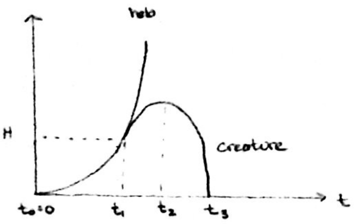
A helicopter, initially at rest, accelerates upward from the ground at a rate a_1. When it reaches a height H, a small, furry, wingless creature is released from a compartment on the bottom of the helicopter.

- 1a) (5 points) On a single plot, sketch (qualitatively) y vs. t for the helicopter and the furry creature.



- 1b) (10 points) What is the greatest height (measured with respect to the ground) reached by the creature?

Helo: $v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0)$
$v_{1y}^2 = 0 + 2a_1(H - 0)$
$v_{1y} = \sqrt{2a_1 H}$

Creature: $v_{2y}^2 = v_{1y}^2 + 2a_{cy}(y_2 - y_1)$
$0 = 2a_1H - 2g(y_2 - H)$
$0 = (a_1 + g)H - gy_2$

$$\boxed{y_2 = H\left(1 + \frac{a_1}{g}\right)}$$

- 1c) (10 points) Where will the helicopter be (with respect to the ground) when the creature reaches its maximum height?

Creature: $v_{2y} = v_{1y} + a_{cy}(t_2 - t_1)$
$0 = \sqrt{2a_1H} - g(t_2 - t_1)$
$t_2 - t_1 = \sqrt{\frac{2a_1H}{g^2}}$

Helo: $y_2 = y_1 + v_{1y}(t_2 - t_1) + \frac{1}{2}a_y(t_2 - t_1)^2$
$y_2 = H + \sqrt{2a_1H}\sqrt{\frac{2a_1H}{g^2}} + \frac{1}{2}a_1\frac{2a_1H}{g^2}$
$y_2 = H + \frac{2a_1H}{g} + \frac{a_1^2H}{g^2}$

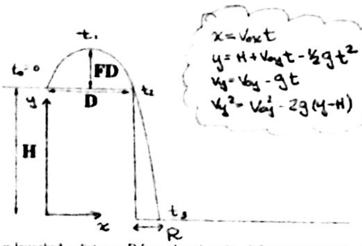
$$\boxed{y_2 = H\left(1 + 2\frac{a_1}{g} + \frac{a_1^2}{g^2}\right)}$$

- 1d) (5 points) For how long is the creature in free-fall?

Creature: $y_3 = y_1 + v_{1y}(t_3 - t_1) + \frac{1}{2}a_c(t_3 - t_1)^2$
$0 = H + \sqrt{2a_1H}(t_3 - t_1) - \frac{1}{2}g(t_3 - t_1)^2$

$t_3 - t_1 = \frac{\sqrt{2a_1H} \pm \sqrt{2a_1H + 2gH}}{g}$ ← $t_3 - t_1 > 0$ why?

$$\boxed{t_3 - t_1 = \sqrt{\frac{2a_1H}{g^2}}\left(1 + \sqrt{1 + g/a_1}\right)}$$



$x = v_{0x}t$
$y = H + v_{0y}t - \frac{1}{2}gt^2$
$v_y = v_{0y} - gt$
$v_y^2 = v_{0y}^2 - 2g(y - H)$

A small, fuzzy bunny is launched a distance D from the edge of a cliff of height H. The little rabbit reaches a maximum height FD above its launch point (F is some constant), and *narrowly* misses the edge of the cliff as it passes below its launch point.

- 2a) (10 points) Find the horizontal and vertical components of the rabbit's initial velocity

$\cancel{v_{1y}^2} = v_{0y}^2 - 2g(FD + H - H)$
$v_{0y} = \sqrt{2gFD}$

$y_2 = y_0 + v_{0y}t_2 - \frac{1}{2}gt_2^2$
$0 = t_2(v_{0y} - \frac{1}{2}gt_2)$
$t_2 = \frac{2v_{0y}}{g}$
$t_2 = 2\sqrt{\frac{2FD}{g}}$

$x_2 = v_{0x}t_2$
$v_{0x} = D/t_2$
$v_{0x} = \frac{1}{2}D\sqrt{\frac{g}{2FD}}$

$$\boxed{\begin{aligned} v_{0x} &= \tfrac{1}{2}\sqrt{\frac{gD}{2F}} \\ v_{0y} &= \sqrt{2gFD} \end{aligned}}$$

- 2b) (10 points) How far from the base of the cliff did the rabbit land?

$y_3 = y_0 + v_{0y}t_3 - \frac{1}{2}gt_3^2$
$0 = H + v_{0y}t_3 - \frac{1}{2}gt_3^2$
$t_3 = \frac{v_{0y} \pm \sqrt{v_{0y}^2 + 2gH}}{g}$
$t_3 = \frac{v_{0y}}{g}\left(1 + \sqrt{1 + \frac{2gH}{v_{0y}^2}}\right)$

$x_3 = v_{0x}t_3$
$x_3 = \frac{v_{0x}v_{0y}}{g}\left(1 + \sqrt{1 + \frac{2gH}{v_{0y}^2}}\right)$

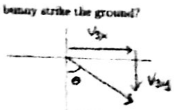
$R = x_3 - D$
$R = \frac{1}{2}D\left(1 + \sqrt{1 + \frac{H}{FD}}\right) - D$

$$\boxed{R = \frac{1}{2}D\left(\sqrt{1 + \frac{H}{FD}} - 1\right)}$$

- 2b) *(Continued...)*

- 2c) (10 points) At what angle (with respect to the vertical direction) did the bunny strike the ground?

$v_{3x} = v_{0x} = \frac{1}{2}\sqrt{\frac{gD}{2F}}$

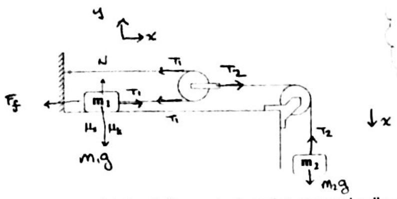


$v_{3y}^2 = v_{0y}^2 - 2g(0 - H)$
$v_{3y}^2 = 2gFD + 2gH$
$v_{3y} = \sqrt{2g(FD + H)}$

$\tan\theta = \frac{v_{3x}}{v_{3y}}$

$\tan\theta = \frac{1}{2}\sqrt{\frac{gD}{2F}}\frac{1}{\sqrt{2g(FD + H)}}$

$$\boxed{\tan\theta = \frac{1}{4}\sqrt{\frac{D}{F(FD + H)}}}$$



if (2) drops x
the moving pulley
pulls x on top
and bottom...
(1) moves 2x

3) Masses m_1 and m_2 are connected indirectly by a pair of massless ropes and pulleys as shown. The coefficients of static and kinetic friction between m_1 and the horizontal table on which it sits are μ_s and μ_k, respectively.

- 3a) (5 points) How large must m_2 be in order to set the system in motion? $\Sigma \vec{F} = m\vec{a}$

$T_1 - F_f = m_1 a_{1x}$
$N - m_1 g = 0$
$T_2 - 2T_1 = 0$
$m_2 g - T_2 = m_2 a_{2x}$

Static: $a_{1x} = a_{2x} = 0$
$F_f \le \mu_s N$

$F_s = T_1 \le \mu_s m_1 g$

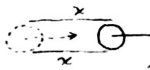
$T_1 = \frac{1}{2} T_2 \le \mu_s m_1 g$
$T_2 \le 2\mu_s m_1 g$

$T_2 = m_2 g \le 2\mu_s m_1 g$

$m_2 \le 2\mu_s m_1$
to remain static

To set the system in motion...
$\boxed{m_2 > 2\mu_s m_1}$

- 3b) (20 points) Assume m_2 is sufficiently large to set the system in motion and find the acceleration of each block and the tension in each rope.



If the pulley moves x to the right, 2x worth of rope are pulled over!
$x_1 = 2x_2$
$a_{1x} = 2a_{2x}$

$F_f = \mu_k N$
$N = m_1 g$
$T_2 = 2T_1$
$a_{1x} = 2a_{2x}$

$T_1 - \mu_k m_1 g = 2m_1 a_{2x}$
$m_2 g - 2T_1 = m_2 a_{2x}$

$(m_2 - 2\mu_k m_1) g = (4m_1 + m_2) a_{2x}$

$a_{2x} = g \frac{m_2 - 2\mu_k m_1}{4m_1 + m_2}$

$a_{1x} = 2g \frac{m_2 - 2\mu_k m_1}{4m_1 + m_2}$

$T_1 = m_1 g \left(\mu_k + \frac{2(m_2 - 2\mu_k m_1)}{4m_1 + m_2} \right)$

$T_2 = 2m_1 g \left(\mu_k + \frac{2(m_2 - 2\mu_k m_1)}{4m_1 + m_2} \right)$

$$\boxed{\begin{aligned} a_{1x} &= \frac{2g(m_2 - 2\mu_k m_1)}{4m_1 + m_2} \\ a_{2x} &= \frac{g(m_2 - 2\mu_k m_1)}{4m_1 + m_2} \\ T_1 &= \frac{m_1 m_2 g (2 + \mu_k)}{4m_1 + m_2} \\ T_2 &= \frac{2m_1 m_2 g (2 + \mu_k)}{4m_1 + m_2} \end{aligned}}$$

- 3b) (Continued...)

- 3c) (5 points) Evaluate the acceleration of m_1 in the limit that $m_1 \to 0$ and in the limit $m_2 \to 0$. Discuss your results (particularly if the result seems nonsensical or weird).

$m_1 \to 0$: $a_{1x} = 2g$ $\Rightarrow$ pure geometry: with no additional inertia, m_2 is in free-fall: m_1 follows with twice the acceleration $\frac{d^2}{dt^2}(x_1 = 2x_2)$

$m_2 \to 0$ $a_{1x} = -4\mu_k g$ $\Rightarrow$ Huh! If $m_2 = 0$, m_1 isn't getting pulled to the right – F_k is replaced by F_s ($\le \mu_s N$). With no relative motion to oppose, $F_s = 0$, which is just fine...