Physics 1A - Winter 2016

# Lecture 3
# Midterm Exam #1 - Solutions

**Problem 1.**

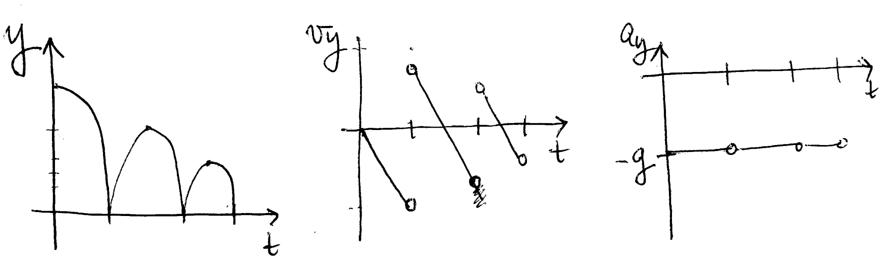
Novak drops a tennis ball from rest onto a tennis court. The ball bounces up and down, but because of the impact with the ground, the height to which rises after each bounce is 2/3 of what is was before that last bounce. You may neglect air resistance, and you may treat each bounce as if it were instantaneous.

**(a)** Sketch graphs of the $y$-component of position, velocity, and acceleration of the tennis ball as functions of time from the moment when it's dropped, to the moment when it bounces for the third time. Explain how you generated your plots.

**(b)** Sketch a graph of the speed of the tennis ball as a function of time from the moment when it's dropped, to the moment when it bounces for the third time. Explain how you generated your plot.

**Solution.**

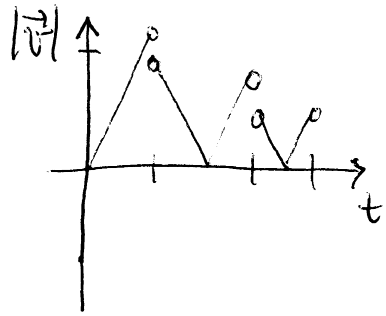
**(a)** When the ball is in the air, its $y$-position as a function of time between bounces looks like a downward-facing parabola because $y(t) = y_0 + v_{0,y}t - \frac{1}{2}gt^2$ is a parabola with a negative coefficient in front of the $t^2$ term. The apex of each successive parabola gets lower and lower by a multiplicative factor 2/3 between successive bounces. The $y$-component of velocity is a downward-sloping linear function with slope $-g$ between each bounce since $v_y(t) = v_{y,0} - gt$. Since the ball doesn't go as high each time, its max velocity between each bounce also decreases by a factor of $\sqrt{2/3}$ (this can be demonstrated using the kinematics equation $v_{y,f}^2 = v_{y,i}^2 - 2g(y_f - y_i)$. Finally, the acceleration of the ball is a constant $-g$ between bounces. The bounces themselves are instantaneous, so strictly speaking the acceleration there would be "infinite" in our idealization, but since we haven't learned how to mathematically deal with this sort of thing, the bounce points themselves can simply be omitted from the plots.



**(b)** Recall that the speed is the magnitude of the velocity, so in this case we have

$$|\mathbf{v}(t)| = \sqrt{v_x(t)^2 + v_y(t)^2} = \sqrt{v_x(t)^2 + 0} = |v_x(t)| \tag{1}$$

Thus, to generate the speed plot, we need only take the absolute value of the $v_x$ plot at every point in time.




**Problem 2.**

Ally and Bernard are pulling on opposite ends of a rope. They are each pulling with a force of about 500 N. The rope is everywhere parallel to the ground to good approximation.

**(a)** If the rope is massless, what is the tension in the middle of the rope?

**(b)** If the rope has nonzero mass $M$ and length $\ell$, then does the answer change? If so, explain. If not, explain why not.

**Solution.**

**(a)** Consider drawing a free body diagram of the left-hand half of the rope (where we assume Ally is pulling on the left). There will be two forces on it: the force $F_{\text{Ally}}$ of Ally pulling to the left, and the tension $T_{\text{middle}}$ of the right-hand half of the rope pulling to the right. On the other hand, the whole rope, including the left half, has zero acceleration since Ally and Bernard are pulling with equal forces on either side. Therefore, applying Newton's Second Law to the left-hand half of the rope gives

$$T_{\text{middle}} - F_{\text{Ally}} = M_{\text{left-half}}a = 0 \cdot 0 = 0. \tag{2}$$

It follows that

$$T_{\text{middle}} = F_{\text{Ally}} = \boxed{500\,\text{N}} \tag{3}$$

**(b)** **The precise answer.** If the cable has nonzero mass $M$, then no matter how hard one pulls on either side of the rope, it can never be completely horizontal. To see why, consider a small segment of the rope in the middle of mass $\Delta m$. The acceleration of this piece of the rope is zero which means that there is no net force on it. However, since it has mass, this piece has a nonzero weight, and if the rope were completely horizontal, there would be no force to cancel out that weight in the $y$-direction. Therefore, the rope must sag a little bit so that in the center, the tension has some component in the vertical direction that cancels the force due to gravity. This issue did not arise in the case of the massless rope because the rope, having no mass, is not influenced by gravity. Therfore, in the case of the massive rope, the answer does change (it's not necessary to compute it in this case for the purposes of this question – it's actually a rather tricky calculus problem to do that.)

**Another answer we accepted.** If the rope is assumed completely horizontal in this case as well (even though strictly speaking this is not possible), then one can reason as follows:

If the cable has nonzero mass $M$ and length $\ell$, then this does not change the answer. Even though the mass of the left-half is now nonzero, its acceleration is still zero because the rope as a whole is still not accelerating due to the opposing force of the two spacecraft. It follows that the same Newton's Second Law calculation will hold and the force in the middle will remain unchanged.

**Problem 3.**

A massless pulley hangs from the ceiling of an elevator by a uniform chain of mass $M$ and length $\ell$ connected to the pulley's center. Masses $m_1$ and $m_2$ are connected by a massless rope and hang on either side of the pulley. The elevator has acceleration $A$ relative to the ground. Let $T(y)$ denote the tension in the chain a distance $y$ above the point at which it is connected to the pulley.

**(a)** What would you predict $T(0)$ would be if $m_1 = m$, $m_2 = m$ and $A = 0$? Explain using physical reasoning.

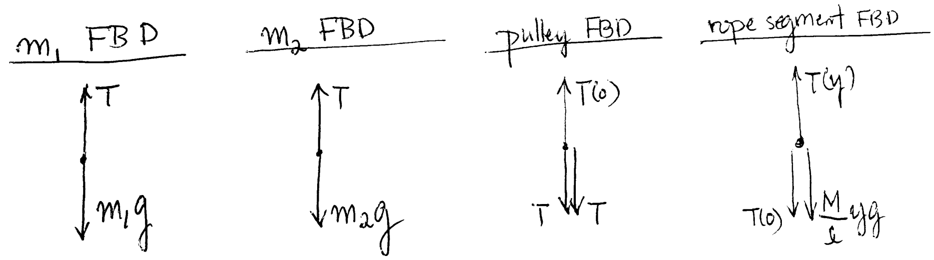
**(b)** Determine the general expression for $T(y)$ in terms of the given variables.

**Solution.**

**(a)** When the acceleration of the elevator is zero, and when the masses on both sides are equal, the tension in the rope becomes the same as that of an Atwood's machine with equal masses fixed to a ceiling (strictly speaking in this case the elevator could also have constant velocity, but the same conclusion would apply in that case as well). In that circumstance, the masses would not be accelerating either because they balance one another, and the tension holding the pulley up would simply be the sum of the weights hanging from it, namely

$$\boxed{T(0) = 2mg}. \tag{4}$$

**(b)** We draw four free body diagrams: one for each mass hanging, one for the pulley, and one for the section of rope between where its is attached to the pulley and a point a distance $y$ above that.




Applying Newton's Second Law to each of these objects in the $y$-direction (but omitting $y$ subscripts for simplicity) gives

$$T - m_1 g = m_1 a_1, \quad T - m_2 g = m_2 a_2 \quad T(0) - 2T = 0, \quad T(y) - \frac{M}{\ell} y g - T(0) = \frac{M}{\ell} y A \tag{5}$$

This is four equations in five unknowns $a_1, a_2, T, T(0), T(y)$. We have already implicitly used Newton's Third Law as much as we can because we noticed that $T(0)$ is the magnitude of the force of the rope on the pulley and the magnitude of the force of pulley on the rope. Therefore, what remains is to find an appropriate constraint. Noting that the rope is of constant length, one can systematically obtain the following constraint:

$$2A = a_1 + a_2. \tag{6}$$

We now have five equations in five unknowns, and we can solve. First, let's use the first two Newton's Second Law equations and the constraint to get rid of $a_1$ and $a_2$. Dividing the first NSL equation by $m_1$ and the second by $m_2$, adding them together, and then invoking the constraint, we obtain

$$T\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - 2g = 2A. \tag{7}$$

Solving for $T$ and then plugging the result into the NSL equation for the pulley gives

$$T(0) = \frac{4m_1 m_2}{m_1 + m_2}(g + A) \tag{8}$$

Finally, we plug this into the NSL equation for the segment of rope and solve for $T(y)$ to obtain the desired result:

$$\boxed{T(y) = \left(\frac{4m_1 m_2}{m_1 + m_2} + \frac{M}{\ell} y\right)(g + A)} \tag{9}$$

If we take the limit $A \to 0$ and set $m_1 = m_2 = m$, then this gives

$$T(0) = \left( \frac{4m \cdot m}{m + m} + 0 \right) (g + 0) = 2mg \tag{10}$$

as we had predicted in part (a) on physical grounds.