Physics 1A - Lecture 3 Final

Michael Shuang Zhang

TOTAL POINTS

108 / 113

QUESTION 1

1 Problem 1 (24 / 24)

- 1 (a) Idea to use strategy of finding acceleration and using this to determine velocity at bottom.
- 3 (a) FBD x-direction and resulting equation
- 3 (a) FBD y-direction and resulting equation
- 1 (a) Relationship between friction and normal force
- 4 (a) Kinematics + algebra to solve for final velocity as function of angle or equivalent
- 3 (a) Argument that statement 1 is true appealing to math.
- 4 (b) Energy conservation + equation
- 2 (b) Solve for final speed
- 3 (c) Take mathematical limit and discuss

+ 0 Point adjustment

QUESTION 2

2 Problem 2 (27 / 27)

- 3 (a) Reasonable argument that mass m_A will free fall
- 4 (b) FBD + NSL equation mass A
- 4 (b) FBD + NSL equation mass B
- 4 (b) FBD + NSL equation for bottom pulley
- 4 (b) Constraint relating accelerations of both masses (can be obtained via combination of more than one constraint as in solution)
- 3 (b) Algebra and solve for a_{A,y}
- 3 (c) Plug in and solve for tension
- 2 (d) Take limit and compare to prediction

+ 0 Point adjustment

QUESTION 3

3 Problem 3 (24 / 24)

+ 3 (a) Convincing argument that block on string will require more initial speed.

+ 2 (b) Momentum conservation

+ 3 (b) Mechanical energy conservation

+ 3 (b) force analysis at top with recognition of condition on tension to just make it around

+ 3 (b) Solve for desired speed

+ 3 (c) Angular momentum conseravation (linear momentum also works)

+ 3 (c) Mechanical energy conservation with recognition that velocity zero at top to just make it around circle

+ 2 (c) Solve for desired speed

+ 2 (d) Determine from math with speed is greater and comment on whether agrees with prediction

+ 1 (a) plausible but incorrect reasoning (partial credit)

+ 2 (b) speed solved but with arithmetic error

+ 1 (c) speed solved but with arithmetic error

+ 2 energy conservation but used incorrectly

+ 2 momentum conservation used incorrectly

+ 0 no credit

QUESTION 4

4 Problem 4 (20 / 24)

+ 3 (a.1) horizontal total force is zero (no external horizontal force actually)

+ 1 (a.2) vertical net forces is close to zero, as long as the system center of mass doesn't change much in the vertical direction

+ 3 (b) Argue stays at rest using momentum conservation and initial condition

+ 3 (c) Argue net external torque in parallel-direction zero since external forces are vertical, so ang. mom. in that direction conserved

+ 3 (d.1) angular momentum conservation equation

+ 2 (d.2) the correct result (c+1)w

+ 3 (e.1) Compute kinetic energy change result

expression correctly

+ 2 (e.2) give reasonable argument that why the energy increases

+ 4 (f) when c goes to zero, all the changes are negligible. relate math with physics.

+ 0 zero

QUESTION 5

5 Problem 5 (13 / 14)

+ 5 (a) Kinematics or equivalent argument to obtain the correct range expression and simplify

+ 3 (b) Compute correct Taylor expansion coefficients and put together to write answer.

+ 6 (c)

+ 4 (c) setup correct, but arithmetic errors

+ 4 (a) correct setup, but arithmetic errors/missing coefficients

+ 2 (b) unsimplified or arithmetic errors

+ 3 (c) half credit for giving two answers/good attempt

+ 2 (a) incorrect attempt

+ 1 (b) incorrect attempt

+ 2 (c) qualitative attempt/incorrect attempt

+ 5 (c) correct but with minor arithmetic error

+ 0 no credit

gradescope

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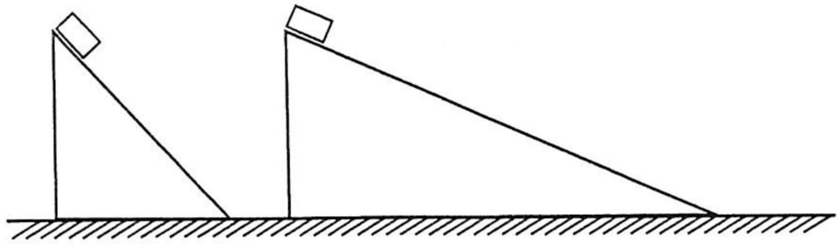
Physics 1A - Winter 2016

Lecture 3

FINAL EXAM

Problem 1.



Two identical boxes slide down different ramps pictured above having started at rest from their tops. The boxes start at the same height. The coefficient of kinetic friction μ_k between the boxes and the ramps is the same in both cases, and both ramps are fixed to the ground. The coefficient of static friction is not large enough to prevent the blocks from sliding down the ramps.

Let box A be the box on the left, and let box B be the box on the right. Consider the following statements:

I. The speed of box A is greater than the speed of box B when they reach the bottoms of their ramps.

II. The speed of box A is less than the speed of box B when they reach the bottoms of their ramps.

III. The speed of box A is the same as the speed of box B when they reach the bottoms of their ramps.

Questions.

(a) Use force methods (energy methods not allowed!) to determine which if these statements is true when $\mu_k \neq 0$. You'll find it useful to name your own relevant variables so that you can answer this question mathematically.

(b) Use energy methods to determine which if these statements is true when $\mu_k \neq 0$. You'll find it useful to name your own relevant variables so that you can answer this question mathematically.

(c) Does the answer change when $\mu_k = 0$? Justify using your answers from the previous parts.

Extra Space

a)

$\sin\theta = \frac{h}{x}$

$\frac{x}{h} = \frac{1}{\sin}$

the general free body diagram on the block

switch axis so its parallel / perpendicular to the plane

NSL

$F_y = N - mg\cos\theta = 0 \rightarrow N = mg\cos\theta$

$F_x = mg\sin\theta - F_f = ma_x$

$ma_x = mg\sin\theta - \mu_k N$

$ma_x = mg\sin\theta - \mu_k mg\cos\theta$

M - mass of block

θ - angle of slope

N - normal force

h - height of slope

F_f - force of friction

$a_x = g\sin\theta - \mu_k g\cos\theta$

$a_x = g(\sin\theta - \mu_k\cos\theta)$

$0 \leq \sin\theta \leq 1$

$V_f^2 = V_0^2 + 2ad$

$V_f^2 = 0 + 2g(\sin\theta - \mu_k\cos\theta)\left(\frac{h}{\sin\theta}\right)$

$V_f^2 = 2gh - \frac{2gh\mu_k\cos\theta}{\sin\theta}$

b) Energy $U_i + K_i = U_f + K_f - W_{nc}$

$V_f^2 = 2gh(1 - \mu_k\tan^{-1}\theta)$

$mgh = \frac{1}{2}mv^2 + F_f \cdot \frac{h}{\sin\theta}$

$\frac{1}{2}mv^2 = mgh - \frac{F_f h}{\sin\theta}$

$v^2 = 2\left(gh - \frac{\mu_k gh\cos\theta}{\sin\theta}\right)$

$v^2 = 2gh(1 - \mu_k\tan^{-1}\theta)$

$v^2 = \frac{2}{m}\left(mgh - \frac{F_f h}{\sin\theta}\right)$

$v^2 = \frac{2}{m}\left(mgh - \frac{\mu_k mg\cos\theta\, h}{\sin\theta}\right)$

in both cases, as θ gets bigger $\tan^{-1}\theta$ gets smaller, so the speed gets greater. Thus, for the same height h, the greater the angle, the faster the speed → I true, II false, III false

c) If μ_k is 0, then I - false, II - false
and III is true

equation from part a) and part b)

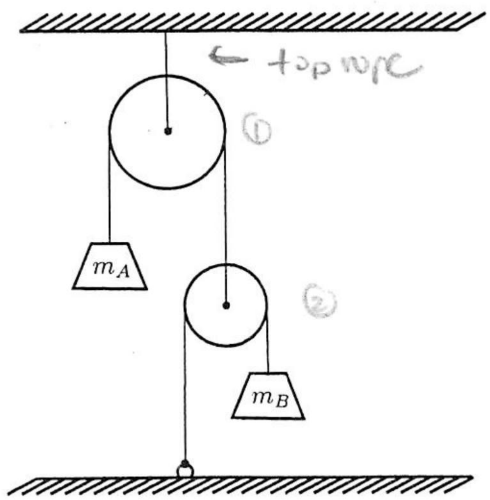
$$v_f^2 = 2gh(1 - \cancel{\mu_k \tan^{-1}\theta})$$

$$v_f^2 = 2gh$$

- v will depend solely on h - which is the same for A and B!
then III is true

Extra Space

Problem 2.



Consider the apparatus above. All pulleys and ropes are massless.

(a) In limit $m_B \to 0$, would you expect the magnitude of the acceleration of mass A to be greater than, equal to, or less than g? Explain using physical reasoning.

(b) Determine an expression for the acceleration of mass A in terms of the given variables.

(c) Determine an expression for the tension in the top rope in terms of the given variables.

(d) Does your mathematical answer in part (b) agree with your answer in part (a)? Explicitly verify this mathematically. If the answers don't agree, you should consider re-evaluating either your math, or your intuition, or both.

a) acceleration of a would be equal to $-g$. There would be no tension, allowing it to essentially freefall.

b) FBD m_A — T_A, $m_A g$

FBD m_B — T_B, $m_B g$

Bottom pulley — T_A, T_B, T_B

Top pulley — T_{top}, T_A, T_A

$m_a a_{a,y} = T_A - m_A g$

$m_B a_{B,y} = T_B - m_B g$

$\cancel{m_{BP}} a_{BP} = T_A - 2T_B \quad (\to 0)$

$T_A = 2T_B$

by constraints

$2a_{BP} = a_B$

$a_{BP} = -a_{a,y}$

$-2a_{a,y} = a_B$

$T_A = 2T_B$

Extra Space

$-2a_{ay} = a_B$

$a_a = -\frac{1}{2}a_B$

$m_a a_a = T_A - m_a g$

$-2m_B a_a = \frac{1}{2}T_A - m_B g$

$T_A = 2m_B g - 4m_B a_a$

$m_a a_a = 2m_B g - 4m_B a_a - m_a g$

$m_a a_a + 4m_B a_a = 2m_B g - m_a g$

$a_a(m_a + 4m_B) = 2m_B g - m_a g$

$a_a = \frac{2m_B g - m_a g}{(m_a + 4m_B)}$

c) $T_{top} = 2T_A$

$m_a a_a = T_A - m_a g$

$m_a\left(\frac{2m_B g - m_a g}{(m_a + 4m_B)}\right) = T_A - m_a g$

$T_A = m_a\left(\frac{2m_B g - m_a g}{(m_a + 4m_B)}\right) + m_a g$

$T_A = m_a g\left(\frac{2m_B - m_a}{m_a + 4m_B} + 1\right)$

$T_{top} = 2m_a g\left(\frac{2m_B - m_a}{m_a + 4m_B} + 1\right)$

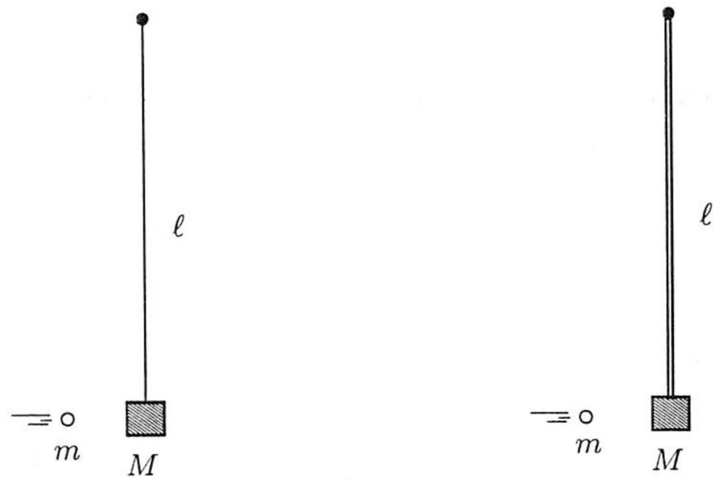
d) yes - if $m_B = 0$

$a_a = \frac{2\cancel{m_B}^{0} g - m_a g}{m_a + \cancel{4m_B}^{0}}$ $\quad a_a = \frac{-m_a g}{m_a} \rightarrow a_a = -g$

Extra Space

Extra Space

Problem 3.



In the diagram on the left, a small block of mass M is connected to a massless string of length ℓ that is frictionlessly pivoted at its end so it can spin around in a vertical circle. In the diagram on the right, an identical block of mass M is connected to a rigid, massless rod of length ℓ that is frictionlessly pivoted at its end so it can spin around in a vertical circle. In both cases, the block starts out hanging at rest, and a clay pellet of mass m is fired horizontally at the block and gets lodged inside.

(a) In which case would you expect the pellet needs to be shot with a higher speed for the block to move all the way around in a vertical circle with radius ℓ?

(b) In the case on the left, with what speed does the pellet need to be shot at the block so the block will make it all the way around in a vertical circle of radius ℓ?

(c) In the case on the right, with what speed does the pellet need to be shot at the block so the block will make it all the way around in a vertical circle of radius ℓ?

(d) According to your answers to parts (b) and (c), in which case does the speed of the pellet need to be greater? Does your mathematics agree with your intuition from part (a)?

see reasoning on next page under c)

a) Case with the string

b) $F_c = \frac{mv^2}{r}$

$a_c = \frac{v^2}{r}$

at the top → $mg = \frac{mv^2}{\ell}$

$g = \frac{v^2}{\ell}$

$V_{top} = \sqrt{g\ell}$

energy just after collision → energy at top

$\frac{1}{2}\cancel{(m+M)}v_0^2 = \frac{1}{2}\cancel{(m+M)}V_{top}^2 + \cancel{(m+M)}g2\ell$

$\frac{1}{2}V_0^2 = \frac{1}{2}g\ell + 2g\ell$

$\frac{1}{2}V_0^2 = \frac{5}{2}g\ell$

$V_0 = \sqrt{5g\ell}$

$mv = (m+M)\sqrt{5g\ell}$

$V = \frac{(m+M)\sqrt{5g\ell}}{m}$

c) If there's a rigid rod - all the block has to do is reach the top, as long as it just "clears" the top the rod will ensure it maintains a circular path on the way down, vs. falling as with the string.

also reasoning for part A

Essentially, with the rod, the velocity at the top has to be just over 0, while with the string, the velocity must match the equation for centripetal acceleration

$$\frac{1}{2}(\cancel{m+M})v_0^2 = (\cancel{m+M})g2l$$

$$\frac{1}{2}v_0^2 = 2gl$$

$$v_0^2 = 4gl$$

$$v_0 = 2\sqrt{gl}$$

$$mv_{\substack{before \\ collision}} = (m+M)2\sqrt{gl}$$

$$v_{\substack{before \\ collision}} = \frac{2(m+M)\sqrt{gl}}{m}$$

d) case with the string $\rightarrow$ which matches part a!

Extra Space

Extra Space

tank

Problem 4.

A large, uniform solid disk of mass M and radius R initially spins on the surface of a flat, frictionless surface at an angular speed ω. Its center of mass is initially at rest relative to the table. Recall that the moment of inertia of a uniform solid disk for rotations about an axis passing through its center of mass and perpendicular to its face is $MR^2/2$.

Josh and Nancy are initially standing diametrically opposite one another on the edge of the disk (so the initial distance between them is $2R$). Next they walk directly toward one another along the diameter joining them until they meet at the disk's center.

They both move with the same speed as a function of time and they both have mass $(c/4)M$ where c is a unitless constant. Let $\mathbf{P}$ denote the total momentum of the Josh + Nancy + disk system. Let $\mathbf{L}_{\parallel}$ denote the angular momentum of the Josh +Nancy + disk system in the direction parallel to the axis of rotation.

(a) Is $\mathbf{P}$ conserved as Josh and Nancy walk to the center?

(b) What is the motion of the center of mass of the disk as Josh and Nancy walk to the center?

(c) Is $\mathbf{L}_{\parallel}$ conserved as Josh and Nancy walk to the center?

(d) Determine the angular speed of the disk when Josh and Nancy are at its center.

(e) Is the mechanical energy of the system conserved as Josh and Nancy walk to the center? If it is conserved, prove it. If not, compute the change in mechanical energy and show that it's nonzero. In both cases, give physical reasoning to explain why your answer makes sense as well. If it doesn't make sense, you may consider re-evaluating either your intuition about this scenario, or your math, or both.

(f) What would you expect the answer to parts (d) and (e) would be in the limit $c \to 0$? Do your mathematical answers agree with these expectations?

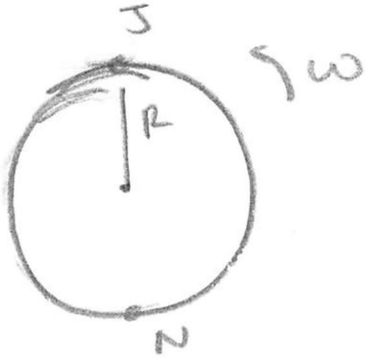
a) P is conserved. The disk has no translational momentum → so that remains 0. Because Josh + Nancy have the same speed + mass and move in opposite directions,

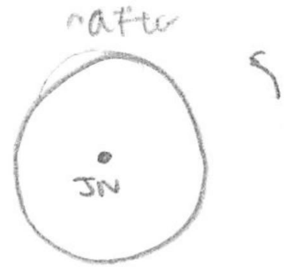
$mv - mv = 0$

b) the $a_{CM} = 0$, center of mass remains constant furthermore, this supports a) as $F_{ext} = \frac{\partial P}{\partial t}$
↓
Ma_{CM}

c) Yes - any force that Josh/Nancy's feet apply is in the same direction as R → meaning that the torque applied would be 0 (F is not perpendicular) and since $\tau_{net} = \frac{\partial L}{\partial t}$, momentum would be conserved!

d)





$V = r\omega$

angular momentum is conserved

$L_{before} = L_{after}$

$$\frac{1}{2}MR^2\omega + R\times\left(\frac{c}{4}\right)M(R\omega) + R\times\left(\frac{c}{4}\right)M(R\omega) = \frac{1}{2}MR^2\omega_{final} + 0 + 0$$

$$\frac{1}{2}MR^2\omega + \frac{c}{4}MR^2\omega + \frac{c}{4}MR^2\omega = \frac{1}{2}MR^2\omega_{final}$$

$$\cancel{MR^2}\omega\left(\frac{1}{2} + \frac{c}{2}\right) = \frac{1}{2}\cancel{MR^2}\omega_{final}$$

$$\omega_{final} = 2\omega\left(\frac{1}{2} + \frac{c}{2}\right) = \omega + c\omega$$

e) NO, E is not conserved - by walking towards the center - J + N are applying energy (calories) so it should increase

E_{before}

$$\underbrace{\frac{1}{2}I\omega^2}_{disk} + \frac{c}{8}M(R\omega)^2 + \frac{c}{8}M(R\omega)^2$$

$$\frac{1}{4}MR^2\omega^2 + \frac{c}{4}MR^2\omega^2$$

$$E_{before} = \frac{1}{4}MR^2\omega^2(1+c)$$

E_{after}

John + Nancy can be considered one particle rotating in the center with $\frac{c}{2}M$ and ω_{final}

$$\frac{1}{2}I\omega^2 + \frac{1}{2}I\omega^2$$

$$\frac{1}{2}\left(\frac{1}{2}MR^2\right)(\omega+c\omega)^2 + \frac{1}{2}\cdot\frac{c}{2}MR^2\cdot(\omega+c\omega)^2$$

If they are at the center, we can assume that ω << than the radius of the disk so E_{after} is

$$\frac{1}{4}MR^2(\omega+c\omega)^2$$

change in KE

$$\frac{1}{4}MR^2(\omega+c\omega)^2 - \frac{1}{4}MR^2\omega^2(1+c)$$

$$\frac{1}{4}MR^2\left((\omega+c\omega)^2 - (\omega^2 + c\omega^2)\right)$$

f) If $c \to 0$, then the disk would essentially be spinning before and after by itself (Josh+Nancy wouldn't exist). Therefore

ω should stay the same

and Mechanical energy should be conserved, or

$\Delta ME = 0$

$$\omega_{final} = \omega + \cancel{c\omega}^{\to 0} \checkmark$$

change in KE

$$\frac{1}{4} MR^2 \left((\omega + \cancel{c\omega})^2 - (\omega^2 + \cancel{c\omega^2}) \right)$$

$$\frac{1}{2} MR^2 (\omega^2 - \omega^2) \checkmark = 0 \checkmark$$

the MATH works out!

Extra Space

Problem 5.

(a) Consider a projectile launched at speed v angle θ relative to the horizontal on flat ground near the surface of a planet with gravitational acceleration g_P. Derive an expression for the range r_P of the projectile on this planet.

(b) If an object of mass m is a distance r away from the center of a planet of mass M, then it experiences an attractive gravitational force of magnitude

$$F = \frac{GMm}{r^2} \tag{1}$$

where G is Newton's gravitational constant. Using Newton's Second Law to set this equal to the object's mass times the magnitude a of its acceleration, we find that the object's gravitational acceleration is independent of its mass, but depends on G, M and r:

$$a = \frac{GM}{r^2} \tag{2}$$

In other words, it depends only on a fundamental physical constant, the mass of the planet, and the distance to the planet's center. If the object is at a height h above the planet's surface, and if the planet's radius is R, then the gravitational acceleration becomes

$$a = \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(1+\frac{h}{R}\right)^{-2} \tag{3}$$

What is the Taylor expansion of the acceleration due to gravity in the variable $x = h/R$ about $x = 0$ including only the first three nonzero terms?

Useful observations. You should find that the first non-vanishing order equals GM/R^2. When you are close to the surface of the planet, only this first nonzero term is significant because $x = h/R$ will be extremely small, so this expression gives the acceleration due to gravity near the planet's surface.

$$g_P = \frac{GM}{R^2}. \tag{4}$$

In the case of Earth, one can for example show that by using this term to compute the acceleration due to gravity, the quantity GM_E/R_E^2 gives a value very close to g, where M_E is the Earth's mass, and R_E is the Earth's radius. In other words, one can predict the acceleration due to gravity near the Earth's surface using its mass and its radius!

(c) Alice is on planet A whose mass is $M_E/2$ and whose radius is R_E. Bob is on planet B whose mass is $\sqrt{3}M_E$ and whose radius is $\sqrt{2}R_E$.

Alice throws a ball at a speed v at a certain angle relative to the ground that maximizes the range of the thrown object. Bob throws a ball at speed $\sqrt{2}v$ and at an angle θ relative to the ground, and the ball ends up having the same range as Alice's ball.

At what angle θ did Bob throw the ball?

a)

g_P

When it hits the ground, y component of velocity will be equal but opposite

$$-v\sin\theta = v\sin\theta - g_p t$$

$$-2v\sin\theta = -g_p t$$

$$t = \frac{2v\sin\theta}{g_P}$$

So $r_p = vt$

$$r_p = \frac{2v^2\sin\theta\cos\theta}{g_P}$$

$$r_p = \frac{v^2\sin 2\theta}{g_P}$$

$$g_{PA} = \frac{G\left(\frac{M_E}{2}\right)}{R_E^2} = \frac{1}{2}g_{EARTH}$$

$$g_{PB} = \frac{G\sqrt{3}M_E}{2R_E^2} = \frac{\sqrt{3}}{2}g_{EARTH}$$

$$g_{EARTH} = \frac{GM_E}{R_E^2}$$

b)

$$a(x) = \frac{GM}{R^2}(1+x)^{-2}$$

$$a'(x) = \frac{GM}{R^2}\cdot -2(1+x)^{-3}$$

$$a''(x) = \frac{GM}{R^2}6(1+x)^{-4}$$

$$\frac{GM}{R^2} - 2\frac{GM}{R^2} + \frac{6GM}{R^2}$$

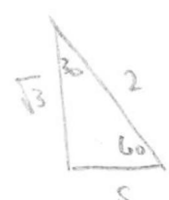
c) $r_p = \dfrac{v^2 \sin 2\theta}{g_p}$

$r'_p = \dfrac{2v^2 \cos 2\theta}{g_p}$ $\Rightarrow$ maximized angle is at $\theta = \frac{\pi}{4}$

ALICE $\rightarrow$ $r_p = \dfrac{v^2(1)}{\frac{1}{2} g_{EARTH}}$ $\qquad r_p = \dfrac{2v^2}{g_{EARTH}}$

BOB $\rightarrow$ $\dfrac{2v^2}{\cancel{g_{EARTH}}} = \dfrac{(\sqrt{2}v)^2 \sin 2\theta}{\frac{\sqrt{3}}{2}\cancel{g_{EARTH}}}$

$\cancel{2v^2} = \dfrac{4\cancel{v^2} \sin 2\theta}{\sqrt{3}}$



$1 = \dfrac{2 \sin 2\theta}{\sqrt{3}}$

$\dfrac{\sqrt{3}}{2} = \sin 2\theta$

$\theta = \dfrac{\pi}{6}$

Extra Space

Extra Space