First: 2 short questions (10 points in total)

(1) (4 points)

A 50 kg firefighter hears the fire alarm and to get fast to the firetruck jumps from the second floor of the firehouse while grabbing a vertical pole.

1-a) If she grabs the pole strongly enough to fall with constant velocity which is the direction and magnitude of the friction force the pole exerts on her while she is falling? (Neglect air resistance)

1-b If the firefighter lets go of the pole for a moment, which is her "apparent weight" during that moment?

a) $\vec{F}_{net} = 0$ So F_friction= mg pointing vertically up

b) $\vec{a} = \vec{g}$, the firefighter is "weight less"

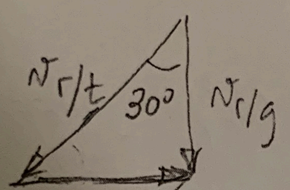
(apparent weight) = 0

(2) (6 points)
When a train's velocity is 12.0 m/s eastward, raindrops that are falling vertically with respect to the earth make traces that are inclined 30.0 degrees to the vertical on the windows of the train ($\sin 30^o = 0.500$, $\cos 30^o = 0.866$, $\tan 30^o = 0.577$)

2-a) What is the magnitude of the velocity of the raindrop with respect to the train?

2-b) What is the magnitude of the velocity of the raindrop with respect to the earth?

$$\vec{v}_{r/g} = \vec{v}_{r/t} + \vec{v}_{t/g}$$



$v_{t/g} = 12.0\, m/s$

a) $v_{r/t} = \dfrac{v_{t/g}}{\sin 30^\circ} = 2 v_{t/g} = 24.0 \dfrac{m}{s}$

b) $v_{r/g} = v_{r/t} \cos 30^\circ = 20.8 \dfrac{m}{s}$

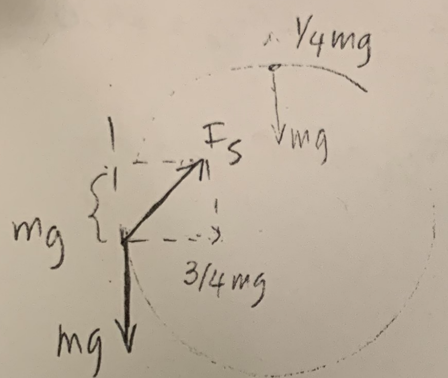
(3) A problem (20 points total)

Jack sits in the chair of a Ferris wheel that is rotating at a constant number n of revolutions per minute (n rev/min). As Jack passes through the highest point of his circular part, the upward force that the chair exerts on him is equal to one-fourth of his weight.

3.a) (8 points) What is the radius of the circle in which Jack travels? Treat him as a point mass

3.b) (6 points) What is the magnitude of the force of support acting on Jack when he is at one quarter of revolution with respect to the highest (or the lowest) point of his circular path?

3.c) (6 points) By which factor should the angular velocity of the Ferris wheel increase for Jack to be "weightless" at the highest point of the circular path?



a) $\frac{3}{4}\cancel{m}g = \cancel{m} R\,\omega_o^2$ $\qquad \omega_0 = \frac{2\pi n}{60 s} = \frac{n}{9.55 s}$

$$\boxed{R = \frac{3}{4}\frac{g}{\omega_o^2} = \frac{3}{4}\frac{g(9.55)^2 s^2}{n^2} = \frac{670.3}{n^2} m}$$

b) $\boxed{F_s = \sqrt{\left(\frac{3}{4}mg\right)^2 + (mg)^2} = mg\sqrt{\frac{9}{16}+1} = \frac{5}{4}mg}$

c) weightless $\Rightarrow$ $R\omega^2 = g$ $\qquad \frac{R\omega^2}{R\omega_0^2} = \frac{4}{3} \Rightarrow \boxed{\omega = \sqrt{\frac{4}{3}}\,\omega_0}$

$= 1.16\,\omega_0$

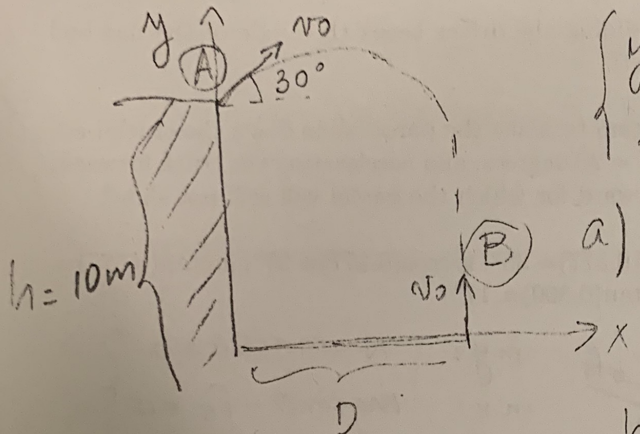
(4) A problem (20 points total-10 each question)

A projectile is sent with speed 20 m/s from the top of a cliff at a 30 degrees angle from the horizontal direction. At the same moment another projectile is sent upwards to intercept the first, from a hight 10 meters below and at a horizontal distance D with respect to the launch site of the first projectile.

4.a) How long will both projectiles be in the air before colliding?

4.b) Which is the horizontal distance D from which the second projectile must be launched for the two projectiles to collide?

(You may need some of of these: $\sin 30^o = 0.500$, $\cos 30^o = 0.866$, $\tan 30^o = 0.577$)



$$\begin{cases} y_A = h + v_0 \sin\theta\, t - \frac{g t^2}{2} \\ x_A = v_0 \cos\theta\, t \end{cases}$$

a) $y_B = v_0 t_{ai} - \frac{g}{2} t^2$, $x_B = D$

Collision $y_A = y_B(t_{air})$

$h + v_0 \sin\theta\, t_{air} - \cancel{\frac{g}{2} t^2_{air}} = v_0 t_{air} - \cancel{\frac{g}{2} t^2_{air}}$

$\sin\theta = \frac{1}{2}$

$\Rightarrow \boxed{t_{air} = \frac{2h}{v_0} = \frac{20 m}{20 m/s} = 1s}$

b) $x_A = x_B$ at collision $\Rightarrow v_0 \cos\theta\, t_{air} = D = 20\frac{m}{s}\, 0.866\ 1s = 17.3\ m$

$\boxed{D = 17\ m}$

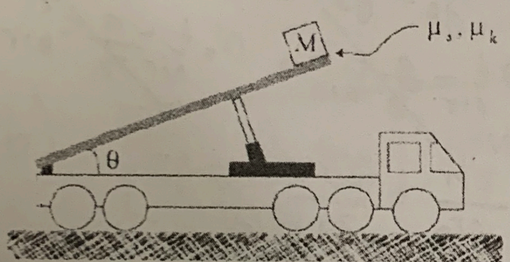
(5) A problem (30 points- 10 each question)
A parcel of mass M is on the raisable freight bed on a delivery truck. The coefficients of static and kinetic friction between the surfaces of the parcel and the truck bed are $\mu_s = 0.577$ and $\mu_k = 0.300$ respectively.

5.a) The driver wants to deliver the parcel by raising the bed by and angle θ until the parcel starts sliding towards the back of the truck. Which is the largest angle for which the parcel will still not slide? (Show the force diagram and show the derivation of your result, starting from Newton's second law)

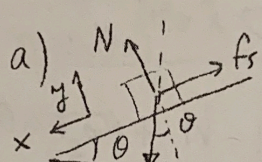
5.b) Assume that as soon as the parcel starts sliding the driver keeps the angle of the flat bed fixed. Which is now the acceleration of the parcel?

5.c) If the driver wants instead to try a new system to make the parcel slide down the truck-bed, consisting in fixing the inclination of the bed at $\theta = 20$ degrees, and accelerating the truck forward. In this case, which is the highest acceleration forward for which the parcel will still not slide?

(You may need some of of these functions: arcsin(0.577)= 35°, arccos(0.577)= 57°, arctan(0.577)= 30°, arcsin(0.300)= 17°, arccos(0.300)= 73°, arctan(0.300)= 17°)



a)



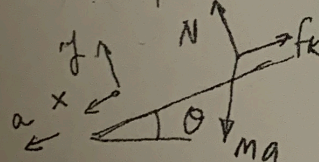
in y: $N - mg\cos\theta = 0$
in x: $mg\sin\theta - f_s = 0$

$\theta = \theta_{max}$ for $f_s = f_{s_{max}} = \mu_s N$

$\Rightarrow mg\sin\theta_{max} = \mu_s mg\cos\theta_{max} \Rightarrow$

$\Rightarrow \boxed{\tan\theta_{max} = \mu_s = 0.577} \Rightarrow \boxed{\theta_{max} = 30°}$

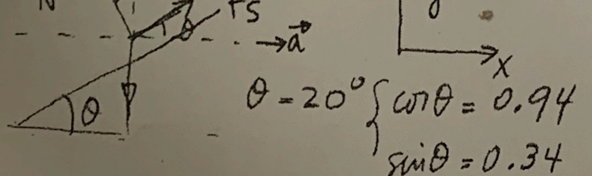
b) f is kinetic, $\theta = 30°$, $\vec{a}$ along the incline, $f_k = \mu_k N = \mu_k mg\cos\theta$



in x: $\boxed{\cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta = \cancel{m}a}$

$\boxed{a = g(0.500 - 0.300 \times 0.866) = 0.24g = 2.35\ m/s^2}$

c) Now the acceleration is horizontal $\longrightarrow \vec{a}$



$\theta = 20°$ $\begin{cases} \cos\theta = 0.94 \\ \sin\theta = 0.34 \end{cases}$

in y: $N\cos\theta - mg + f_s\sin\theta = 0$
in x: $-N\sin\theta + f_s\cos\theta = ma$

a_{max} for $f_s = f_{s_{max}} = \mu_s N \Rightarrow$

$\Rightarrow$ in y: $N(\cos\theta + \mu_s\sin\theta) = mg$ ①
in x: $N(-\sin\theta + \mu_s\cos\theta) = m\,a_{max}$ ②

$\Rightarrow a_{max} = \dfrac{(-\sin\theta + \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}g \Rightarrow$

Divide ②/①

$\Rightarrow \boxed{a_{max} = 0.18\,g = 1.7\ m/s^2}$