Physics 1A
Spring 2015

Midterm Exam 1

April 27, 2015
4:00 PM - 4:50 PM
PAB 1-425

READ THIS BEFORE YOU BEGIN

- You are allowed to use only yourself and a writing instrument on the exam.
- Print your name on the top right of your exam.
- If you finish more than 5 minutes before the end of the exam period, then please raise your hand and a proctor will collect your exam. Otherwise, please stay in your seat until the end of time is called.
- When the exam is finished, please remain in your seat, and the proctor(s) will come around and collect your exam. Once your exam is collected, you may leave the room.
- **Show all work.** The purpose of this exam is primarily to test how you think; you will get more partial credit for a logical, well-thought-out response.
- Please **box all of your final answers** to computational problems.

ADVICE

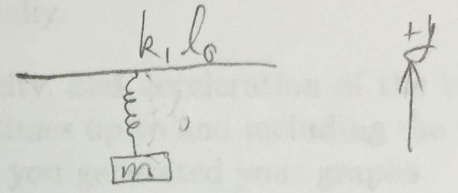
Do not just attempt to blindly calculate answers to computational questions. Use your physical and geometric intuition first to try and determine as much about the answer as you can before you launch into computation!

Problem 1. (5 points)

A mass m is suspended from the ceiling of an office building elevator by a Hooke's Law spring of spring constant k and natural length ℓ_0. Let the positive y-direction be the direction in which the floor numbers in the building increase.

Assuming that the y-component of the acceleration of the elevator is a, which of the following is the length of the spring as the elevator accelerates?

Circle one answer, then explain the reasoning behind your answer. Reasoning may include calculation, but it doesn't have to.



(a) $\ell_0 + \frac{m}{k}(a+g)$

(b) $\ell_0 - \frac{m}{k}(g-a)$

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(c) $\ell_0 + \frac{m}{k}(g-a)$

(d) $\ell_0 - \frac{m}{k}(a+g)$

The springs length would be $> \ell_0$ when at rest because of the weight mg on the bottom end. When the elevator is accelerating, this places another force on the opposite end of the spring, which will stretch even further before bringing m with it.

Problem 2. (15 points)

Suppose that at time $t = 0$, Novak Djokovic drops a tennis ball from rest onto a tennis court. Let h_0 be the initial height from which the ball is dropped. For $i = 1, 2, 3, \ldots$, let v_i denote the speed of the ball just after its i^{th} bounce, and let h_i denote its maximum height after its i^{th} bounce. Assume that each bounce is instantaneous.

(a) After each bounce, the subsequent maximum height reached by the ball is reduced by a factor of 9/16 . Write this as a mathematical condition relating h_{i+1} and h_i.

(b) What is the ratio of v_{i+1} to v_i? Justify mathematically.

(c) Draw graphs of the y-component of position, velocity, and acceleration of the tennis ball as functions of time. Draw your graphs for all times up to and including the third bounce. Write a short explanation in words of how you generated your graphs.

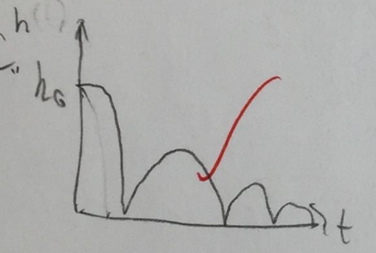
(d) Extra Credit. (5 points) Let Δt_i be the time between bounce i and bounce $i+1$. Determine Δt_i in terms of h_0.

a. $h_{i+1} = \frac{9}{16} h_i \quad h(i) = \frac{9}{16} i h_0$ +1

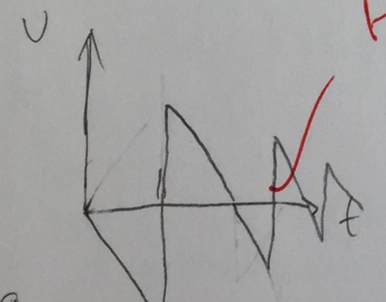
b. $V_{i+1}^2 = V_i^2 - 2g\left(\frac{9}{16} i h_0\right)$ +1

$V_{i+1} = \sqrt{V_i - 2g\left(\frac{9}{16} h_i\right)}$

c.

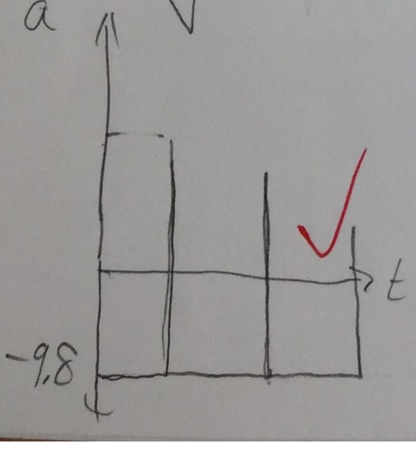


Ball starts at h_0, only goes $\frac{9}{16} h_{i-1}$ everytime afterward.



+9

Ball gains negative velocity until it hits the ground, then loses positive velocity until it reaches h_i, then repeat.

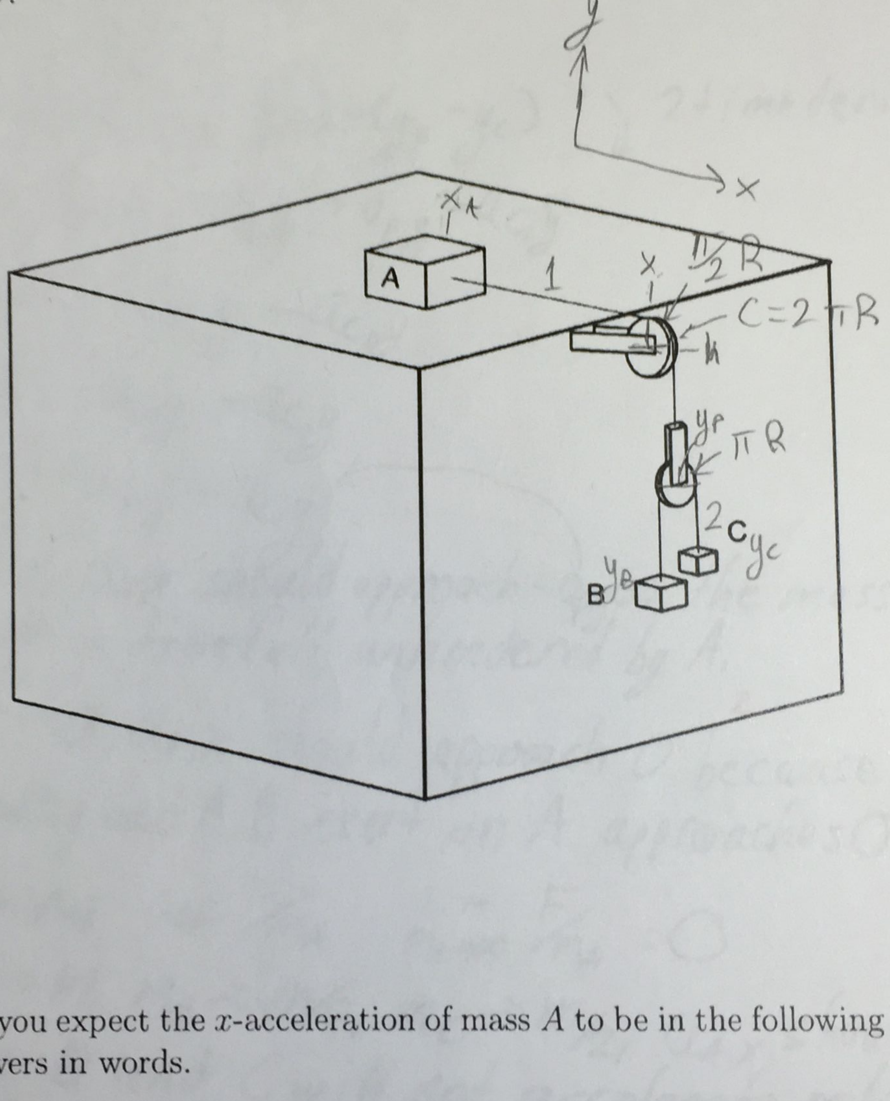


Acceleration is always $-g$. except when the ball hits the concrete with less force in every bounce.

Problem 3. (22 points)

Consider the following system. All ropes and pulleys are massless, and all surfaces are frictionless. The mass m_A slides on an *immovable* table and is pulled by two masses m_B and m_C dangling over the edge of the table and connected by a rope threaded over a pulley.

Take the positive x-direction to be the direction of the resulting motion of mass A, and take the positive y-direction to be the direction opposite the motion of the pulley holding masses B and C.



(a) What do you expect the x-acceleration of mass A to be in the following limits? Explain your answers in words.

(i) $m_A \to 0$

(ii) $m_A \to \infty$

(iii) $m_A \to m$, $m_B \to m/2$ and $m_C \to m/2$. In this case, apply all of these limits simultaneously.

(b) Determine an expression for the x-acceleration of mass m_A in terms of the masses m_A, m_B, m_C and g.

Extra Space

a. i. $L_1 = \pi/2 R + (x - x_A) + (h - y_P)$ 2 time derivs

This is way too real, Ambitious though!

$0 = -a_{A,x} - a_{P,y}$

$a_{A,x} = -a_{P,y}$

$L_2 = \pi R + (y_P - y_B) + (y_P - y_C)$ 2 time derivs

$0 = a_{P,y} - a_{B,y} + a_{P,y} - a_{C,y}$

$-2a_{P,y} = -a_{B,y} - a_{C,y}$

$2a_{P,y} = a_{B,y} + a_{C,y}$

$a_{A,x} = -a_{B,y} - a_{C,y}$

As $m_A \to 0$, $a_{A,x}$ should approach $-a_{P,y}$ as the masses on P and P itself are in freefall, unhindered by A.

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ii. As $m_A \to \infty$, $a_{A,x}$ should approach 0 because the force that the pulley and A, B exert on A approaches 0.

$F = m_A a_{A,x}$ $a_{A,x} = F/m_A$ $\lim_{m_A \to \infty} F/m_A = 0$

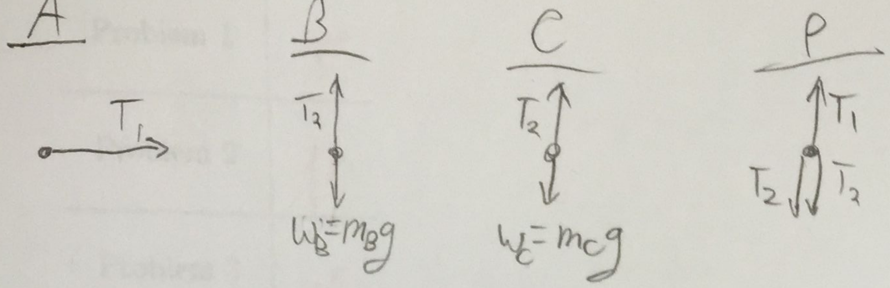
iii. As $m_A \to m$, $m_B \to m/2$, $m_C \to m/2$, $a_{A,x}$ should approach g because B and C will not accelerate relative to each other and can be considered a single body with mass m. Here the weight of that body will be mg.

$F = m a_{A,x} \longrightarrow \cancel{m}g = \cancel{m}a_{A,x}$

$a_{A,x} = g$ 2

Extra Space

b.



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$a_{A,x} = -a_{P,y} = -a_{B,y} - a_{C,y}$

$a_{P,y} = a_{B,y} + a_{C,y}$

$T_1 / m_A = a_{Ax}$

A: $\sum F_x = T_1 = m_A a_{Ax}$

$a_{By} = \frac{m_A a_{Ax}}{2} - m_B g$

B: $\sum F_y = T_2 - m_B g = m_B a_{By}$

C: $\sum F_y = T_2 - m_C g = m_C a_{Cy} = m_C (a_{By} - a_{Ax})$

P: $\sum F_y = T_1 - 2T_2 = 0 \quad T_1 = 2T_2$

$2T_2 = m_B(a_{By} + g) + m_C(a_{By} - a_{Ax} + g) = a_{By}(m_B - m_C) - m_C a_{Ax} + m_B m_C g$

??

$a_{Ax}(m_A + m_C) = a_{By}(m_B - m_C) + m_B m_C g = m_A a_{Ax}/2 - m_B g + m_B m_C g$

$\boxed{a_{Ax} = \frac{-m_B g + m_B m_C g}{m_A + m_C - m_A}}$