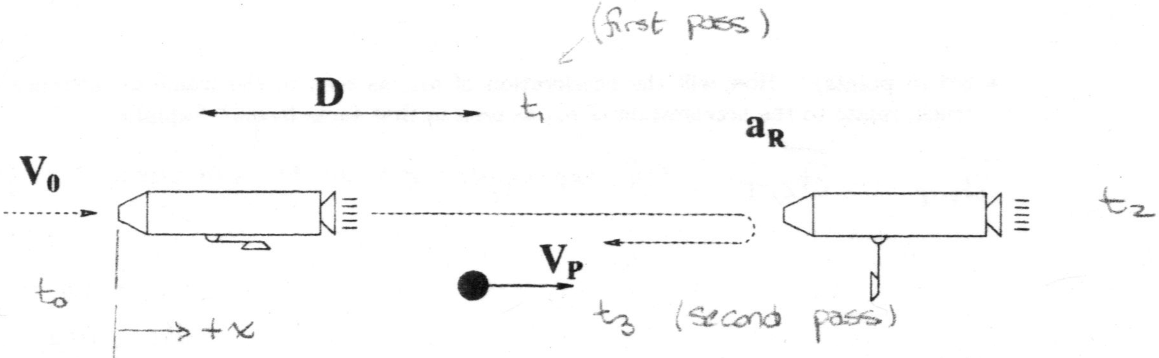


1) A pod-retrieval rocket is designed to overshoot its target and scoop it up on the rebound. It approaches the pod with an initial velocity $\vec{V_0}$ parallel to the pod's (constant) velocity $\vec{V_p}$. When it is a distance D away from the pod (still on approach), the rocket turns on retro-thrusters which give it a constant acceleration of magnitude a_r.

- 1a) (10 points) How much time elapses between the moment the rocket passes the pod and the instant it scoops the pod up? At t_1 and t_3 $x_{ROCKET} = x_{POD}$

Rocket: $x_R(t) = V_0 t - \frac{1}{2} a_R t^2$

Pod: $x_P(t) = D + V_P t$

at t_1 and t_3...

$$x_P(t) = x_R(t)$$

$$D + V_P t = V_0 t - \tfrac{1}{2} a_R t^2$$

$$\frac{1}{2} a_R t^2 - (V_0 - V_P) t + D = 0$$

$$t_1, t_3 = \frac{V_0 - V_P \pm \sqrt{(V_0 - V_P)^2 - 2 a_R D}}{a_R}$$

$t_3 > t_1$, So...

$$\boxed{t_3 - t_1 = \frac{2(V_0 - V_P)}{a_r} \sqrt{1 - \frac{2 a_r D}{(V_0 - V_P)^2}}}$$

- 1b) (10 points) What is the greatest distance ahead of the pod that the rocket will ever get?

$$\Delta X(t) = x_R(t) - x_P(t)$$

$$\Delta X(t) = (V_0 - V_P) t - \tfrac{1}{2} a_R t^2 - D$$

Note: the instant when $V_R = V_P$!

$$\frac{d(\Delta X)}{dt} = (V_0 - V_P) - a_R t = 0$$

$$t = \frac{V_0 - V_P}{a_R}$$ ← when the greatest distance occurs...

$$\boxed{\Delta X_{max} = \frac{(V_0 - V_P)^2}{2 a_R} - D}$$

Plug this back in to $\Delta X(t)$...

• 1c) (10 points) **How fast is the pod moving relative to the rocket when it gets scooped?**

The pod gets scooped at t_3... from part a:

$$t_3 = \frac{V_0 - V_p}{a_R}\left[1 + \sqrt{1 - \frac{2a_R D}{(V_0 - V_p)^2}}\right]$$

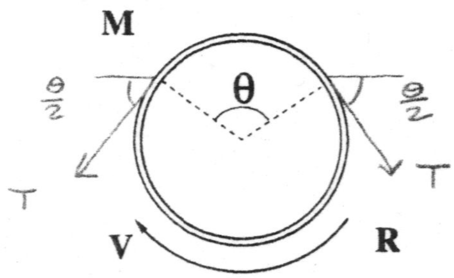
$$\vec{V}_{P,R} = \vec{V}_{P,g} - \vec{V}_{R,g} \qquad V_{xR,g} = \frac{d}{dt}(x_R(t)) = V_0 - a_R t$$

$$= \vec{V}_P - \vec{V}_R(t)$$

$$V_{x\,P,R}(t_3) = V_p - V_0 + a_R t_3$$

$$V_{xPR}(t_3) = -(V_0 - V_p) + (V_0 - V_p) + (V_0 - V_p)\sqrt{1 - \frac{2a_R D}{(V_0 - V_p)^2}}$$

$$\boxed{V_{xP,R}(t_3) = (V_0 - V_p)\sqrt{1 - \frac{2a_R D}{(V_0 - V_p)^2}}}$$



2) A thin uniform ring of mass M and radius R rotates about an axis through its center and perpendicular to the plane of the ring in such a way that the points in the ring all move with a speed v. This happens in a region of space where the effects of gravity can be safely ignored...

- 2a) (5 points) How much mass is contained in a segment of the ring that spans an angle θ?

$$M(\theta) = \frac{M}{2\pi R}(R\theta)$$

$$\boxed{M(\theta) = \frac{M}{2\pi}\theta}$$

- 2b) (5 points) If θ is small, we can treat the segment as a point-mass moving in a circle of radius R. What is the magnitude and direction of the net force that acts on such a small segment?

$$\Sigma F_r = ma_r$$

$$\Sigma F_r = M(\theta)\frac{v^2}{R}$$

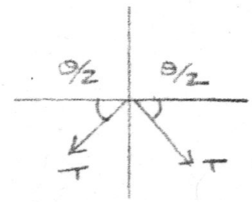
$$\Sigma F_r = \frac{Mv^2}{2\pi R}\theta$$

The net force points in towards the center of the ring with a magnitude

$$\frac{Mv^2\theta}{2\pi R}$$

- 2c) (10 points) What is the force that keeps the ring from flying apart as it spins? Assume θ is small enough to treat the segment as a point-mass moving in a circle of radius R but large enough to explore the geometry in the problem and find the magnitude of the force that holds the segment in place as a function of the angular size of the segment (θ).

Tension holds the ring together (see the picture on the previous page)



$$\Sigma F_R = 2T\sin\left(\frac{\theta}{2}\right)$$

Since $\Sigma F_R = \frac{mv^2\theta}{2\pi R}$

$$2T\sin\left(\frac{\theta}{2}\right) = \frac{mv^2\theta}{2\pi R}$$

$$\boxed{T = \frac{mv^2 \frac{\theta}{2}}{2\pi R \sin(\theta/2)}}$$

- 2d) (5 points) To get the correct answer for part c, we have to assume that θ is really, really small. Evaluate your answer for part c in the limit $\theta \to 0$.

$$\lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$$

$$\boxed{T = \frac{mv^2}{2\pi R}}$$

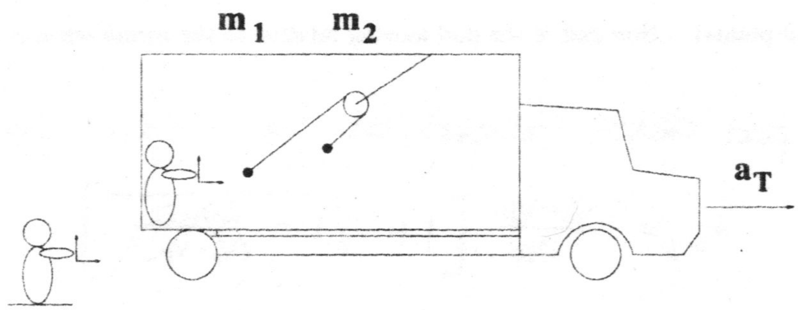
- 2e) (5 points) Suppose the ring is made of string. If we were to perturb the string slightly by introducing a small bump, that bump would travel around the string with a speed (relative to the string) equal to $\sqrt{T/\mu}$, where T is the tension in the string and μ is the linear mass density of the string. Using the information you derived earlier, describe how a bump traveling through a spinning string would look like to an outside observer. (Hint: there are two cases to consider.)

$$V_x = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \sqrt{\frac{T2\pi R}{m}} = V$$:)

The perturbation travels with a speed V relative to the string

* If it moves in the same direction as the string, an outside observer will see it whip around the string twice as fast (in the observer's frame) as the string is moving

* If it moves in the opposite direction, the observer will see the bump stand still in her frame! (Cool!)



3) An "Atwood Machine" is constructed using two small masses, a light rope and a massless pulley. The device is hung in the back of a large truck and the truck, in turn, is given a horizontal acceleration a_T in the forward direction. The Atwood machine tilts back, in response to the truck's acceleration, until the center of the pulley appears to be at rest *with respect to the truck*. In this configuration, let's call the mass attached to the upper portion of the rope m_1 and the mass attached to the lower portion, m_2.

- 3a) (5 points) While Newton's laws are only truly relevant in inertial frames of reference, Galileo's relative velocity relationship holds for all frames (so long as they aren't moving *too* fast) - and so, by extension, would an expression for relative acceleration. Write vector expressions that relate the acceleration of each mass as seen in a coordinate system attached to the ground to the acceleration of each mass as seen relative to a coordinate system attached to the truck. Evaluate each of these expressions for components taken along the horizontal and vertical direction.

$$\vec{a}_{1,g} = \vec{a}_{1,T} + \vec{a}_{T,g} \qquad \vec{a}_{2,g} = \vec{a}_{2,T} + \vec{a}_{T,g}$$

$$a_{1x,g} = a_{1x,T} + a_T \qquad a_{2x,g} = a_{2x,T} + a_T$$

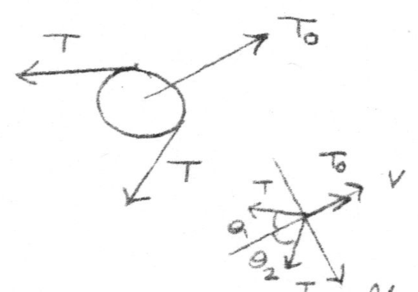
$$a_{1y,g} = a_{1y,T} \qquad a_{2y,g} = a_{2y,T}$$

- 3b) (5 points) Consider how Newton's laws apply to the massless pulley and show that the upper and lower portions of the rope that connects m_1 to m_2 must be parallel.

$$\Sigma F_u = \cancel{m}\vec{a}_u \qquad \Sigma F_v = \cancel{m}\vec{a}_v$$

$$T\sin\theta_2 - T\sin\theta_1 = 0 \qquad T_0 - 2T\cos\theta_1 = 0$$

$$\theta_1 = \theta_2 \qquad T\cos\theta_1 = \tfrac{1}{2}T_0$$



Hmm - this shows the portions have to come off symmetrically around the line defined by T_0, but that's not quite enough... (Corbin gets 3/5 ☺)

T is smallest when $\theta_1 = \theta_2 = 0$ (parallel portions) ☺

- 3c) (5 points) How will the acceleration of m_1, as seen in the frame of reference attached to the truck, relate to the acceleration of m_2 as seen in that same frame? Explain.

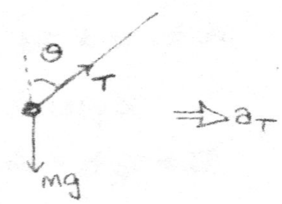
$\vec{a}_{1,T} = -\vec{a}_{2,T}$ (if the upper and lower portions are parallel)

$$L_1 + L_2 = L$$
$$\frac{dL_1}{dt} + \frac{dL_2}{dt} = 0$$
$$-V_{1,v} - V_{2,v} = 0$$
$$a_{1,v} = -a_{2,v}$$

relative to u,v axis in truck

- 3d) (15 points) Find the tension in the rope that connects m_1 to m_2. For full credit, your answer should be clear and follow logically from first-principles.

$$\left.\begin{aligned} T\sin\theta &= m_1 a_{1x,g} & T\cos\theta - m_1 g &= m_1 a_{1y,g} \\ T\sin\theta &= M_2 a_{2x,g} & T\cos\theta - M_2 g &= M_2 a_{2y,g} \end{aligned}\right\} N2$$



$$T\sin\theta = m_1 (a_{1xT} + a_T)$$
$$T\sin\theta = M_2 (a_{2xT} + a_T)$$
$$\left(\frac{1}{m_1} + \frac{1}{M_2}\right) T\sin\theta = 2a_T$$

$$a_{1xT} = -a_{2xT}$$
$$a_{1yT} = -a_{2yT}$$

$$T\cos\theta = m_1 (g + a_{1yT})$$
$$T\cos\theta = M_2 (g + a_{2yT})$$
$$\left(\frac{1}{m_1} + \frac{1}{M_2}\right) T\cos\theta = 2g$$

$$a_T = g\tan\theta$$

Not Too Suprising

$$T\sin\theta = \frac{m_1 M_2}{M_1 + M_2} 2g\tan\theta$$

$$T = \frac{M_1 M_2}{M_1 + M_2} \frac{2g}{\cos\theta}$$

$$\boxed{T = \frac{2M_1 M_2}{M_1 + M_2}\sqrt{a_T^2 + g^2}}$$