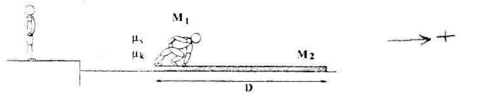



1) A long wooden platform of length $D$ and mass $M_2$ floats on the surface of a pond. A sprinter of mass $M_1$ stands at the edge of the platform nearest to the shore. When the observer on the shore fires her starter's pistol, the sprinter is to dash to the other end of the platform as rapidly as she can.

For the following, we'll assume that the sprinter and the platform start at rest relative to the shore, there is but negligible friction and drag between the platform and the pond and the coefficients of kinetic and static friction between the sprinter's shoes and the platform are $\mu_k$ and $\mu_s$, respectively.

- 1a) (5 points) Explain, in terms of the physics involved, how this can be achieved. In particular, address issues like: *Where does the force that propels the runner come from? How is that force obtained? Under what conditions will the sprinter have her greatest acceleration? What is the largest acceleration she can have, and why?* The graders will reward you for providing (correct) detail, being concise, precise and clear.

The sprinter has to push backwards (towards the shore) against the platform with her feet. Newton's third law results in an equal and opposite forward force from the platform on the sprinter that propels her forward. The force in play is friction - if she can push against the platform without slipping on it the sprinter can take advantage of the fact that $\mu_s > \mu_k$ - If she pushes hard enough to almost slip, she will be propelled forward at the fastest rate available...

$\Sigma F_x = ma_x$ $\quad \mu_s N = \cancel{m} a_x$

$F_{s,max} = ma_x$ $\quad \mu_s \cancel{m} g = \cancel{m} a_x$ $\Rightarrow$ $\boxed{a_{x,max} = \mu_s g}$

for the sprinter...

- 1b) (5 points) We'll assume for the rest of this that the sprinter is running with the greatest acceleration available to her. How fast, and in what direction is the platform accelerating as she does so?

for the platform: $\vec{F}_{S,P} = -\vec{F}_{P,S}$

$\Rightarrow \quad F_{SP} = F_{PS} = \mu_s M_1 g$ (magnitude!)

$\Sigma F_x = ma_x$

$-F_{SP} = M_2 a_x$

$-\mu_s M_1 g = M_2 a_x$

$$\boxed{a_x = -\frac{M_1}{M_2}\mu_s g}$$

for the platform... $a = \frac{M_1}{M_2}\mu_s g$ directed towards the shore

- 1c) (10 points) How long will it take for the sprinter to reach the other end of the platform?

Sprinter: $a_x = \mu_s g$

$t_0 = 0$, $x_0 = 0$, $V_{0x} = 0$ $\qquad$ $t_1$, $x_1 = \tilde{x}_1$, $V_{1x}$

Platform (far side): $\tilde{a}_x = -\mu_s g\, m_1/m_2$

$t_0 = 0$, $\tilde{x}_0 = D$, $\tilde{V}_{0x} = 0$ $\qquad$ $t_1$, $\tilde{x}_1 = x_1$, $\tilde{V}_{1x}$

$x_1 = \tilde{x}_1$

x) $x_0 + V_{0x}t_1 + \frac{1}{2}a_x t_1^2 = \tilde{x}_0 + \tilde{V}_{0x}t_1 + \frac{1}{2}\tilde{a}_x t_1^2$

$\frac{1}{2}\mu_s g t_1^2 = D - \frac{1}{2}\frac{M_1}{M_2}\mu_s g t_1^2$ $\Rightarrow$ $\boxed{t_1 = \sqrt{\frac{2m_2 D}{\mu_s g(m_1+m_2)}}}$

- 1d) (5 points) How far will the sprinter have moved relative to the observer on the shore in this time?

(1x) $x_1 = x_0 + V_{0x}t_1 + \frac{1}{2}a_x t_1^2$

$x_1 - x_0 = \frac{1}{2}\mu_s g \cdot \frac{2m_2 D}{\mu_s g(m_1+m_2)}$

$$\boxed{x_1 - x_0 = \frac{M_2}{M_1+M_2}D}$$

- 1e) (5 points) How far will the *platform* have moved relative to the observer on the shore in this time?

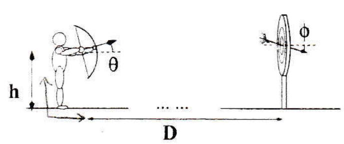
(1x) $\tilde{x}_1 = \tilde{x}_0 + \tilde{V}_{0x}t_1 + \frac{1}{2}\tilde{a}_x t_1^2$

$\tilde{x}_1 - \tilde{x}_0 = -\frac{1}{2}\frac{M_1}{M_2}\mu_s g \cdot \frac{2m_2 D}{\mu_s g(M_1+M_2)}$

$$\boxed{\tilde{x}_1 - \tilde{x}_0 = -\frac{M_1}{M_1+M_2}D}$$

That is... $\frac{M_1}{M_1+M_2}D$ towards the Shore

Notes:
- As an exercise, you should evaluate these answers in the limits $M_1 \ll M_2$ and $M_2 \ll M_1$ and see if you can interpret the results
- If you're feeling some deja vu ~ this problem is very closely related to center-of-mass and Momentum. We'll cover those soon - this is really about planting some seeds early :)




2) An archer stands a distance $D$ from her target. She aims her bow at an angle $\theta$ above the horizontal and shoots. The arrow strikes the target and sticks, making an angle $\phi$ with the horizontal, as shown.

Take $\theta$ and $\phi$ to be positive quantities (that is, magnitudes of angular deflection) and assume the arrow starts its flight at a height $h$ above the ground...

- 2a) (10 points) With what speed did the arrow leave the bow?

$t_0 = 0$: $x_0 = 0$, $y_0 = h$, $V_{0x} = V_0\cos\theta$, $V_{0y} = V_0\sin\theta$

$t_1 =$: $x_1 =$, $y_1 =$, $V_{1x} =$, $V_{1y} = 0$

$t_2$: $x_2 = D$, $y_2 =$, $V_{2x} = V_2\cos\phi$, $V_{2y} = -V_2\sin\phi$

(2x) $V_x = V_{0x} + \cancel{a_x} t$

$V_{2x} = V_{0x}$

$V_2\cos\phi = V_0\cos\theta$

(2y) $V_y = V_{0y} + a_y t$

$V_{2y} = V_{0y} - gt_2$

$-V_2\sin\phi = V_0\sin\theta - gt_2$

$V_2\sin\phi = \frac{gD}{V_0\cos\theta} - V_0\sin\theta$

(1x) $x = \cancel{x_0} + V_{0x}t + \frac{1}{2}\cancel{a_x}t^2$

$x_2 = V_0\cos\theta\, t_2$

$D = V_0\cos\theta\, t_2$

$\tan\phi = \frac{V_2\sin\phi}{V_2\cos\phi}$

$\tan\phi = \frac{gD}{V_0^2\cos^2\theta} - \tan\theta$

$\frac{gD}{V_0^2\cos^2\theta} = \tan\theta + \tan\phi$

$$\boxed{V_0 = \frac{1}{\cos\theta}\sqrt{\frac{gD}{\tan\theta + \tan\phi}}}$$

Check this in the case $\theta = \phi$ :)

- 2b) (10 points) How far above the ground did the arrow strike the target?

(1y) $y_2 = y_0 + V_{0y}t_2 - \frac{1}{2}gt_2^2$

$y_2 = h + V_0\sin\theta\, t_2 - \frac{1}{2}gt_2^2$

(1x) $x_2 = \cancel{x_0} + V_{0x}t_2$

$D = V_0\cos\theta\, t_2$

$y_2 = h + D\tan\theta - \frac{gD^2}{2V_0^2\cos^2\theta}$

$y_2 = h + D\tan\theta - \frac{1}{2}gD^2 \cdot \frac{\tan\theta + \tan\phi}{gD}$

$$\boxed{y_2 = h + \frac{D}{2}(\tan\theta - \tan\phi)}$$

again... it's good to check this in the case $\theta = \phi$

- 2c) (10 points) At the moment when the arrow was oriented along the horizontal direction, how high was it above the ground?

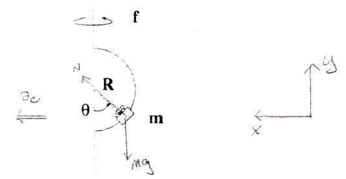
(3y) $\cancel{V_{1y}^2} = V_{0y}^2 - 2g(y_1 - y_0)$

$y_1 = h + \frac{V_0^2\sin^2\theta}{2g}$

$y_1 = h + \frac{1}{2g}\frac{\sin^2\theta}{\cos^2\theta}\left(\frac{gD}{\tan\theta + \tan\phi}\right)$

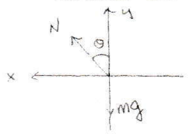
$$\boxed{y_1 = h + \frac{1}{2}D \cdot \frac{\tan^2\theta}{\tan\theta + \tan\phi}}$$

Since you're in the habit of it now... Check this one too :)




3) A new ride at the Tragic Kingdom involves placing people in a cart of mass $m$ that is free to slide (without friction) along a semicircular rail of radius $R$. The rail, in turn, rotates about a vertical axis at a rate $f$ revolutions per unit time. We are interested in finding the angle (made with the vertical) to which the cart will rise when it reaches a stationary position relative to the rotating rail.

- 3a) (5 points) Draw a free-body diagram showing the forces that act on the cart.




easy enough :)

- 3b) (5 points) How fast would the cart have to be moving to keep up with the rotating rail if it were located at the angle $\theta$ drawn on the diagram? (Hint - this part is a geometry problem, not dynamics)

The cart is traveling in a circle of radius $r = R\sin\theta$... Its speed...

$$V = \frac{\text{distance traveled in one revolution}}{\text{time for one revolution..}}$$

$$V = \frac{2\pi r}{1/f}$$

$$\boxed{V = 2\pi f R\sin\theta}$$

- 3c) (10 points) Will the cart accelerate? If so, in what direction and with what magnitude? [It may help the grader if you also sketch the acceleration vector onto the diagram above].

Yes... It will have an acceleration of magnitude $a_c = \frac{v^2}{r} = \frac{(2\pi f R\sin\theta)^2}{R\sin\theta} = 4\pi^2 f^2 R\sin\theta$ pointed in towards the center of its orbit (along a line perpendicular to the vertical axis)

- 3d) (10 points) We're ready - find the value of $\theta$ at which the cart remains stationary relative to the rail.

$\Sigma F_x = ma_x$

$N\sin\theta = m4\pi^2 f^2 R\sin\theta$

$\Sigma F_y = ma_y$

$N\cos\theta - mg = 0$

$$\frac{N\sin\theta}{N\cos\theta} = \frac{4\pi^2 f^2 R\sin\theta}{g}$$

$$\tan\theta = \frac{Rf^2}{g} \cdot 4\pi^2\sin\theta$$

$$\boxed{\cos\theta = \frac{g}{4\pi^2 R f^2}}$$