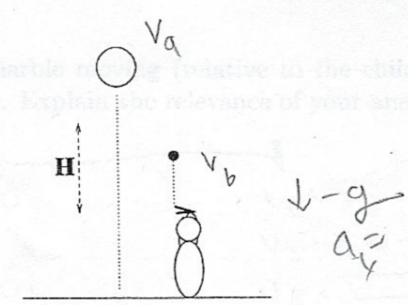


1) A small hot-air balloon slowly rises from the surface of the Earth at a constant speed v_a. Nearby, a young child holds a loaded slingshot above his head, pointed straight up. When the balloon reaches a height H above the slingshot, the child fires a marble with a large velocity v_b along a vertical path adjacent to that of the balloon.

+10/10

- 1a) (10 points) How fast is the marble moving (relative to the child) when it first overtakes the balloon?

$\cancel{X_1 = X_0}$ $\quad Y_f = Y_0 + V_0 t$

$Y_1 = H + h$ $\quad Y_1 = H + V_a(t_1)$

$Y_1 = Y_1 + V_0 t + \frac{1}{2} a(t)^2$

$Y_1 = V_b(t_1) + \frac{1}{2}(-g(t_1^2))$

$H + V_a(t_1) = V_b(t_1) + \frac{1}{2}(-g)(t_1^2)$

$V_F^2 = V_0^2 + 2a\Delta X$

$V_1^2 = V_b^2 + 2(-g)(H+h)$

$V_1 = \sqrt{V_b^2 + (-2g)\left(H + V_a\left(\frac{-V_b + V_a + \sqrt{(V_b - V_a)^2 - 2gH}}{-g}\right)\right)}$

~~you plugged in a factor~~

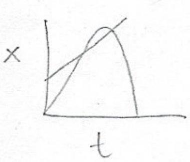
$\cancel{\frac{1}{2} m v_0^2 = \frac{1}{2} m v_1^2 + mgh}$

$\frac{-g}{2} = a \quad (V_b - V_a) = b \quad -H = C$ ✓

$$\frac{-(V_b - V_a) \pm \sqrt{(V_b - V_a)^2 - 4\left(\frac{-g}{2}\right)(-H)}}{-g} = \frac{-V_b + V_a \pm \sqrt{(V_b - V_a)^2 - 2gH}}{-g} = t_1$$

10
+10/10

- 1b) (10 points) How far above the balloon will the marble appear to go?



when $V_m = V_a$

$t_0 = 0 \qquad t_2 =$

$V_0 = V_b \quad a = -g \quad V_2 = 0$

$Y_0 = 0 \qquad Y_2 = \boxed{}$

$V_2^2 = V_0^2 + 2a(\Delta Y)$

$0 = V_b^2 - 2g(Y_2)$

$Y_2 = \frac{V_b^2}{2g}$

$V_2 = V_0 + a(t_2)$

$0 = V_b + -g(t_2)$

$t_2 = \frac{-V_b}{-g} = \frac{V_b}{g}$

$t_3 =$

$V_3 = V_a$

$Y_3 = \boxed{}$

$Y_2 - Y_B =$

$\frac{V_b^2}{2g} - \frac{V_a V_b}{g} + H$

SORRY!

work on 1.c. shown

answer: 1.b. $\frac{V_a - V_b^2}{-2g} - H + V_a\left(\frac{V_a - V_b}{-g}\right)$

$\Delta Y = V_a(t)$

$Y_B - H = V_a\left(\frac{V_b}{g}\right)$

$Y_B = V_a\left(\frac{V_B}{g}\right) + H$

+5/5 • 1c) (5 points) How fast is the marble moving (relative to the child) when the marble reaches its greatest distance above the balloon. Explain the relevance of your answer.

$t_3 =$

$V_3 = V_a$

$Y_3 = \boxed{}$

V_B

Answer to 1.c.

It's moving V_a because it will stop gaining height relative to the balloon once it's velocity is ~~less~~ less than the balloon because then the balloon's relative distance from marble will decrease since balloon's velocity > marble velocity.

$$V_a^2 = V_B^2 + 2(-g)(\Delta y)$$

height marble

$$\rightarrow \Delta y = \frac{V_a^2 - V_b^2}{(-2g)}$$

$$V_a = V_b + 2(-g)(t_3)$$

$$t_3 = \frac{V_a - V_b}{-g}$$

$$Y_{HB} = H + V_a(t_3)$$

$$\cancel{Y} = H + V_a\left(\frac{V_a - V_b}{-g}\right)$$

Answer to 1.b.

$$Y_3 - Y_{HB} = \frac{V_a^2 - V_b^2}{-2g} - H + V_a\left(\frac{V_a - V_b}{-g}\right)$$

+5/5 • 1d) (5 points) How much time will elapse between the marble's first encounter with the balloon and it's last?

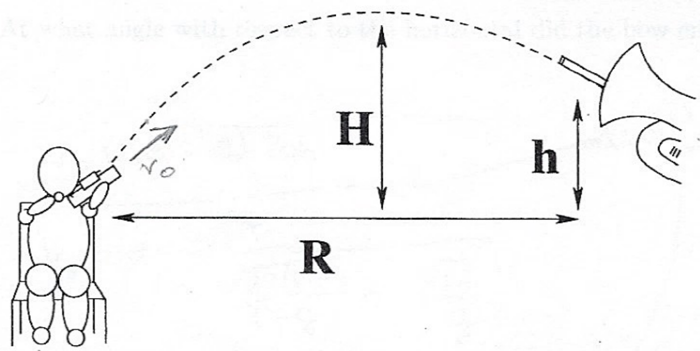
* between the positive & negative roots of t_1

$$t_1 = \frac{-V_b + V_a \pm \sqrt{(V_b - V_a)^2 - 2gh}}{-g}$$

$$|(t - root) - (t + root)|$$

$$= \left\| \left(\frac{-V_b + V_a - \sqrt{(V_b - V_a)^2 - 2gh}}{-g} - \frac{-V_b + V_a + \sqrt{(V_b - V_a)^2 - 2gh}}{-g} \right) \right\|$$

25



2) During a particularly lively solo the bow gets away from our virtuoso violinist. It rises to a maximum height H above the violin then descends into a nearby tuba. Assume the opening of the tuba lies a horizontal distance R from and a vertical height h above the violin and answer the following questions...

10 • 2a) (10 points) For how long was the bow in flight?

$v_y = 0$ ~~$v_y=0$~~

t_0 t_1 t_2

$v_0\sin\theta$, $v_0\cos\theta$

$R = v_0\cos\theta\,(t_2)$

$t_2 = \frac{R}{v_0\cos\theta}$

$H = v_0\sin\theta\,(t_1) + \frac{1}{2}a(t_1^2)$

$0 = (v_0\sin\theta)^2 + 2a(H)$

$v_0\sin\theta = \sqrt{2gH}$

$H = \sqrt{2gH}\,(t_1) + \frac{1}{2}(-g)(t_1)^2$

$a = \frac{-g}{2}$ $b = \sqrt{2gH}$ $c = -H$

$t_1 = \frac{-\sqrt{2gH} \pm \sqrt{2gH - 4(\frac{-g}{2})(-H)}}{-g}$

$h = H + 0 + \frac{1}{2}a(t_2 - t_1)^2$

$\sqrt{\frac{2(h-H)}{-g}} + t_1 = t_2$

$t_2 = \sqrt{\frac{2(h-H)}{-g}} + \frac{-\sqrt{2gH} + \cancel{\sqrt{2gH - 2gH}}}{-g}$

10 • 2b) (10 points) With what speed did the bow leave the violin?

$v_0\sin\theta = \sqrt{2gh}$

$v_0\cos\theta = \frac{R}{\sqrt{\frac{2(h-H)}{-g}} + \frac{\sqrt{2gh}}{g}}$

~~$\frac{R}{2} = v_0\cos\theta$~~

$\frac{R}{2} = v_0\cos\theta\,(t_1)$

$a^2 + b^2 = c^2$

$\sqrt{(v_0\sin\theta)^2 + (v_0\cos\theta)^2} = v_0$

$\sqrt{2gh + \left(\frac{R}{\sqrt{\frac{2(h-H)}{-g}} + \frac{\sqrt{2gh}}{g}}\right)^2} = v_0$

- 2c) (10 points) At what angle with respect to the horizontal did the bow enter the tuba?

$$\frac{v_0 \sin\theta = \sqrt{2gh}}{v_0 \cos\theta = \dfrac{R}{\sqrt{\frac{2(h-H)}{-g}} + \frac{\sqrt{2gh}}{g}}}$$

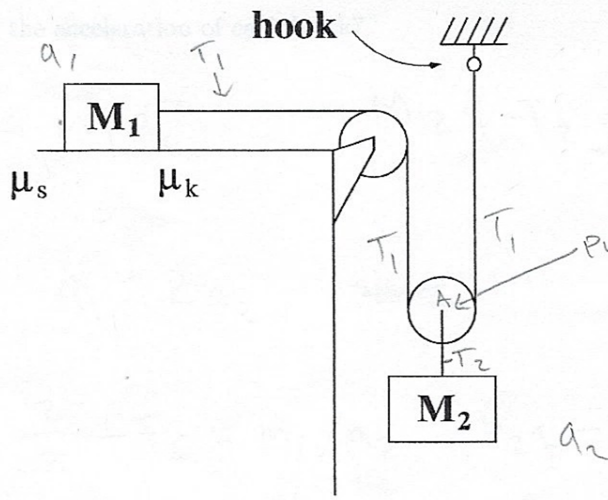
divide

$$\tan\theta = \frac{\sqrt{2gh}}{\dfrac{R}{\sqrt{\frac{2(h-H)}{-g}} + \frac{\sqrt{2gh}}{g}}}$$

$$\theta_0 = \tan^{-1}\left(\frac{\sqrt{2gh}}{\dfrac{R}{\sqrt{\frac{2(h-H)}{-g}} + \frac{\sqrt{2gh}}{g}}}\right)$$

5

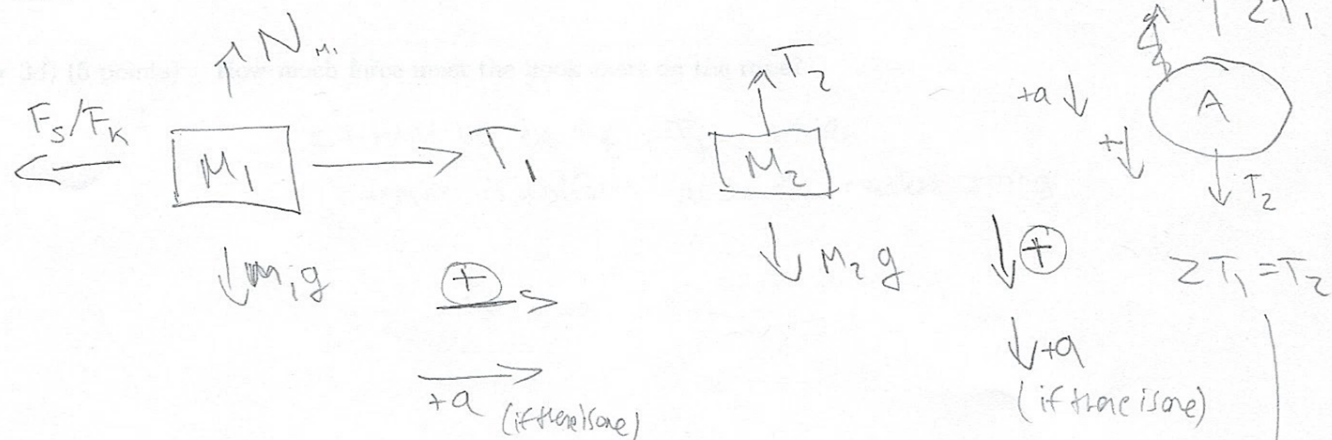
$\theta_f = ?$



Consider the apparatus shown above. The coefficients of friction between block one and the table are both known, as are the masses M_1 and M_2.

- 3a) (5 points) Identify the object (or objects) you're interested in, and draw free-body diagrams for each.

5



- 3b) (10 points) Use Newton's laws to obtain equations that describe the dynamics of each of the objects of interest. Describe friction as F_f, and allow for acceleration of the body or bodies.

10

$N_{m1} = m_1 g$

$M_2 g - T_2 = m_2 \vec{a}_2$

$T_1 - F_f = m_1 \vec{a}_1$

(since massless pulley)

$2T_1 = T_2$

$a_1 = 2a_2$ ✓

$F_f = \mu_K F_N$

$= \mu_K m_1 g$

- 3c) (10 points) What is the acceleration of each block?

$T_1 - F_f = m_1 \vec{a_1}$ $M_2 g - T_2 = m_2 \vec{a_2}$

4 5/10

$2T_1 = T_2$

$a_1 = 2a_2$ ~~since~~

$\frac{T_2}{2} - F_f = m_1 2a_2$ $M_2 g - T_2 = m_2 a_2$

$T_2 = M_2 g - m_2 a_2$

$\cancel{= 2m_1}$

$\frac{m_2(g - a_2)}{2} - F_f = m_1 2a_2$

- 3d) (5 points) How much force must the hook exert on the rope?

3/5

It must be exerting T_1 since tension is uniform across the massless string.