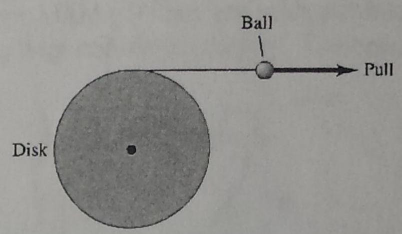
SungHyun Yoon

Problem 1

A disk of radius R=25.0 cm is free to turn about an axle perpendicular to it through its center. It has a very thin but strong string wrapped around its rim, and the string is attached to a ball that is pulled tangentially away from the rim of the disk (see figure). The pull increases in magnitude and produces a linear acceleration of the ball that obeys the equation $a(t)=At$, where t is in seconds and A is a constant. The cylinder starts from rest, at t=3.00 s, the ball's acceleration is $a = 1.80$ m/s^2.



(5 Points) a) What is A? $\boxed{A = 0.6\ m/s^3}$

(5 Points) b) Express the angular acceleration of the disk as function of time. $\boxed{\alpha(t) = 2.4t}$

(8 Points) c) How long after it begins to turn does it take the disk to reach an angular speed of 15.0 rad/s? $\boxed{2.5s}$

(7 Points) d) Through what angle has the disk turned when it reaches an angular speed of 15.0 rad/s? $\boxed{18.75\ rad}$

(a) $a(3.00s) = A(3.00s) = 1.80 m/s^2$. → $\boxed{A = 0.6 m/s^3}$ +5

(b) $a = 0.6t$. m/s^2

$R\alpha = a$. → $\alpha = \frac{a}{R} = \frac{0.6t}{0.250m} = 2.4t$. $\boxed{\alpha = 2.4t}$ +5

(c)

$\omega = 15.0$ rad/s,

$\omega = \omega_0 + \alpha t = \omega_0 + 2.4t^2$. +1

$t^2 = \frac{\omega - \omega_0}{2.4} = \frac{15.0 rad/s}{2.4} = 6.25s^2$ → $\boxed{t = 2.5s}$

(d) $\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t$

$= \frac{1}{2}(0 + 15.0 rad/s)\ 2.5s$ +1

$= \boxed{18.75\ rad}$

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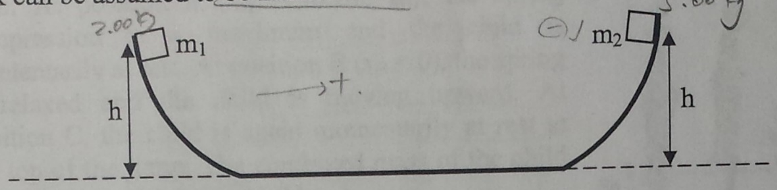
Problem 2

Two blocks are a height $h = 1.50$ m above a flat surface. Block 1 has a mass $m_1 = 2.00$ kg and the other has a mass $m_2 = 5.00$ kg. They are released from rest. At some point along the flat region, they collide elastically. The entire track can be assumed to be frictionless.



(9 Points) a) How high does the 2.00 kg block reach after the collision? $\boxed{5.17\text{m}}$

(8 Points) b) How high does the 5.00 kg block reach after the collision? $\boxed{0.03\text{m}}$

(8 Points) c) If the two blocks collided and stuck together, how high would the combined blocks reach? and which side?

0.27 m
m_1's side (left)

(a) m_1: $W_{other} = 0$, $U_i + K_i = U_2 + K_2$.

$m_1 g h = \frac{1}{2} m_1 v_{1i}^2$

$v_{1i} = \sqrt{2gh}$

m_2: $U_1 + K_1 = U_2 + K_2$.

$v_{2i} = -\sqrt{2gh}$

$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} - \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$

$= \sqrt{2gh}\left(\frac{m_1 - 3m_2}{m_1 + m_2}\right) = 10.07$ m/s

$v_{2f} = \left(\frac{2m_1 - m_2 + m_1}{m_1 + m_2}\right)\sqrt{2gh} = \left(\frac{3m_1 - m_2}{m_1 + m_2}\right)\sqrt{2gh} = 0.77$

m_1 has ~~$v_{1f} = \sqrt{2gh}$~~ immediately ~~after~~ collision.

~~Because m_1's $|v|$ doesn't change after the collision, it goes back up to the point it started, which is a height of $\boxed{1.50\text{m}}$~~

$|v_{1f}| = 10.07$ m/s.

$\frac{1}{2} \cancel{m_1} v_{1f}^2 = \cancel{m_1} g h$ $h_1 = 5.17$ m

(b) The same with m_2. ~~Its $|v|$ hasn't changed before and after the collision, so it goes back up to the point it started,~~ which is a ~~height of $\boxed{1.50\text{m}}$~~

$\frac{1}{2} \cancel{m_2} (v_{2f})^2 = \cancel{m_2} g h$. $h_2 = 0.03$ m

0.77

(c) $m_1 v_{1B} + m_2 v_{2B} = (m_1 + m_2) v_f$

$v_f = \frac{m_1 v_{1B} + m_2 v_{2B}}{m_1 + m_2} = \cancel{\frac{(m_1 + m_2)\sqrt{2gh}}{m_1 + m_2}\sqrt{2gh}} = \frac{-16.27}{7} = -2.32$ m/s

~~$\frac{1}{2}(m_1 + m_2) v^2 = (m_1 + m_2) g H$.~~

~~$H = \frac{v^2}{2g} = \frac{2gh}{2g} = h$.~~

Since $P_{m_2} = m_2\sqrt{2gh} > m_1\sqrt{2gh} = P_{m_1}$, in m_2's direction.

$\therefore$ The combined blocks reach a height of $\boxed{0.27\text{m}}$ on m_1's original side, which is on the left side of the diagram provided.

$\frac{1}{2} \cancel{m} v^2 = \cancel{m} g h$. $h = 0.27$ m

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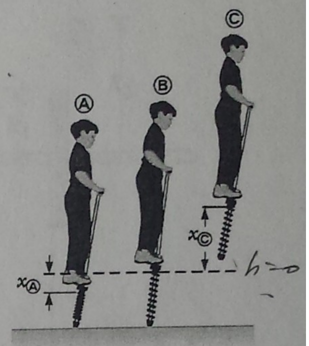
Problem 3

A child's pogo stick as shown on the right stores energy in a spring with a force constant of 2.50×10^4 N/m. At position *A* (x_A = -0.100 m), the spring compression is a maximum and the child is momentarily at rest. At position B ($x_B = 0$), the spring is relaxed and the child is moving upward. At position C, the child is again momentarily at rest at the top of the jump. The combined mass of the child and the pogo stick is m=25.0 kg.



(5 Points) a) Calculate the total energy of the child-stick system if both gravitational and elastic potential energies are zero at position B. $\boxed{100.5\text{J}}$

(5 Points) b) Determine x_C. $\boxed{0.410\text{m}}$

(8 Points) c) Determine the value of x for which the kinetic energy of the system is a maximum. $\boxed{-0.0098\text{m}}$

(7 Points) d) Calculate the child's maximum upward speed. $\boxed{7.944\text{m/s}}$

(a) at B, $U_{total} = 0$, at A, $x_A = -0.100\text{m}$ $\}$ $U_A + \cancel{K_A} = U_B + K_B$.

$mg(x_A) + \frac{1}{2}k(x_A)^2 = E_{tot} = -24.5\text{J} + 125\text{J}$.

$= \boxed{100.5\text{J}}$

(b) $100.5\text{J} = U_C + K_C$

$= mg x_C + \emptyset$.

$x_C = \dfrac{100.5\text{J}}{mg} = \boxed{0.410\text{m}}$

(c) $E_{total} = U + K = \text{constant}$.

$K \Rightarrow$ maximum when $U = $ minimum.

$U = \frac{1}{2}kx^2 + mgx$.

$= 1.25 \times 10^4 x^2 + 245x$.

$U' = 2.5 \times 10^4 x + 245 = 0$.

$x = \dfrac{-245}{2.5 \times 10^4} = \underline{-0.0098\text{m}}$

(d) $100.5\text{J} = U + K$.

$K = 100.5 - U = 100.5 - (1.25 \times 10^4(-0.0098)^2 + 245(-0.0098))$.

$= 99.2995\text{J} = \frac{1}{2}mv^2$. -3

$v = \dfrac{2(99.2995\text{J})}{m} = 7.944\text{m/s}$.

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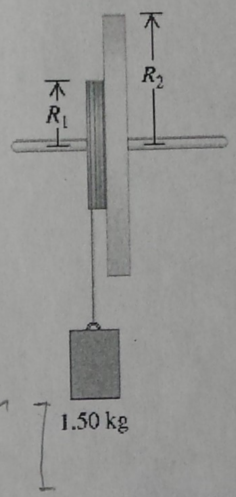
Problem 4

$R_1 = 0.025m$

Two metal disks, one with radius $R_1 = 2.50$ cm and mass $M_1 = 0.80$ kg and the other with radius $R_2 = 5.00$ cm and mass $M_2 = 1.60$ kg, are welded together and mounted on a frictionless axis through their common center as shown.

$R_2 = 0.05m$

(moment of inertia of a disk is $I = \frac{1}{2}MR^2$)



(8 Points) a) What is the total moment of inertia of the two disks? $\boxed{0.00225 \text{ kg}\cdot\text{m}^2}$

(10 Points) b) A light string is wrapped around the edge of the smaller disk, and a 1.50-kg block, suspended from the free end of the string. If the block is released from rest at a distance of 2.00 m above the floor, what is its speed just before it strikes the floor? $\boxed{3.40 m/s}$

(7 Points) c) Repeat the calculation of part b, this time with the string wrapped around the edge of the larger disk. $\boxed{4.95 m/s}$

(a) $I_{tot} = I_1 + I_2 = \frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2 = \frac{1}{2}(M_1R_1^2 + M_2R_2^2) = \frac{1}{2}(0.8kg(0.025m)^2 + 1.6kg(0.05m)^2)$

$= \boxed{0.00225 \text{ kg}\cdot\text{m}^2}$

×8

×10

(b) $v = ?$ $U_1 + K_1 + \cancel{W_{other}} = U_2 + K_2$

$\frac{1}{2}I\omega_0^2 + \cancel{\frac{1}{2}mv_0^2} + mgh = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \longrightarrow R\omega = v.$ $\omega = \frac{v}{R_1}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R_1^2}$

$v^2(\frac{1}{2}m + \frac{1}{2}I/R_1^2) = mgh.$

$v = \sqrt{\frac{2mgh}{m + I/R_1^2}} = \boxed{3.40 m/s.}$

(c) $v = \sqrt{\frac{2mgh}{m_1 + I/R_2^2}} = \boxed{4.95 m/s}$

×7