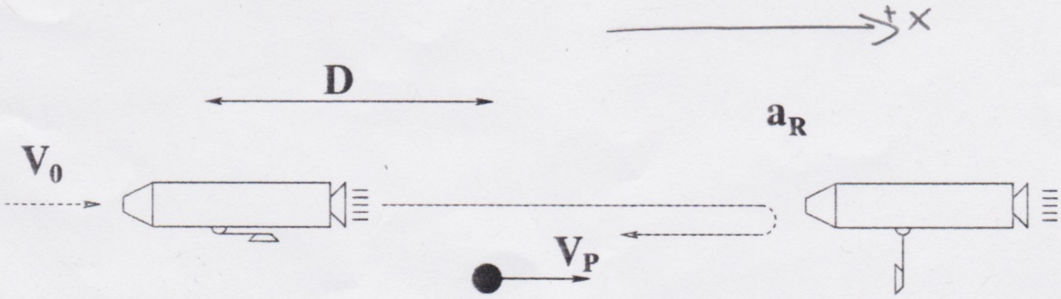


1) A pod-retrieval rocket is designed to overshoot its target and scoop it up on the rebound. It approaches the pod with an initial velocity $\vec{V_0}$ parallel to the pod's (constant) velocity $\vec{V_p}$. When it is a distance D away from the pod (still on approach), the rocket turns on retro-thrusters which give it a constant acceleration of magnitude a_r.

- 1a) (10 points) How much time elapses between the moment the rocket passes the pod and the instant it scoops the pod up?

$$X_R = V_0 t - \frac{1}{2} a_r t^2$$

$$X_P = D + V_p t$$

$$V_0 t - \frac{1}{2} a_r t^2 = D + V_p t$$

$$\frac{1}{2} a_r t^2 + (V_p - V_0) t + D = 0$$

The smaller value from the quad. eq. is when the rocket first passes the pod, the larger value is when the rocket scoops the pod

$$t = \frac{-(V_p - V_0) \pm \sqrt{(V_p - V_0)^2 - 2 a_r D}}{a_r}$$

$t_{pass} = - \sqrt{\sim}$

$t_{scoop} = + \sqrt{\sim}$

$$t_{scoop} - t_{pass} = \boxed{\frac{2\sqrt{(V_p - V_0)^2 - 2a_r D}}{a_r}}$$

10/10

- 1b) (10 points) What is the greatest distance ahead of the pod that the rocket will ever get?

Greatest distance occurs when V_{Rocket} $(V_r) = V_p$, since after that the pod will catch up to be closer to the rocket.

t_{max}

$$V_0 - a_r t_{max} = V_p$$

$$t_{max} = \frac{V_0 - V_p}{a_r}$$

$$X_R - X_P = -\frac{1}{2} a_r t^2 + (V_0 - V_p) t - D$$

$$X_R - X_P (t_{max}) = -\frac{1}{2} a_r (t_{max})^2 + (V_0 - V_p)(t_{max}) - D$$

$$= -\frac{1}{2}\left(\frac{(V_0 - V_p)^2}{a_r}\right) + \frac{(V_0 - V_p)^2}{a_r} - D = \boxed{\frac{(V_0 - V_p)^2}{2a_r} - D}$$

10/10

• 1c) (10 points) How fast is the pod moving relative to the rocket when it gets scooped?

$$t_{scoop} = \frac{-(v_p - v_0) + \sqrt{(v_p - v_0)^2 - 2a_r D}}{a_r}$$

$$\cancel{V_{p,g} = V_{p,r} + V_{r,g}}$$

$$V_{r,g} = V_{r,p} + V_{r,g}$$

$$V_{p,r} = V_{p,g} - V_{r,g}$$

$$= V_p - V_r$$

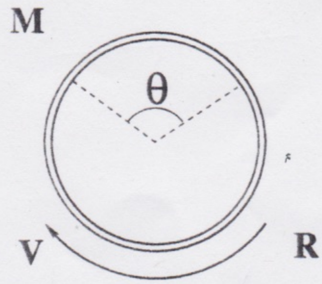
$$V_r = V_0 + \left(\frac{-v_p - v_0 + \sqrt{(v_p - v_0)^2 - 2a_r D}}{a_r} \right)(a_r)$$

$$= +V_p \mp \sqrt{(v_p - v_0)^2 - 2a_r D}$$

$$V_p - V_r = V_p - \left(-V_p + \sqrt{(v_p - v_0)^2 - 2a_r D}\right)$$

$$V_{p,g} = \cancel{2}V_p - \sqrt{(v_p - v_0) - 2a_r D}$$

8/10



2) A thin uniform ring of mass M and radius R rotates about an axis through its center and perpendicular to the plane of the ring in such a way that the points in the ring all move with a speed v. This happens in a region of space where the effects of gravity can be safely ignored...

- 2a) (5 points) How much mass is contained in a segment of the ring that spans an angle θ?

Total Mass = M

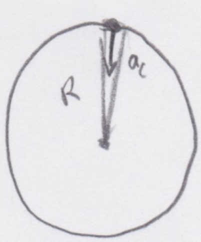
$$\text{Mass}_\theta = \left[\frac{\theta}{2\pi}\right]M = \boxed{\frac{\theta M}{2\pi}}$$

✓

- 2b) (5 points) If θ is small, we can treat the segment as a point-mass moving in a circle of radius R. What is the magnitude and direction of the net force that acts on such a small segment?

all move with speed v

– no tangential acceleration, only centripetal acceleration

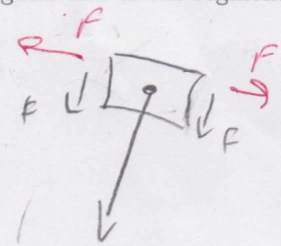


$$|F| = m a_c = \boxed{\frac{mV^2}{R}}$$, m = mass of the point-mass = ?

−1

Direction = towards the center of the circle

- 2c) (10 points) What is the force that keeps the ring from flying apart as it spins? Assume θ is small enough to treat the segment as a point-mass moving in a circle of radius R but large enough to explore the geometry in the problem and find the magnitude of the force that holds the segment in place as a function of the angular size of the segment (θ).



Force that holds the segment in place is coming from the sides of the point mass as it is held in by the rest of the circle. it points towards the center of the circle ~~force~~ since it contributes to the centripetal acceleration.

$$\frac{mv^2}{R} = \left(\frac{\theta}{2\pi R} v^2\right) M$$

$$m = \frac{\theta}{2\pi} M$$

-6

Magnitude = $\frac{\theta M}{2\pi R} v^2$

Direction = center of the circle

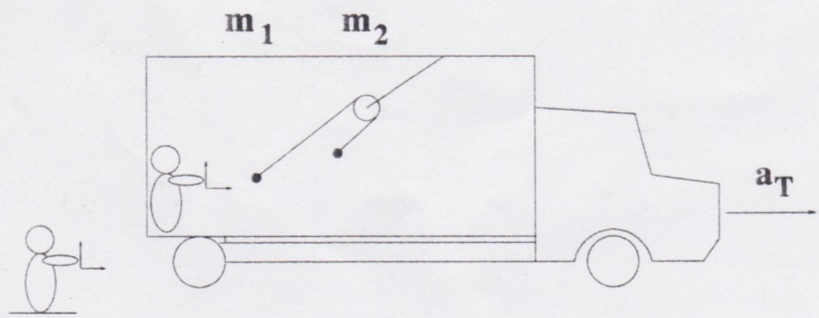
- 2d) (5 points) To get the correct answer for part c, we have to assume that θ is really, really small. Evaluate your answer for part c in the limit $\theta \to 0$.

$$d\theta\left(\frac{M}{2\pi R} v^2\right)$$

? -5

- 2e) (5 points) Suppose the ring is made of string. If we were to perturb the string slightly by introducing a small bump, that bump would travel around the string with a speed (relative to the string) equal to $\sqrt{T/\mu}$, where T is the tension in the string and μ is the linear mass density of the string. Using the information you derived earlier, describe how a bump traveling through a spinning string would look like to an outside observer. (Hint: there are two cases to consider.)

-5



3) An "Atwood Machine" is constructed using two small masses, a light rope and a massless pulley. The device is hung in the back of a large truck and the truck, in turn, is given a horizontal acceleration a_T in the forward direction. The Atwood machine tilts back, in response to the truck's acceleration, until the center of the pulley appears to be at rest *with respect to the truck.* In this configuration, let's call the mass attached to the upper portion of the rope m_1 and the mass attached to the lower portion, m_2.

✓

- 3a) (5 points) While Newton's laws are only truly relevant in inertial frames of reference, Galileo's relative velocity relationship holds for all frames (so long as they aren't moving *too* fast) - and so, by extension, would an expression for relative acceleration. Write vector expressions that relate the acceleration of each mass as seen in a coordinate system attached to the ground to the acceleration of each mass as seen relative to a coordinate system attached to the truck. Evaluate each of these expressions for components taken along the horizontal and vertical direction.

m_1

$a_{1x,g} = a_{1x,t} + a_{tx,g}$

$a_{1y,g} = a_{1y,t} + a_{ty,g}$

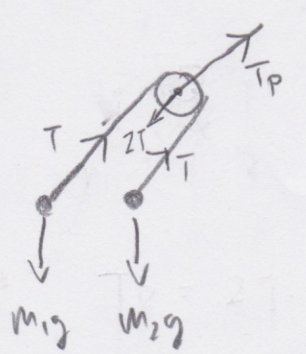
m_2

$a_{2x,g} = a_{2x,t} + a_{tx,g}$

$a_{2y,g} = a_{2y,t} + a_{ty,g}$

- 3b) (5 points) Consider how Newton's laws apply to the massless pulley and show that the upper and lower portions of the rope that connects m_1 to m_2 must be parallel.

−4



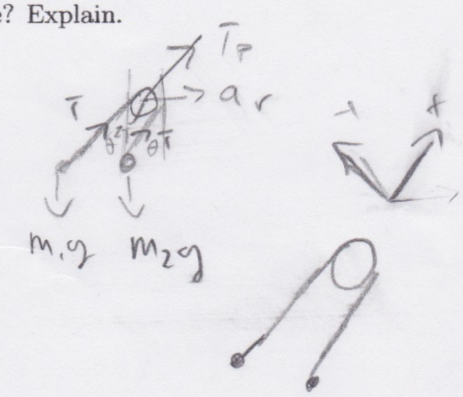
The pulley has two forces acting on it: T_p, which is tension of the rope attached the pulley & the truck, and $2T$, which comes from the rope holding the two masses. $2T$ has to cancel the y-component of T_p so that the pulley doesn't accelerate in the y-direction. To do this with the smallest amount of tension in the rope holding the masses, both T must point in the same direction.

- 3c) (5 points) How will the acceleration of m_1, as seen in the frame of reference attached to the truck, relate to the acceleration of m_2 as seen in that same frame? Explain.

-4

$$\Sigma F_{1x,g} = T\sin\theta = m_1 a_{1x,g}$$

$$\Sigma F_{1y} = T\cos\theta - m_1 g = m_1 a_{1y,g}$$



$$\Sigma F_{2x} = T\sin\theta = m_2 a_{2x,g}$$

$$\Sigma F_{2y} = T\cos\theta - m_2 g = m_1 a_{2y,g}$$

- 3d) (15 points) Find the tension in the rope that connects m_1 to m_2. For full credit, your answer should be clear and follow logically from first-principles.

-15