^2}{3(1.35)^2}}$