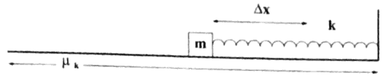


1) A block of mass m is set at rest on a rough surface (μ_k) up against a spring (k) that is initially uncompressed. Someone then comes along and pushes the mass into the spring until the spring is compressed an amount Δx, and holds it there...

- 1a) (5 points) What is the total amount of work done on the block during the compression?

$\Delta K = W_{TOT}$

The block starts and ends at rest, So...

$$\boxed{W_{TOT} = 0}$$

- 1b) (5 points) How much of that work (done on the block during compression) was done by the spring?

$W_{sp} = -\Delta U_{sp}$

$W_{sp} = -\frac{1}{2}k\,(x_f^2 - x_i^2)$ $x_i = 0$, $x_f = \Delta x$

$$\boxed{W_{sp} = -\tfrac{1}{2}k\,\Delta x^2}$$

- 1c) (10 points) How much work was done (in total) by the someone who compressed the spring?

$W_{TOT} = W_N + W_g + W_{sp} + W_{fric} + W_{ext}$

$0 = 0 + 0 + (-\frac{1}{2}k\Delta x^2) + (-\mu_k mg\Delta x) + W_{ext}$

$W_{fric} = \int |\vec{f}|\,|d\vec{r}|\cos(180°) = \mu_k\, mg\,\Delta x\,(-1)$

$$\boxed{W_{ext} = \tfrac{1}{2}k\Delta x^2 + \mu_k mg\,\Delta x}$$

That is - Work was done to Compress the spring and move the block over friction.

- 1d) (10 points) If the block is then released, how far from its initial position (when the spring was compressed) will it move before coming to rest?

$\Delta E = W_{NC}$

$\Delta K + \Delta U_g + \Delta U_s = W_{fric}$

$0 + 0 + \frac{1}{2}k(0 - \Delta x^2) = -\mu_k mg\,d$

$$\boxed{d = \frac{k\,\Delta x^2}{2\mu_k mg}}$$

All surfaces are frictionless

2) A pair of identical ramps of mass M_2 sit at rest, facing one another, on a frictionless horizontal surface, as shown. A small block of mass M_1 is placed at a height h on the left ramp and released from rest. Assuming the contact between the block and each ramp is also frictionless...

- 2a) (10 points) How fast is the block moving when it reaches the bottom of the left ramp? (It might be a good idea to check your answer in the limits $M_1 << M_2$ and $M_1 >> M_2$).

$\Sigma P_{xi} = \Sigma P_{xf}$

$0 = m_1 V_{1Ax} + m_2 V_{2Ax}$

$V_{2Ax} = -\frac{m_1}{m_2} V_{1Ax}$

$\Delta E = \cancel{W_{NC}}^{\,0}$

$m_1 gh = \frac{1}{2}m_1 V_{1Ax}^2 + \frac{1}{2}m_2 V_{2Ax}^2$

$m_1 gh = \frac{1}{2}m_1 V_{1Ax}^2\,(1 + \frac{m_1}{m_2})$

(1-D) (Components OK)

$$\boxed{V_{1Ax} = \sqrt{2gh\,\frac{m_2}{m_1+m_2}}}$$

Check:
$m_1 >> m_2$ $V_{1x} \sim 0$
$m_1 << m_2$ $V_{1x} \sim \sqrt{2gh}$

- 2b) (10 points) How are the horizontal components of the block's and the right ramp's velocities related when the block has traveled as far up the right ramp as it's going to go? Find the greatest height to which the block climbs on the right ramp.

At the instant the block reaches its highest point, it is no longer moving (up or down) with respect to the ramp, $V_{1Bx} = V_{2Bx}$

$\Sigma P_{xi} = \Sigma P_{xf}$

$m_1 V_{1Ax} = (m_1 + m_2)\,V_{12Bx}$

$V_{12Bx} = \frac{m_1}{m_1+m_2} V_{1Ax}$

$\Delta E = \cancel{W_{NC}}$

$\frac{1}{2}m_1 V_{1Ax}^2 = \frac{1}{2}(m_1+m_2)V_{12Bx}^2 + m_1 g h_B$

$\frac{1}{2}m_1 V_{1Ax}^2 = \frac{1}{2}m_1 V_{1Ax}^2\,(\frac{m_1}{m_1+m_2}) + m_1 g h_B$

$h_B = \frac{1}{2g} V_{1Ax}^2\,(1 - \frac{m_1}{m_1+m_2})$

$$\boxed{h_B = h\left(\frac{m_2}{m_1+m_2}\right)^2}$$

This is actually kind of Cool!

- 2b) (*continued*)

- 2c) (10 points) How fast will the block be traveling when it leaves the bottom of the right ramp? Under what condition(s) will it return to the left ramp?

We can find the velocity of the block as it leaves the right ramp by analyzing the elastic collision it made with the ramp (that is, consider the incident block, the departing block and ramp, ignore the moments when the block is climbing the ramp)

$\Sigma P_{xi} = \Sigma P_{xf}$

$m_1 V_{1Ax} = m_1 V_{1Cx} + m_2 V_{2Cx}$

$m_1 V_{1Ax} = m_1 V_{1Cx} + m_2(V_{1Ax} + V_{1Cx})$

$V_{1Cx} = \frac{M_1 - M_2}{M_1 + M_2} V_{1Ax}$

$\Sigma K_i = \Sigma K_f$ 1D ELASTIC Shortcut

$V_{1Ax} + V_{1Cx} = \cancel{V_{2Ax}} + V_{2Cx}$

(from part a) $V_{2Ax} = -\frac{M_1}{M_2} V_{1Ax}$

Both masses have negative components. To Catch up - V_{1Cx} needs to be more negative than V_{2Ax}

$\frac{M_1 - M_2}{M_1 + M_2} < -\frac{M_1}{M_2}$

The 'easiest' answer is that $M_1 < M_2$

But you can do better (for full credit ☺)

$m_1^2 + 2m_2 m_1 - m_2^2 < 0$

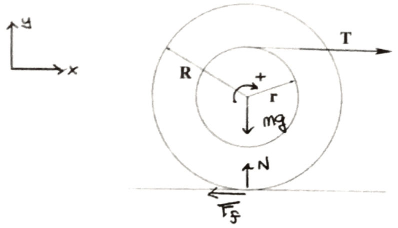
$m_1^2 + 2m_1 m_2 - m_2^2 = 0$

$m_1 = m_2(-1 \pm \sqrt{2})$

$(m_1 - m_2(-1-\sqrt{2}))(m_1 - m_2(-1+\sqrt{2})) < 0$

$$\boxed{M_1 < M_2(\sqrt{2} - 1)}$$

FOR FULL CREDIT



No-Slip
$a_x = R\alpha$

3) A heavy, uniform cylinder has a mass m and a radius R. It is accelerated by a force of magnitude T that is applied via a rope wound around a light (read negligible) drum of radius r that is attached to the cylinder. The coefficient of static friction is sufficient to allow the cylinder to roll without slipping.

- 3a) (15 pts) Find the frictional force acting on the cylinder.

$\Sigma F_x = ma_x$

$T - F_f = ma_x$

$\Sigma F_y = ma_y$

$N - mg = 0$

$\Sigma \vec{\tau} = I\vec{\alpha}$

$a_x = R\alpha$

$rT + RF_f = \frac{1}{2}mR^2\left(\frac{a_x}{R}\right)$

$rT + RF_f = \frac{1}{2}mR\left(\frac{T}{m} - \frac{F_f}{m}\right)$

$rT + RF_f = \frac{1}{2}RT - \frac{1}{2}RF_f$

$\frac{3}{2}RF_f = \left(\frac{1}{2}R - r\right)T$

$$\boxed{F_f = T\,\tfrac{1}{3}\left(1 - 2\tfrac{r}{R}\right)}$$

- 3b) (5 pts) Find the acceleration of the center-of-mass of the cylinder.

$a_x = \frac{1}{m}T\left(1 - \frac{F_f}{T}\right)$

$a_x = \frac{T}{m}\left[1 - \frac{1}{3} + \frac{2}{3}\frac{r}{R}\right]$

$$\boxed{a_x = \tfrac{T}{m}\,\tfrac{2}{3}\left(1 + \tfrac{r}{R}\right)}$$

- 3c) (5 pts) If you were to pull on a block that has the same mass as the cylinder with a horizontal force T, what would be the largest acceleration you could give it? Show that, with a proper choice of r, you can give the cylinder a larger acceleration than the block!

$\boxed{\text{The maximum acceleration you could give a block: } T/m}$

For the cylinder, we can exceed this if:

$\frac{2}{3}\left(1 + \frac{r}{R}\right) > 1$

$\Rightarrow \boxed{r > R/2}$

- 3d) (5 pts) In what direction will the force of friction point when the cylinder is accelerating under the circumstances laid out in part c?

> if $r > R/2$, $F_f \rightarrow$ negative... that is, it points opposite the direction we assumed – it is pointing forward (why?? :') → This, of course is what gives the center-of-mass its extra boost!