(1) (20 points-5 each question)

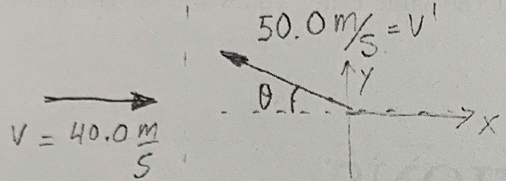
A bat strikes a 0.200-kg baseball. Just before impact, the ball is traveling horizontally to the right at 40.0 m/s; when it leaves the bat, the ball is traveling to the left at an angle of 30.0 degrees above horizontal with a speed of 50.0 m/s. The ball and bat are in contact for 2.00 ms.

1-a) What was the magnitude of the average force vector exerted by the bat on the ball?

1-b) What was the direction of the average force vector exerted by the bat on the ball? (indicate clearly your reference frame if you give an angle)

1-c) What was the work done by this force?

1-d) What was the average power delivered by the bat?



$\theta = 30°$ $\qquad$ $\Delta t = 2.00\ \text{ms}$

$I_x = \Delta p_x = -m v' \cos\theta - m v$

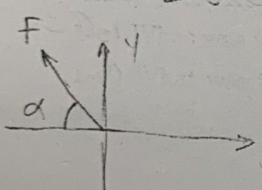
$= 0.200\text{kg}\ (-43.30 - 40.00)\ \text{m/s}$

$I_x = -16.66\ \text{kg}\ \frac{m}{s}$

$I_y = m v' \sin\theta = 0.200 \times 25.0\ \text{kg}\ \frac{m}{s}$

$= 5.00\ \text{kg}\ \frac{m}{s}$

$F_x = \frac{I_x}{\Delta t} = -8.33\ 10^3 N$ $\qquad$ $F_y = \frac{I_y}{\Delta t} = 2.50 10^3\ N$



a) $F = \sqrt{F_x^2 + F_y^2} = \sqrt{69.4 + 6.25\ ''} = 8.70\ N$

b) $\tan\alpha = \frac{F_y}{|F_x|} = \frac{2.50}{8.33} = 0.30 \Rightarrow \alpha = 17°$

1.c) $\text{Work} = \Delta E_K = \frac{m}{2}(v'^2 - v^2) = 0.100\,(2500.0 - 1600.00)^2\ J = 90.0\ J$

1.d) $\bar{P} = \frac{\text{Work}}{\Delta t} = 4.5 \times 10^4\ W$

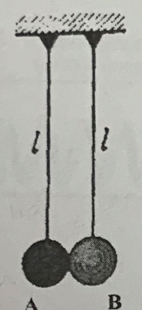
(2) (20 points)

Two small balls (consider them particles) A and B, are suspended side by side from two strings of length ℓ= 1m so that they touch when they are in equilibrium, as seen in the figure. The body A has a mass $m = 0.20$ kg and the body B has mass $2m = 0.40$ kg. The left ball, A, is pulled aside with its string extended and released from rest from a height $h = 0.20$ m with respect to where it was. The ball A then swings as a pendulum and at the lowest point of its path collides elastically with B.

2-a) (7 points) How high does the ball B swing after the collision and in which direction (before turning back down).

2-b) (7 points) How high does the ball A swing after the collision and in which direction (before turning again back down)

2-c) (6 points) Both balls return back and collide elastically again at the lowest point where they collided first. How high will each ball swing after this second collision?



a) $\frac{m}{2} v_A^2 = mgh \Rightarrow v_A = \sqrt{2gh} = 1.98\ m/s^2$

① $m v_A = m v_A' + 2m v_B'$ ① $+ m$② $\Rightarrow 2m v_A = 3m v_B'$

② $v_A = -v_A' + v_B'$ $\Rightarrow v_B' = \frac{2}{3} v_A$ B moves to the right

$\frac{2m v_B'^2}{2} = 2mgh_B' \Rightarrow \boxed{h_B' = \frac{v_B'^2}{2g} = \frac{4}{9}\frac{v_A^2}{2g} = \frac{4}{9}h}$

$\boxed{h_B' = 0.88\ m}$

b) ① $- 2m$ ② $\Rightarrow -m v_A = 3m v_A' \Rightarrow v_A' = -\frac{v_A}{3}$

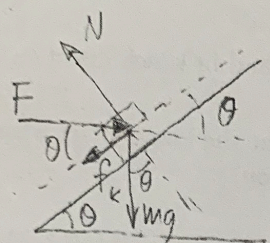
$\frac{m}{2} v_A'^2 = mgh_A' \Rightarrow \boxed{h_A' = \frac{1}{9}\frac{v_A^2}{2g} = \frac{h}{9} = 0.22}$ A moves to the left after 1st collision

c) Since the collision is elastic the 2nd collision reproduces the initial configuration, with reversed signs of the velocities. So A goes left with speed v_A and gets back to height h while B stops just after the collision.

(3) (10 points)

A crate of mass m is pushed up a ramp inclined upward at an angle θ above the horizontal, by exerting a constant horizontal force of magnitude F on the crate. The coefficient of kinetic friction between the ramp and crate is μ. Use work-energy considerations to find the crate's speed a distance L up along the ramp, if its speed at the bottom of the ramp is v_0. Give your answer in terms of $\theta, L, F, m, \mu, v_0$ and g as necessary.

Only $F\cos\theta$ does work, $W_{other} = \Delta(E_k + U_g)$



$W_{other} = (F\cos\theta - \mu mg\cos\theta)L =$

$= \frac{m}{2}v^2 + mgL\sin\theta - \frac{m}{2}v_0^2$

$N = mg\cos\theta$, $f_K = \mu_K mg\cos\theta$

$$\boxed{\sqrt{\frac{2L}{m}\cos\theta(F - \mu mg) - 2gL\sin\theta + v_0^2} = v}$$

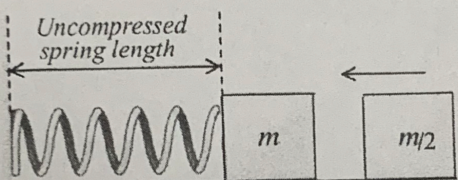
(4) **(30 points)**

A block of mass m is on a horizontal surface to the right of an unextended and uncompressed horizontal spring attached to a wall, as shown in the figure. The block is not attached to the spring. Another block of mass $m/2$ comes horizontally from the right and hits the first block with a speed v_0, after which the spring is compressed. There is a coefficient of kinetic friction μ between the blocks and the surface. Treat the blocks as particles. Express your answers in terms of some or all the mentioned parameters and g, as necessary.

4-a) Assuming that you know the spring constant k of the spring, find the maximum compression of the spring.

4-b) Assume now that you do not know the spring constant but observe that after the blocks compress the spring maximally, the blocks move back right and stop exactly when the spring reaches again its natural (unextended and uncompressed) length. What is the maximum compression of the spring in this case?

4-c) What is the spring constant k of the spring in the case of question b)?



a) k is known

Collision: $\frac{m}{2}V_0 = \frac{3m}{2}V \Rightarrow V = \frac{V_0}{3}$

Work other $= -\mu\frac{3}{2}mgx = \frac{k}{2}x^2 - \frac{3m}{4}\frac{V_0^2}{9}$

$\Rightarrow kx^2 + 3\mu mgx - \frac{m}{2}\frac{V_0^2}{3} = 0$

Solve quadratic equation: $x = \frac{-3\mu mg + \sqrt{9\mu^2m^2g^2 + 2km(V_0^2/3)}}{2k}$

Important: need to choose + sign in the $\sqrt{\ }$, because x should go to zero as V_0 goes to zero.

b) Now we do not know k but we have two equations:

① compression of spring: $-\mu\frac{3}{2}mgx = \frac{k}{2}x^2 - \frac{3m}{4}\frac{V_0^2}{9}$ (as before)

② expansion of spring: $-\mu\frac{3}{2}mgx = 0 - \frac{k}{2}x^2 \Rightarrow \frac{k}{2}x^2 = \mu\frac{3}{2}mgx$

$\Rightarrow$ in ① $+2\mu\frac{3}{2}mgx = +\frac{3m}{4}\frac{V_0^2}{9} \Rightarrow \boxed{x = \frac{V_0^2}{36\mu g}}$

c) Using 2: $\boxed{k = \frac{\mu 3mg}{x} = \frac{3\mu mg(36\mu g)}{V_0^2} = \frac{108\mu^2g^2m}{V_0^2}}$