# 20W-PHYSICS1A-2 1AW20 FINAL EXAM

TOTAL POINTS

**93 / 180**

QUESTION 1

## 1 30 pts

### 1.1 1a 10 / 10

- ✓ **+ 10 pts Correct**
- **+ 5 pts** Tortoise curve looks like a line and not a parabola...
- **+ 7.5 pts** Tortoise curve looks like exponential and not parabola...
- **- 2 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.

### 1.2 1b 6 / 10

- **+ 10 pts** Correct
- **+ 8 pts** Mostly right
- ✓ **+ 6 pts Correct idea, but not correctly implemented**
- **+ 5 pts** Mostly right, but wonky units
- **+ 4 pts** Some progress
- **+ 2 pts** Minor progress
- **- 2 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **- 2 pts** Not solved for t_2
- **- 1 pts** Incorrect use of quadratic formula
- **- 2 pts** Bad algebra and/or wrong units

### 1.3 1c 3 / 5

- **+ 5 pts** Correct with correct t_2
- ✓ **+ 3 pts Correct continuation**
- **+ 2 pts** Some progress.
- **+ 0 pts** No progress/Not a distance
- **- 1 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **- 1 pts** Wrong units...

### 1.4 1d 2 / 5

- **+ 5 pts** Correct
- **+ 4 pts** Close
- **+ 3 pts** Correct continuation
- ✓ **+ 2 pts Some progress**
- **+ 1 pts** Plus one for actually understanding what to do!
- **+ 1 pts** Generic formulas, with no real connection to problem and/or previous work
- **+ 0 pts** No work/Doesn’t understand relative velocity
- **- 1 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **- 1 pts** Bad algebra/Wrong units

QUESTION 2

## 2 30 pts

### 2.1 2a 2 / 10

- **+ 10 pts** Correct
- **+ 8 pts** Mostly correct
- **+ 6 pts** Right idea, not correctly implemented
- **+ 4 pts** Some progress
- ✓ **+ 2 pts Minor progress**
- **+ 1 pts** Essay in lieu of physics (Basically, you didn't even make a sound essay).
- **+ 1 pts** Plus one for having the correct expression for a_3 (regardless of sign) but not satisfactorily shown why it is so.
- **+ 0 pts** No work/Insignificant progress
- **- 2 pts** Solution not assigned to correct page! Points deducted if I need to search 2 pages or more to find your solution.
- **- 2 pts** Check units

### 2.2 2b **0 / 10**

+ **10 pts** Correct
+ **8 pts** Mostly correct
+ **6 pts** Right idea, but not correctly implemented
+ **4 pts** Some progress
+ **2 pts** Minor progress

✓ + **0 pts** **No work**

- **2 pts** Check units!
- **2 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.

### 2.3 2c **0 / 10**

+ **10 pts** Correct
+ **8 pts** Mostly correct
+ **6 pts** Right idea, but not correctly implemented
+ **4 pts** Some progress
+ **2 pts** Minor progress
+ **1 pts** Essay in lieu of physics

✓ + **0 pts** **No work/Insignificant progress**

- **2 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **2 pts** Check units!

💬 You have time to draw that but you don't have time to at least try and explain an attempt for the solution!?

QUESTION 3

## 3 30 pts

### 3.1 3a **3 / 5**

+ **5 pts** Correct
+ **4 pts** Mostly correct

✓ + **3 pts** **Right idea, but incorrect implementation**

+ **2 pts** Some progress
+ **1 pts** Minor progress
+ **0 pts** No work/insignificant progress

- **1 pts** check units
- **1 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.

### 3.2 3b **4 / 5**

+ **5 pts** Correct

✓ + **4 pts** **Mostly correct/correct continuation**

+ **3 pts** Right idea, but incorrect implementation
+ **2 pts** Some progress/Used CoM instead of CoE
+ **1 pts** Minor progress
+ **0 pts** No work/Insignificant progress

- **1 pts** Check units!
- **1 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.

### 3.3 3c **4 / 5**

+ **5 pts** Correct

✓ + **4 pts** **Mostly correct/Correct continuation**

+ **3 pts** Right idea, but incorrect implementation
+ **2 pts** Some progress/Used CoE instead of CoM
+ **1 pts** Minor progress
+ **0 pts** No work/Insignificant progress/nonsense

- **1 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **1 pts** Check units!

### 3.4 3d **4 / 5**

+ **5 pts** Correct

✓ + **4 pts** **Mostly correct/correct continuation**

+ **3 pts** Right idea, but incorrectly implemented
+ **2 pts** Some progress
+ **1 pts** Minor progress
+ **0 pts** No work

- **1 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **1 pts** Check units

### 3.5 3e **4 / 5**

+ **5 pts** Correct

✓ + **4 pts** **Mostly correct/correct continuation**

+ **3 pts** Right idea, but incorrectly implemented

+ **2 pts** Some progress
+ **1 pts** Minor progress
+ **0 pts** No work
- **1 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **1 pts** Check units

### 3.6 3f 0 / 5

+ **5 pts** Correct
+ **4 pts** Mostly correct
+ **3 pts** Right idea, but not well stated
+ **2 pts** Kind of...
+ **1 pts** Vague
✓ + **0 pts** **No work/Stated energy is the same i.e. elastic collision and/or friction not dissipative**
- **1 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.

It is a perfectly inelastic collision

QUESTION 4

## 4 30 pts

### 4.1 4a 10 / 10

✓ + **10 pts** **Correct**
+ **8 pts** Mostly correct
+ **6 pts** Right idea, but incorrectly implemented
+ **4 pts** Some progress
+ **2 pts** Minor progress
+ **0 pts** No work
- **2 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **2 pts** Check units!

sign error but ok..

### 4.2 4b 1 / 10

+ **10 pts** Correct
+ **8 pts** Mostly correct
+ **6 pts** Right idea, but incorrectly implemented
+ **4 pts** Some progress
+ **2 pts** Minor progress
✓ + **1 pts** **Essay in lieu of physics**
+ **0 pts** No work
- **2 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **2 pts** Check units!

### 4.3 4c 2 / 10

+ **10 pts** Correct
+ **8 pts** Mostly correct
+ **6 pts** Right idea, but incorrectly implemented
+ **4 pts** Some progress
✓ + **2 pts** **Minor progress**
+ **0 pts** No work
- **2 pts** Solution not assigned to correct page! Points deducted if I need to search more than 2 pages to find your solution.
- **2 pts** Check units!

QUESTION 5

## 5 30 pts

### 5.1 5a 3 / 5

✓ + **1 pts** **Mechanical Energy is conserved**
✓ + **1 pts** **Justification for mechanical energy**
+ **1 pts** angular momentum conserved
+ **1 pts** Justification for Ang Mom
✓ + **1 pts** **Translational momentum not conserved because of axle**
+ **0 pts** completely wrong or no answer

### 5.2 5b 5 / 15

+ **4 pts** Conservation of kinetic energy
+ **2 pts** Partial credit for kinetic energy
+ **4 pts** conservation of angular momentum
✓ + **2 pts** **Partial credit for ang mom**
+ **3 pts** Calculating I_axle
+ **1.5 pts** partial credit for I_axle
+ **4 pts** Correct calculation for v_2, omega
✓ + **2 pts** **partial credit for calculation leading to v_2,**

**omega**

+ **0 pts** No answer or incorrect answer

✓ **+ 1 pts extra points**

### 5.3 5c 1 / 10

+ **3 pts** correct formula for kinetic energy

+ **1.5 pts** partial credit for kinetic energy

+ **3 pts** correct formula for gravitational energy

+ **1.5 pts** partial credit for gravitational energy

+ **4 pts** calculations leading to theta ( Dh, etc)

+ **2 pts** partial credit for calculations

+ **0 pts** completely wrong or no answer

✓ **+ 1 pts extra points**

QUESTION 6

## 6 30 pts

### 6.1 6a 5 / 5

✓ **+ 2 pts Correct M1 distance**

✓ **+ 2 pts Correct M2 distance**

✓ **+ 1 pts justification**

+ **1 pts** partial credit for D1

+ **1 pts** partial credit for D2

+ **0 pts** No answer or incorrect answer

### 6.2 6b 5 / 5

✓ **+ 2 pts Correct gravitational force**

✓ **+ 3 pts correct centripetal force**

+ **0 pts** No answer or incorrect answer

+ **1 pts** partial credit for gravitational force

+ **1.5 pts** partial credit for centripetal force

### 6.3 6c 10 / 10

✓ **+ 4 pts Correct L_1**

✓ **+ 4 pts Correct L_2**

✓ **+ 2 pts remains constant because torque is zero**

+ **0 pts** No answer or incorrect answer

+ **2.5 pts** Partial credit for L_1

+ **2.5 pts** Partial credit for L_2

+ **1 pts** remains constant no justification

### 6.4 6d 9 / 10

✓ **+ 4 pts Correct kinetic energy**

✓ **+ 4 pts correct potential energy**

+ **2 pts** constant in time since no non conservative forces

+ **0 pts** no answer or incorrect answer

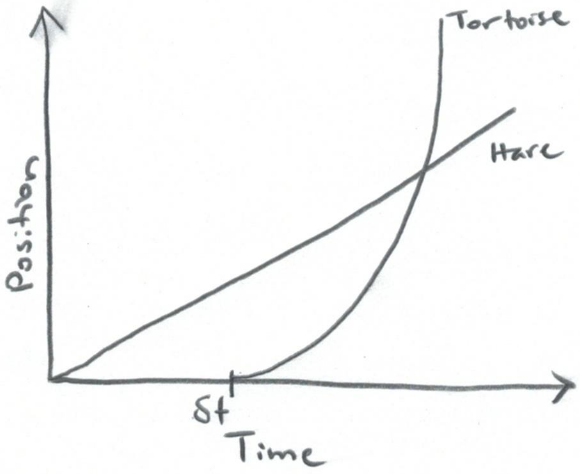
+ **2.5 pts** partial credit for kinetic energy

+ **2.5 pts** partial credit for potential energy (eg putting extra factor of 2)

✓ **+ 1 pts remains constant without justification**

1) A tortoise sleeps by the side of a road. Suddenly, a hare blazes past with a constant speed $v_h$. It takes the tortoise a time $\delta t$ to poke its head out of the shell, observe the goings-on and get moving, but at that moment he starts, the caffeine and sugar-powered boosters kick in and he roars off with a crazy constant acceleration $a_t$, following in the same initial direction as the hare.

- 1a) (10 points) On a single plot of position vs. time, plot the trajectories of the tortoise and the hare. Be very clear about which curve belongs to which animal and label the plot with the information you know.



- 1b) (10 points) How long (after the hare initially passes the tortoise) does it take the tortoise to catch up with the hare? If you have to choose between roots in your final answer, explain why you chose the root you did and explain the presence of the root you discarded.

Hare position at δt:

$x_{h_i} = V_h \delta t$

Tortoise position after δt:

$x_t = \frac{1}{2} a_t (t - \delta t)^2$ (tortoise x=0 at δt)

Hare position after δt:

$x_h = V_h \delta t + V_h (t - \delta t)$

$$x_h = x_t$$

$$V_h \delta t + V_h (t - \delta t) = \frac{1}{2} a_t (t - \delta t)^2$$

$$\frac{1}{2} a_t (t - \delta t)^2 - V_h (t - \delta t) - V_h \delta t = 0$$

$$\frac{V_h \pm \sqrt{V_h^2 + 2 a t V_h \delta t}}{a_t} = t - \delta t$$

$a = \frac{1}{2} a_t$

$b = -V_h$

$c = -V_h \delta t$

$$\boxed{\Delta t = \frac{V_h + \sqrt{V_h^2 + 2 a_t V_h \delta t}}{a_t}}$$

We choose the positive root since the alternative would give a negative time. It has a negative root since, as a quadratic, it is squared to produce the same value.

- 1c) (5 points) How far from its initial resting place was the tortoise when he caught up to the hare?

$$x_t = \frac{1}{2}a_t(t-\delta t)^2 \qquad t-\delta t = \frac{V_h+\sqrt{V_h^2+2a_tV_h\delta t}}{a_t}$$

$$x_t = \frac{1}{2}a_t\left(\frac{V_h+\sqrt{V_h^2+2a_tV_h\delta t}}{a_t}\right)^2$$

$$= \frac{1}{2}\cancel{a_t}\left(\frac{V_h^2+2V_h\sqrt{V_h^2+2a_tV_h\delta t}+V_h^2+2a_tV_h\delta t}{a_t^{\cancel{2}}}\right)$$

$$= \frac{1}{2}\left(\frac{2V_h^2+2V_h\sqrt{V_h^2+2a_tV_h\delta t}+2a_tV_h\delta t}{a_t}\right)$$

$$\boxed{x_t = \frac{V_h^2+V_h\sqrt{V_h^2+2a_tV_h\delta t}+a_tV_h\delta t}{a_t}}$$

- 1d) (5 points) What was the velocity of the *hare* relative to the *tortoise* (magnitude and direction) at the instant the tortoise caught up to the hare?

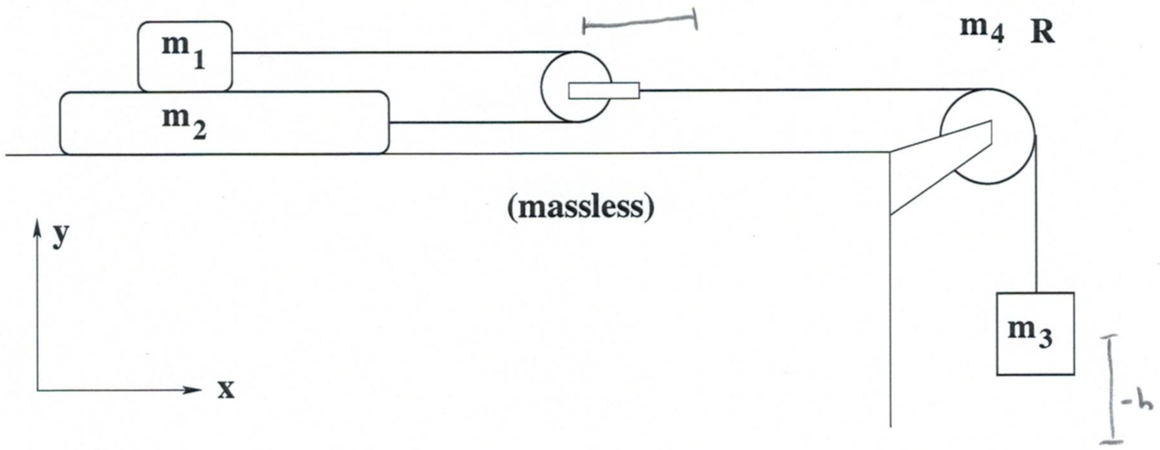
$$V_{h,t} = V_{h,g} - V_{g,t} \qquad V_{h,g} = V_h \qquad t-\delta t = \frac{V_h+\sqrt{V_h^2+2a_tV_h\delta t}}{a_t}$$

$$V_{g,t} = a_t(t-\delta t)$$

$$V_{g,t} = V_h + \sqrt{V_h^2+2a_tV_h\delta t}$$

$$V_{h,t} = V_h - V_h - \sqrt{V_h^2+2a_tV_h\delta t}$$

$$\boxed{V_{h,t} = -\sqrt{V_h^2+2a_tV_h\delta t}}$$



2) Masses $m_1$ and $m_2$ are are connected by way of an ideal massless rope that is draped over a massless pulley, as shown. That massless pulley is connected to a hanging mass of mass $m_3$ by another ideal massless rope that is draped over a massive pulley of mass $m_4$ and radius $R$, as shown. The rotational inertia of the massive pulley around its axle is given by $km_4R^2$.

- 2a) (10 points) How does the translational acceleration of $m_1$ relative to the massless pulley relate to the translational acceleration of $m_2$ relative to the massless pulley? Derive an expression that relates the translational accelerations of $m_1$, $m_2$ and the massless pulley relative to an inertial frame attached to the ground. How does the translational acceleration of the massless pulley relate to the translational acceleration of $m_3$ (as seen relative to the ground)? How does the translational acceleration of $m_3$ relative to the ground relate to the angular acceleration of the massive pulley around its axle (For full credit, you should pick a positive rotational direction for that pulley and properly relate the signs of the quantities)?

$$\overset{0}{\cancel{mgh_i}} = mg(-h) + \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}Km_4R\omega^2$$

$\Sigma F = ma$

$\Sigma \tau = I\alpha$

$a_{ten} = R\alpha$

$a_{tm} = g$

$\Sigma \tau = \frac{g}{R}$

- 2b) (10 points) Find the translational accelerations of each mass and the angular acceleration of the massive pulley as seen in a frame attached to the ground.

$a_1 = a_2$

$$\sum \tau = I\alpha$$

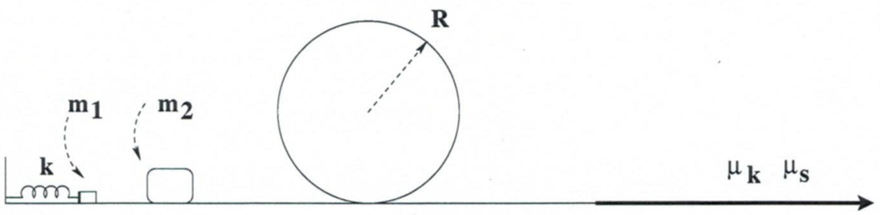
$a_{tan} = R\alpha$

$$\sum \tau = \frac{Ig}{R}$$

Angular acceleration:

$$\alpha = \frac{g}{R}$$

- 2c) (10 points) Find the tension in each rope segment.



3) A mass $m_1$ is shot from a spring of constant $k$ towards another mass, $m_2$. The two masses collide and stick and the combination proceeds through a loop of radius $R$. At the top of the loop, the apparent weight of the combination is three times the actual weight of the combination. After exiting the loop, the combination encounters a long horizontal stretch of friction characterized by the coefficients $\mu_s$ and $\mu_k$.

- 3a) (5 points) How fast was the combined mass moving at the top of the loop?

$\sum \vec{F} = ma$ $\quad a = \frac{v^2}{R}$

$3(m_1+m_2)g = (m_1+m_2)\frac{v^2}{R}$ Weight = 3 × weight

$$\boxed{v=\sqrt{3gR}}$$

- 3b) (5 points) How fast was the combined mass moving **immediately after** the collision?

Use bottom of circle as initial velocity: $V_f=\sqrt{3gR}$

$$\frac{1}{2}(m_1+m_2)V_i^2 = \frac{1}{2}(m_1+m_2)V_f^2 + (m_1+m_2)g2R$$

$$\frac{V_i^2}{2} = \frac{3gR}{2} + 2gR$$

$$\boxed{V_i=\sqrt{7gR}}$$

$$V_i^2 = 3gR + 4gR$$

- 3c) (5 points) How fast was $m_1$ moving **immediately before** the collision?

Perfectly inelastic:

$V_f=\sqrt{7gR}$

$$m_1V_i = (m_1+m_2)V_f$$

$$\boxed{V_i = \frac{(m_1+m_2)\sqrt{7gR}}{m_1}}$$

- 3d) (5 points) By what distance was the spring initially compressed?

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1v^2 \qquad V = \frac{(m_1+m_2)\sqrt{7gR}}{m_1}$$

$$kx^2 = m_1\frac{(m_1+m_2)^2 \cdot 7gR}{m_1^2}$$

$$x^2 = \frac{7gR(m_1+m_2)^2}{m_1 k}$$

$$\boxed{x = \sqrt{\frac{7gR(m_1+m_2)^2}{m_1 k}}}$$

- 3e) (5 points) How far will the combined mass travel over the friction patch before coming to rest?

$$F_k = -\mu_k N$$

$$F_k = -\mu_k(m_1+m_2)g$$

$$\Sigma\vec{F} = ma$$

$$-\mu_k(m_1+m_2)g = (m_1+m_2)a$$

$$a = -\mu_k g$$

$$v^2 = v_0^2 + 2a\Delta x \qquad V_0 = \sqrt{7gR}$$

(bottom of loop)

$$0 = 7gR - 2\mu_k g\Delta x$$

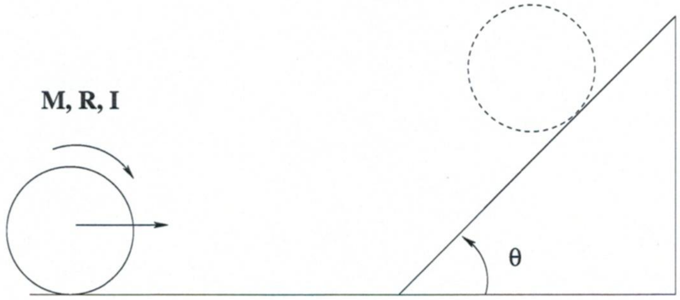
$$2\mu_k g\Delta x = 7gR$$

$$\Delta x = \frac{7gR}{2\mu_k g}$$

$$\boxed{\Delta x = \frac{7R}{2\mu_k}}$$

- 3f) (5 points) How does the energy dissipated by friction compare to the initial energy stored in the spring? Explain. (I'm looking for a qualitative answer, you do not need to do a calculation here.)

Until the combined mass comes to the patch of friction, $\Sigma\vec{F}_{ext} = 0$. Therefore, the system was elastic. The energy dissipated by friction was the same energy from the spring. Therefore, the energy dissipated by friction is equal to the energy stored in the spring.



4) A ball of mass $M$, radius $R$ and rotational inertia $I$ (around an axis through its center-of-mass) rolls without slipping at a constant speed over a horizontal surface. Eventually, the ball encounters an incline sloped at an angle $\theta$ relative to the horizontal. All surfaces (horizontal and sloped) present friction described by the coefficients $\mu_k$ and $\mu_s$ to the ball.

- 4a) (10 points) Assuming the ball continues to roll without slipping as it climbs the incline, find the acceleration of the center-of-mass and the angular acceleration around the center of mass as it makes the climb.

$\sum \vec{\tau} = I\vec{\alpha}$ $F_f$ $F_{g_x} = -mg\sin\theta$ $\tau_g = 0$ $\tau_f = RF_f$

$a_{cm} = R\alpha$

$\alpha = \frac{a_{cm}}{R}$

$\sum F_x = F_f - F_{gx} = m a_{cm}$

$\sum \tau = RF_f = I\vec{\alpha}$

$F_f = \frac{I\vec{\alpha}}{R}$, $F_f = \frac{I a_{cm}}{R^2}$

$$\frac{I a_{cm}}{R^2} + mg\sin\theta = m a_{cm}$$

$$-mg\sin\theta = a_{cm}\left(m - \frac{I}{R^2}\right)$$

$$\boxed{a_{cm} = -\frac{mg\sin\theta}{m - \frac{I}{R^2}} \qquad \alpha = -\frac{mg\sin\theta}{R\left(m - \frac{I}{R^2}\right)}}$$

- 4b) (10 points) Still assuming the ball rolls without slipping, find the magnitude and direction of the force of friction that acts on the ball. Does this friction aid or retard the translational motion of the center-of-mass? Does the friction aid or retard the motion of the ball around its center-of-mass? Given that the ball is still rolling without slipping, what can you tell me about the angle of the incline (that is, what are the limits on its size)?

$F_f = \mu_s mg$ in the positive direction. It aids in the translational motion.

It also aids the rotational motion around its center of mass.

The angle has to be less than 90° in order for it to go up.

- 4c) (10 points) Suppose that it is actually impossible for the ball to roll up the incline without slipping. Find the translational acceleration and the rotational acceleration of the ball around its center-of-mass. How does the ratio of the two accelerations compare to the case where the ball rolls without slipping?

No rotational acceleration if ball can't roll w/o slipping.

$\Sigma F = ma$

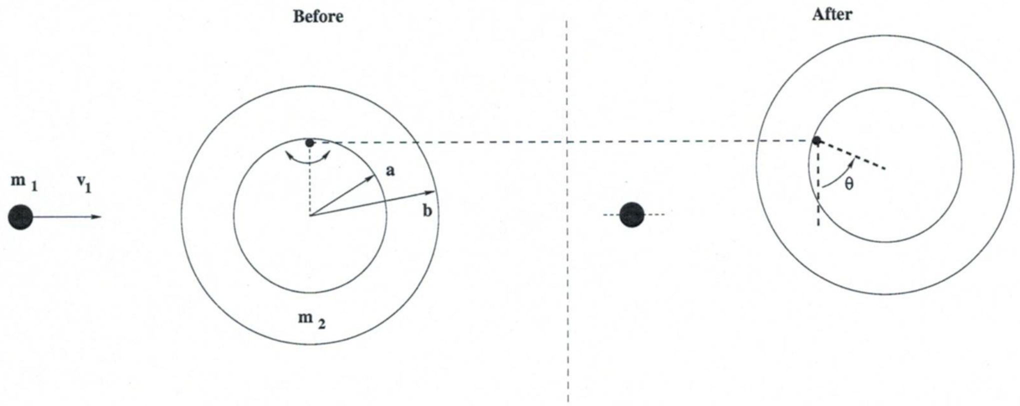
$F_g - F_f = ma$

$-mg\sin\theta + \mu_k mg = ma$

$\mu_k g - g\sin\theta = a$

$$\boxed{a_{cm} = \mu_k g - g\sin\theta}$$

The ratio is much greater since there is no rotational acceleration.



5) A uniform washer of mass $m_2$, inner-radius $a$ and outer-radius $b$ is attached to an axle so that it is free to rotate in the vertical plane. The rotational inertia of the washer **about its center-of-mass** is given by $I_{cm} = \frac{1}{2}m_2(a^2 + b^2)$.

A particle of mass $m_1$ is shot horizontally directly towards the center of mass of the washer with a speed $v_1$. It makes an elastic collision with the washer's outer edge. It is safe to assume that the impulse delivered to the particle by the washer during their brief encounter is horizontal.

- 5a) (5 points) Take the washer and the particle together as a system and consider the temporal interval that starts just before the objects collide and ends right after they collide. Is **mechanical energy** conserved over the interval? Is **translational momentum** conserved in any direction over the interval? Is **angular momentum** conserved around any point during the interval? For full credit, justify your responses.

The collision is elastic as per the question.

Therefore, mechanical energy is conserved.

While the total momentum of the system is conserved,

translational and angular aren't conserved since some

translational momentum is turned into angular momentum

(the before picture has no angular momentum, but the after does)

- 5b) (15 points) How fast is $m_1$ moving immediately after the collision? With what angular speed is the washer rotating about its axle immediately after the collision?

$$m_1 v_{1i} = m_1 v_{1f} + \frac{L}{b-a}$$

$$V = Rw \quad R = b-a$$

$$m_1 v_{1i} = m_1 v_{1f} + \frac{Iw}{b-a} = m_1 v_{1f} + \frac{1}{2}\frac{m_2(a^2+b^2)}{b-a} w$$

$$m_1 v_{1i} = m_1 v_{1f} + \frac{V_1 m_2(a^2+b^2)}{2(b-a)^2}$$

$$m_1 v_{1f} = m_1 v_{1i} - \frac{V_1 m_2(a^2+b^2)}{2(b-a)^2}$$

$$\boxed{V_{1f} = V_1 - \frac{v_1 m_2(a^2+b^2)}{2m_1(b-a)^2} \qquad w = \frac{(m_1 v_{1i} - m_1 v_{1f})(2(b-a))}{m_2(a^2+b^2}}$$

- 5c) (10 points) To what angle ($\theta$, measured with respect to the vertical) will the center-of-mass of the washer rise after the collision?

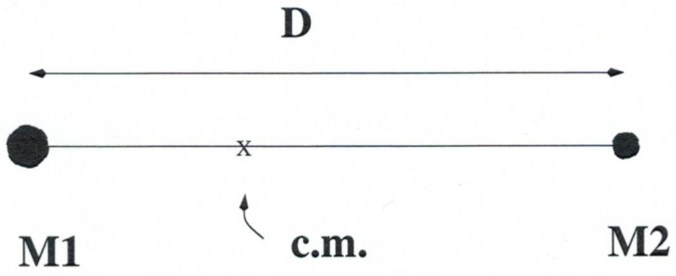
Use $w$ from last question, $\alpha$ from first question.

$$\cancel{\vec{\theta}(t) = \theta_0 + w_0 \Delta t + \frac{1}{2}\alpha \Delta t^2}$$

~~plug in~~

$$w^2 = w_0^2 + 2\alpha\Delta\theta$$

$$\boxed{\Delta\theta = \frac{w^2}{2\alpha}}$$



6) $M_1$ and $M_2$ are two stars that make up a binary star system. They are separated by a distance $D$ and make circular orbits about their common center of mass.

- 6a) (5 points) Find the distances of $M_1$ and $M_2$ from the center of mass of the system (in terms of $G$, $M_1$, $M_2$ and $D$).

$$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$r_1 = \frac{m_1(0) + m_2 D}{m_1 + m_2} = D\frac{m_2}{m_1 + m_2} \qquad D - r_1 = r_2$$

$$\boxed{r_1 = D\frac{m_2}{m_1 + m_2} \qquad r_2 = D\frac{m_1}{m_1 + m_2}}$$

- 6b) (5 points) Find the angular velocity with which each orbits around the center of mass (in terms of $G$, $M_1$, $M_2$ and $D$).

Both have same $w$ $(w_1 = w_2)$

$$\Sigma \vec{F} = m\vec{a} \qquad i = 1, 2$$

$$\frac{Gm_1m_2}{D^2} = m_i\frac{V_i^2}{r_i} \swarrow V_i = r_i w$$

$$\frac{Gm_1m_2}{D^2} = m_i r_i w^2 \qquad m_i r_i = D\frac{m_1 m_2}{m_1 + m_2}$$

$$\frac{Gm_1m_2}{D^2} = \left(\frac{m_1m_2}{m_1 + m_2}\right) R w^2$$

$$\frac{G(m_1 + m_2)}{D^3} = w^2$$

$$\boxed{w = \sqrt{\frac{G(m_1 + m_2)}{D^3}}}$$

- 6c) (10 points) Find the magnitude of the total angular momentum of the system relative to the center-of-mass of the system (in terms of $G$, $M_1$, $M_2$ and $D$). Do you expect it to remain constant? Why or why not?

$\sum \vec{\tau} = \frac{d\vec{L}}{dt} = 0$, so the total angular momentum is constant.

$$\vec{L} = I\vec{\omega}$$

$$\vec{L} = \overset{m_1}{I_{\parallel}}\vec{\omega} + \overset{m_2}{I_{\parallel}}\vec{\omega}$$

$$\vec{L} = (\cancelto{0}{I_{cm}} + m_1 r_1^2)\vec{\omega} + (\cancelto{0}{I_{cm}} + m_2 r_2^2)\vec{\omega}$$

$$\boxed{\vec{L} = m_1\left(D\frac{m_2}{m_1+m_2}\right)^2\vec{\omega} + m_2\left(D\frac{m_1}{m_1+m_2}\right)^2}$$

- 6d) (10 points) Find the total mechanical energy of the system (in terms of $G$, $M_1$, $M_2$ and $D$). Do you expect it to remain constant? Why or why not?

Since the total angular momentum is constant and there are no external torques or forces, mechanical energy is conserved.

$v = wR$ $\quad w = \sqrt{\frac{G(m_1+m_2)}{D^3}}$ $\quad U_g(r) = -\frac{G m_0 m_1}{r}$

$$K_1 + K_2 - U_g = \text{Total energy}$$

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 - \frac{Gm_1m_2}{D}$$

$$\frac{1}{2}m_1(wr_1)^2 + \frac{1}{2}m_2(wr_2)^2 - \frac{Gm_1m_2}{D}$$

$$\frac{w^2}{2}(m_1r_1^2 + m_2r_2^2) - \frac{Gm_1m_2}{D}$$

$$\boxed{= \frac{G(m_1+m_2)}{2D}\left[m_1\left(D\frac{m_2}{m_1+m_2}\right)^2 + m_2\left(D\frac{m_1}{m_1+m_2}\right)^2\right] - \frac{Gm_1m_2}{D}}$$