MT2 Physics 1A F17

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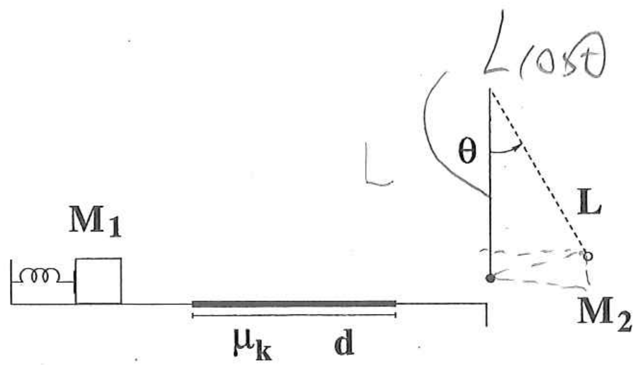
Seat Number ____________________

Problem	Grade
1	20 /30
2	26 /30
3	0 /30
Total	45 /90

46

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



1) A block (M_1) is launched from a spring to travel over a horizontal surface that is frictionless apart from a small patch of length d and coefficient μ_k. After traveling over the patch, the block makes an elastic collision with a point-mass (m_2) hung at rest from a string of length L. Following the collision, the block reverses direction and comes to rest in the friction patch (without having returned to the spring).

- 1a) (10 points) Compare the velocity of M_1 after it strikes the point-mass to the velocity of M_1 before it strikes the point-mass. Compare the mass of M_2 to the mass of M_1.

+5

V₁ after and V₁ before have different direction

$|V_1 \text{after}| < |V_1 \text{before}|$, because the kinetic energy of M₁ reduced

$\frac{1}{2} M_1 V_1 \text{before}^2 = \frac{1}{2} M_1 V_1 \text{after}^2 + \frac{1}{2} M_2 V_2^2$ +2

$M_1 (V_1 \text{before}^2 - V_1 \text{after}^2) = M_2 V_2^2$

$\therefore \frac{M_1}{M_2} = \frac{V_2^2}{V_1 \text{before}^2 - V_1 \text{after}^2}$ +1 close...

according to conservation of momentum $M_1 V_1 \text{before} = M_1 V_1 \text{after} + M_2 V_2$

$\therefore M_1 (V_1 \text{before} - V_1 \text{after}) = M_2 V_2$ +2

$\therefore M_2 V_2 (V_1 \text{before} + V_1 \text{after}) = M_2 V_2^2 \therefore V_1 \text{before} + V_1 \text{after} = V_2$

$\therefore \frac{M_1}{M_2} = \frac{V_2^2}{V_2 (V_1 \text{before} - V_1 \text{after})} = \frac{V_2}{V_1 \text{before} - V_1 \text{after}}$

- 1b) (15 points) Given what you know about where the block comes to rest, find the range of angles (measured with respect to the vertical, as shown) to which the point-mass may rise. +14

$\frac{1}{2} M_1 V_{1after}^2 < \mu M_1 g \cdot d$ ∴ $V_{1after}^2 < 2\mu g d$, $V_{1after} < \sqrt{2\mu g d}$ +6

$\frac{1}{2} M_2 V_2^2 = M_2 g (L - L\cos\theta)$ +2

∴ $L(1-\cos\theta) = \frac{\frac{1}{2}V_2^2}{g}$, $1-\cos\theta = \frac{V_2^2}{2gL}$, $\cos\theta = 1 - \frac{V_2^2}{2gL}$

$\frac{M_1}{M_2} = \frac{V_{1before} + V_{1after}}{V_{1before} - V_{1after}}$

$M_1 V_{1befor} - M_1 V_{1after} = M_2 V_{1before} + M_2 V_{1after}$

∴ $V_{1before}(M_1 - M_2) = V_{1after}(M_2 + M_1)$

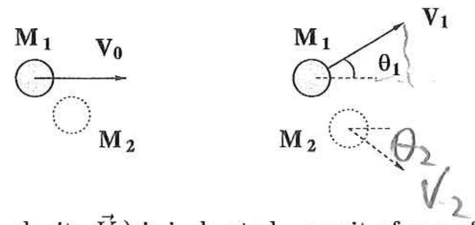
∴ $V_{1befor} = \frac{V_{1after}(M_2 + M_1)}{M_1 - M_2} < \sqrt{2\mu g d}\,\frac{M_1+M_2}{M_1-M_2}$

∴ $V_2 = V_{1before} + V_{1after} < \sqrt{2\mu g d}\left(1 + \frac{M_1+M_2}{M_1-M_2}\right)$ +5

∴ $\cos\theta > 1 - \frac{2\mu g d\left(1+\frac{M_1+M_2}{M_1-M_2}\right)^2}{2gL}$ $= \cos^{-1}\left(1 - \frac{\mu d\left(1+\frac{M_1+M_1}{M_1-M_2}\right)^2}{L}\right)$ +2

- 1c) (5 points) Use a (linear) approximation to simplify the answer to part b in the event that θ is small and briefly discuss the effects μ_k, L, d and M_1 have on θ. +1

L bigger, θ smaller ✓



2) A demo-dog (mass M_1, initial velocity $\vec{V}_0$) is in heated pursuit of one of the slower, meatier, members of the Hawkins Middle School AV Club. Before it can catch up to its prey, it collides with a stationary trash can of mass M_2. After the collision, the demo-dog moves with a speed V_1, deflected by an angle θ_1 from its initial path (as shown).

- 2a) (15 points) How fast was the trash can moving and in what direction (relative to the demo-dog's initial velocity) after the collision?

Conservation of momentum:

X: $M_1 V_0 = M_1 V_1 \cos\theta_1 + M_2 V_2 \cos\theta_2$

y: $0 = M_1 V_1 \sin\theta_1 + M_2 V_2 \sin\theta_2$

$\therefore V_2 \sin\theta_2 = \frac{-M_1 V_1 \sin\theta_1}{M_2}$

$V_2 \cos\theta_2 = \frac{M_1 V_0 - M_1 V_1 \cos\theta_1}{M_2}$

$\theta_2 = \arctan\frac{-V_1 \sin\theta_1}{V_0 - V_1\cos\theta_1}$

+15

$\tan\theta_2 = \frac{-M_1 V_1 \sin\theta_1}{M_1 V_0 - M_1 V_1 \cos\theta_1} = \frac{-V_1\sin\theta_1}{V_0 - V_1\cos\theta_1}$

$|V_2|^2 = (V_2\sin\theta_2)^2 + (V_2\cos\theta_2)^2 = \frac{M_1^2 V_1^2 \sin^2\theta_1 + M_1^2 V_0^2 + M_1^2 V_1^2 \cos^2\theta - 2M_1V_0M_1V_1\cos\theta_1}{M_2}$

$= \frac{M_1^2 V_1^2 + M_1^2 V_0^2 - 2M_1 V_0 M_1 V_1 \cos\theta}{M_2}$

$\therefore |V_2| = \frac{\sqrt{M_1^2 V_1^2 + M_1^2 V_0^2 - 2M_1 V_0 M_1 V_1 \cos\theta}}{M_2}$

- 2b) (10 points) By what fraction of the demo-dog's initial energy did the total energy of the system (demo-dog + trash can) change?

$K_{dog\ i} = \frac{1}{2} M_1 V_0^2$ $K_{system-after} = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2$

, we are looking for

$$\frac{\frac{1}{2}M_1 V_0^2 - \frac{1}{2} M_1 V_1^2 - \frac{1}{2} M_2 V_2^2}{\frac{1}{2} M_1 V_0^2} = \frac{M_1 V_0^2 - M_1 V_1^2 - M_2 V_2^2}{M_1 V_0^2}$$

$$= \frac{M_1 V_0^2 - M_1 V_1^2 - M_1^2 V_1^2 - M_1^2 V_0^2 + 2 M_1 V_0 M_1 V_1 \cos\theta}{M_1 V_0^2}$$

+8

not quiet

$$= \frac{V_0^2 - V_1^2 - M_1 V_1^2 - M_1 V_0^2 + 2 V_0 M_1 V_1 \cos\theta}{V_0^2}$$

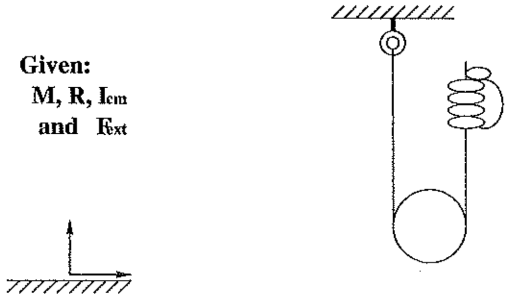
- 2c) (5 points) While in principle, the answer to *b* would seem to allow for the possibility of an elastic collision, the likelihood of that happening is essentially nil - collisions with trash cans are generally noisy, messy affairs. What would your answer to part *b* look like if the collision was elastic, and how does nature manage to avoid this condition?

it would be zero ✓

+3

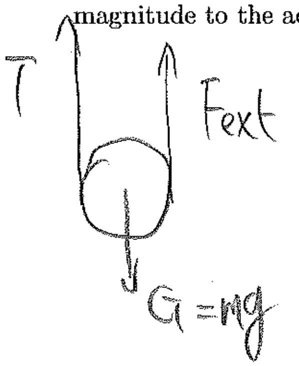
according to math

$V_0^2 - V_1^2 - M_1 V_1^2 - M_1 V_0^2 + 2 V_0 M_1 V_1 \cos\theta \neq 0$



3) A cylindrical pulley, described by the (known) parameters M, R and I_{cm} turns without slipping over a massless rope that is tied to a fixed hook on one end and held by a hand on the other. The hand exerts a force F_{ext} on the rope as the pulley is lowered. You may assume that the segments of rope that appear to be vertical in the diagram are, indeed, vertical.

- 3a) (5 points) Show that the tangential acceleration for points on the rim of the pulley is equal in magnitude to the acceleration of the center-of-mass of the pulley in the ground frame of reference.



$\tau = \bar{I}\alpha$

$\vec{\tau} =$

- 3b) (10 points) What is the acceleration of the center of mass of the pulley?

- 3c) (10 points) How much force does the rope exert on the hook?

- 3d) (5 points) What happens to the answers in parts b and c if I_{cm} is very large? Explain.