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ıplete this exam. One standard 3" x 5" i
Scientific and graphing calculators are a
Both pen and pencil are allowed.

space below the problem. Scratch pape
ꝫasoning to get full credit. For clarity, p

Q1	17
Q2	5
Q3	19
Q4	20
Q5	20
TOTAL	81

Trig Identities

$$\cos\left(\theta \pm \frac{\pi}{2}\right) = \mp\sin(\theta) \qquad \sin\left(\theta \pm \frac{\pi}{2}\right) = \pm\cos(\theta)$$

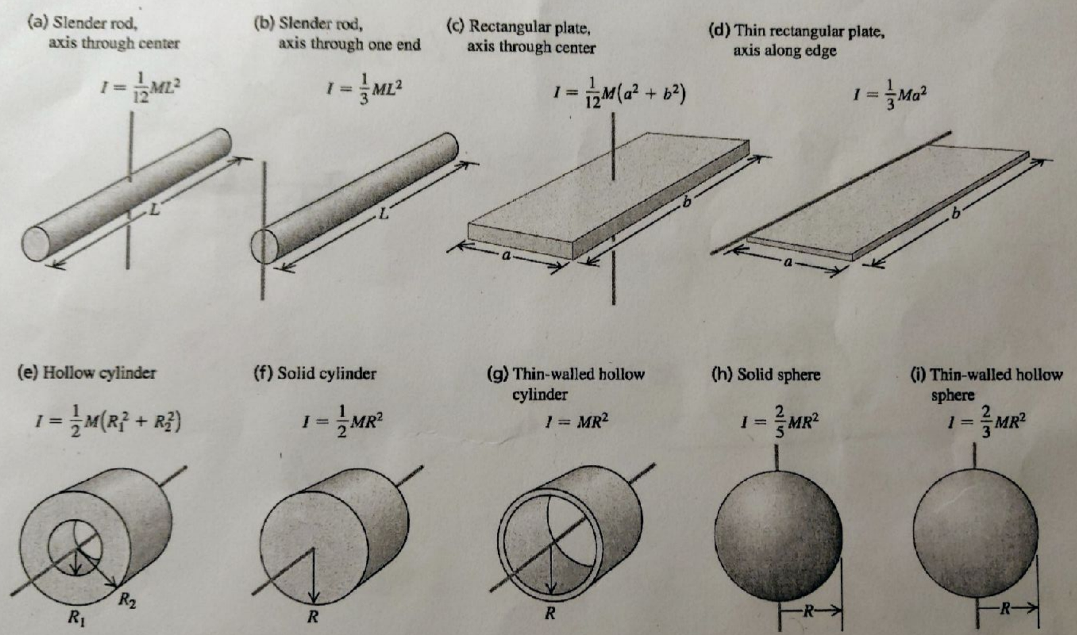
$$\cos(\pi \pm \theta) = -\cos(\theta) \qquad \sin(\pi \pm \theta) = \mp\sin(\theta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \qquad \sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right) + \cos\left(\frac{\alpha-\beta}{2}\right) \qquad \sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right) + \cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right) + \sin\left(\frac{\alpha-\beta}{2}\right) \qquad \sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right) + \sin\left(\frac{\alpha-\beta}{2}\right)$$

Moments of Inertia



Derivatives

$$\frac{d}{dx}e^{\pm cx} = \pm ce^{\pm x} \qquad \frac{d}{dx}e^{\pm(c_1+c_2x)^n} = \pm n(c_1 + c_2x)c_2e^{\pm(c_1+c_2x)^n}$$

Question 1

A 1.80 kg physical pendulum in simple harmonic motion is pivoted 200 cm from its center of mass with a moment of inertia 0.129 kg·m². At some point in time the pendulum is 3.50° from equilibrium, and 0.524 s later it is at -5.59° from equilibrium.

(a) Find the amplitude of oscillations.

(b) Write a function for the angular position in time, ϴ(t), that describes the motion between the given 0.524 s. Set ϴ(0) equal to the first given angle, 3.50°.

(a)

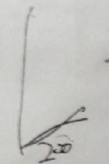
(b) assume amplitude $= A$.

$\theta(t) = A\cos(\omega t + \phi)$ +2

$\theta(0) = A\cos(\phi) = 3.5^\circ \rightarrow \phi = \cos^{-1}\left(\frac{3.5^\circ}{A}\right)$

+10 $\theta(0.524) = A\cos(0.524\omega + \phi) = -5.59^\circ$

(a)



$\cancel{\tau = \alpha I}$

$A\cos(0.524\omega + \phi) = -5.59^\circ$

$0.524\omega + \cos^{-1}\left(\frac{3.5^\circ}{A}\right) = \cos^{-1}\left(\frac{-5.59^\circ}{A}\right)$

$\omega = \sqrt{\frac{mgL}{I}}$

$$\omega = \frac{\cos^{-1}\left(\frac{-5.59^\circ}{A}\right) - \cos^{-1}\left(\frac{3.5^\circ}{A}\right)}{0.524}.$$

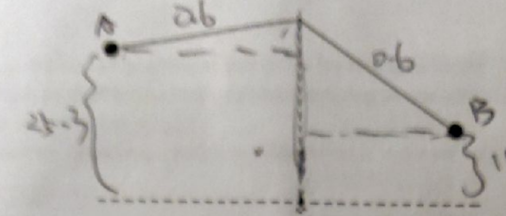
$A = ?$

$$\theta(t) = A\cos\left(\frac{\cos^{-1}\left(\frac{-5.59^\circ}{A}\right) - \cos^{-1}\left(\frac{3.5^\circ}{A}\right)}{0.524} t + \cos^{-1}\left(\frac{3.5^\circ}{A}\right)\right.$$

[illegible]

Question 2

The figure below shows two pendulums at t = 0. At t = 0, the ball on the right has a height of 11.0 cm and the ball on the left has a height of 25.3 cm. The balls are released from rest simultaneously. When they collide, they stick to each other. The strings are both 60.0 cm long. The ball on the right has mass 3.00 kg, and the ball on the left has mass 2.50 kg.



(a) Find the frequency of the motion after collision.
(b) Find the maximum angular displacement of the motion after collision.

+5

(a) $\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\ m\cdot kg/s^2}{0.6 m}} = 4.04\ \cancel{\ } rad/s$

$f = 2\pi\ \omega = 2\pi \cdot 4.0415\ rad/s = 25.39\ s^{-1}$

$f = \frac{\omega}{2\pi}$

(b)

Question 3

(C) ~~is~~ on back

A horizontal rope with some tension is given a pulse. The function that describes the motion of the pulse along the rope is given by:

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$$y(x,t) = Ae^{-\frac{(ax-bt)^2}{c}}$$

(7) $v = \sqrt{\dfrac{\frac{4b^2}{c^2}(ax-bt)^2 - \frac{2b^2}{c}}{\frac{4a^2}{c^2}(ax-bt)^2 - \frac{2a^2}{c}}} = \sqrt{\dfrac{b^2}{a^2}} = \sqrt{\dfrac{b}{a}}$

(a) If constants a, b, and c are all positive, is this pulse travelling to the left or to the right? How can we change this function so that it describes a pulse travelling in the opposite direction?

(b) What is the wave speed of the pulse?

(c) If we make the tension in the rope twice as large, what is the new wave speed?

(a) as t increases, x must increase to keep $\frac{(ax-bt)^2}{c}$ in place. Thus to the right.

+5

for the left: $y(x,t) = Ae^{-\frac{(ax+bt)^2}{c}}$

(5) $\dfrac{d^2y}{dx^2} = \left(\dfrac{4a^2(ax-bt)^2}{c^2} - \dfrac{2a^2}{c}\right)Ae^{-\frac{(ax-bt)^2}{c}}$

(6) $v^2 = \dfrac{d^2y}{dt^2} \div \dfrac{d^2y}{dx^2}$

$= \left(\dfrac{4a^2}{c^2}(ax-bt)^2 - \dfrac{2a^2}{c}\right)^{-1} \cdot \left(\dfrac{4b^2}{c^2}(ax-bt)^2 - \dfrac{2b^2}{c}\right)$

(b) wave speed =

(1) $\dfrac{dy}{dt} = Ae^{-\frac{(ax-bt)^2}{c}} \cdot -\dfrac{2(ax-bt)}{c} \cdot (-b)$

$= \dfrac{2b(ax-bt)}{c} \cdot Ae^{-\frac{(ax-bt)^2}{c}}$

$\dfrac{1}{v^2}\dfrac{d^2y}{dt^2} = \dfrac{d^2y}{dx^2}$

(2) $\dfrac{d^2y}{dt^2} = \dfrac{4b^2(ax-bt)^2}{c^2} \cdot Ae^{-\frac{(ax-bt)^2}{c}} + Ae^{-\frac{(ax-bt)^2}{c}} \cdot -\dfrac{2b^2}{c}$

+7

(3) $\dfrac{dy}{dx} = -\dfrac{2(ax-bt)}{c} \cdot a \cdot Ae^{-\frac{(ax-bt)^2}{c}}$

(4) $\dfrac{d^2y}{dx^2} = \dfrac{4a^2(ax-bt)^2}{c^2} \cdot Ae^{-\frac{(ax-bt)^2}{c}} - \dfrac{2a^2}{c}$

$\dfrac{d^2y}{dx^2} = \dfrac{1}{v^2}\dfrac{d^2y}{dt^2}$. $\dfrac{4a^2(ax-bt)^2}{c^2} = \dfrac{1}{v^2} \cdot \dfrac{4b^2(ax-bt)^2}{c^2}$

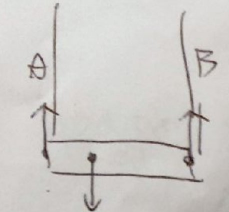
Question 4

A 1800 N irregular beam is hanging horizontally by its ends from the ceiling by two vertical wires (A and B), each 1.20 m long and weighing 0.130 N. The center of gravity of this beam is one-fourth of the way along the beam from the end where wire A is attached.

A 1.20 m long pipe is closed at one and open at the other. A standing air wave in the pipe is in its first overtone. The pipe is held near the hanging wires, causing the strings to vibrate with large amplitude. The speed of sound in air is 340 m/s.

(a) What is the wave speed for each wire?

(b) What mode n is produced in each string? Round to the nearest whole number. Note: if you've chosen an indexing scheme that is different from the textbook(s), write down the formula you are using.



(a) $\cancel{v_A = \sqrt{\frac{F}{\mu}}}$

$$\begin{cases} F_A \cdot \frac{1}{4}L = F_B \cdot \frac{3}{4}L \rightarrow F_A = 3F_B \\ F_B \cdot L - 1800\,N \cdot \frac{1}{4}L = 0 \end{cases}$$

$$F_B = 1800\,N \cdot \frac{1}{4} = 450\,N$$

$$F_A = 3F_B = 1350\,N$$

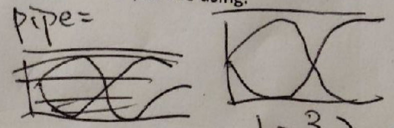
+26

$$\mu = \frac{m}{L_1} = \frac{0.130\,N / 9.8\,kg \cdot m/s^2}{1.20\,m} = 0.01105\,kg/m$$

$$v_A = \sqrt{\frac{F_A}{\mu}} = \sqrt{\frac{1350\,N}{0.01105\,kg/m}} = 349.46\,m/s$$

$$v_B = \sqrt{\frac{F_B}{\mu}} = \sqrt{\frac{450\,N}{0.01105\,kg/m}} = 201.76\,m/s$$

(b) pipe =



$L = \frac{3}{4}\lambda$

$$\lambda = \frac{4}{3}L_p = \frac{4}{3} \cdot 1.20\,m = 1.6\,m$$

$$f = \frac{v}{\lambda} = \frac{340\,m/s}{1.6\,m} = 212.5\,s^{-1}$$

for strings = (closed end)

$$L_s = \frac{n}{2}\lambda, \quad n = 1, 2 \ldots$$

$$\lambda = \frac{2L}{n} = \frac{v}{f}$$

for A =

$$\frac{2 \cdot 1.20\,m}{n} = \frac{349.46\,m/s}{212.5\,s^{-1}}$$

$$\boxed{n_A = 1.45 \approx 1}$$

for B =

$$\frac{2 \cdot 1.20\,m}{n} = \frac{201.76\,m/s}{212.5\,s^{-1}}$$

$$\boxed{n_B = 2.52 \approx 3}$$

Question 5

A police car's siren emits a sinusoidal wave with frequency 350 Hz. The speed of sound is 340 m/s. The police car is moving away from a warehouse at 35 m/s. What frequency does the driver hear reflected from the warehouse?

$$f_{warehouse} = f_{police} \cdot \frac{V}{V + V_{pol}}$$

$$= 350\,Hz \left(\frac{340\,m/s}{340\,m/s + 35\,m/s} \right)$$

$$= 317.33\,Hz$$

$$f_{driver\ ref} = f_{warehouse} \cdot \frac{V - V_{driver}}{V}$$

$$= 317.333\,Hz \cdot \frac{340\,m/s - 35\,m/s}{340\,m/s}$$

$$= 284.67\,Hz$$

$$\boxed{\approx 285\,Hz}$$

+20