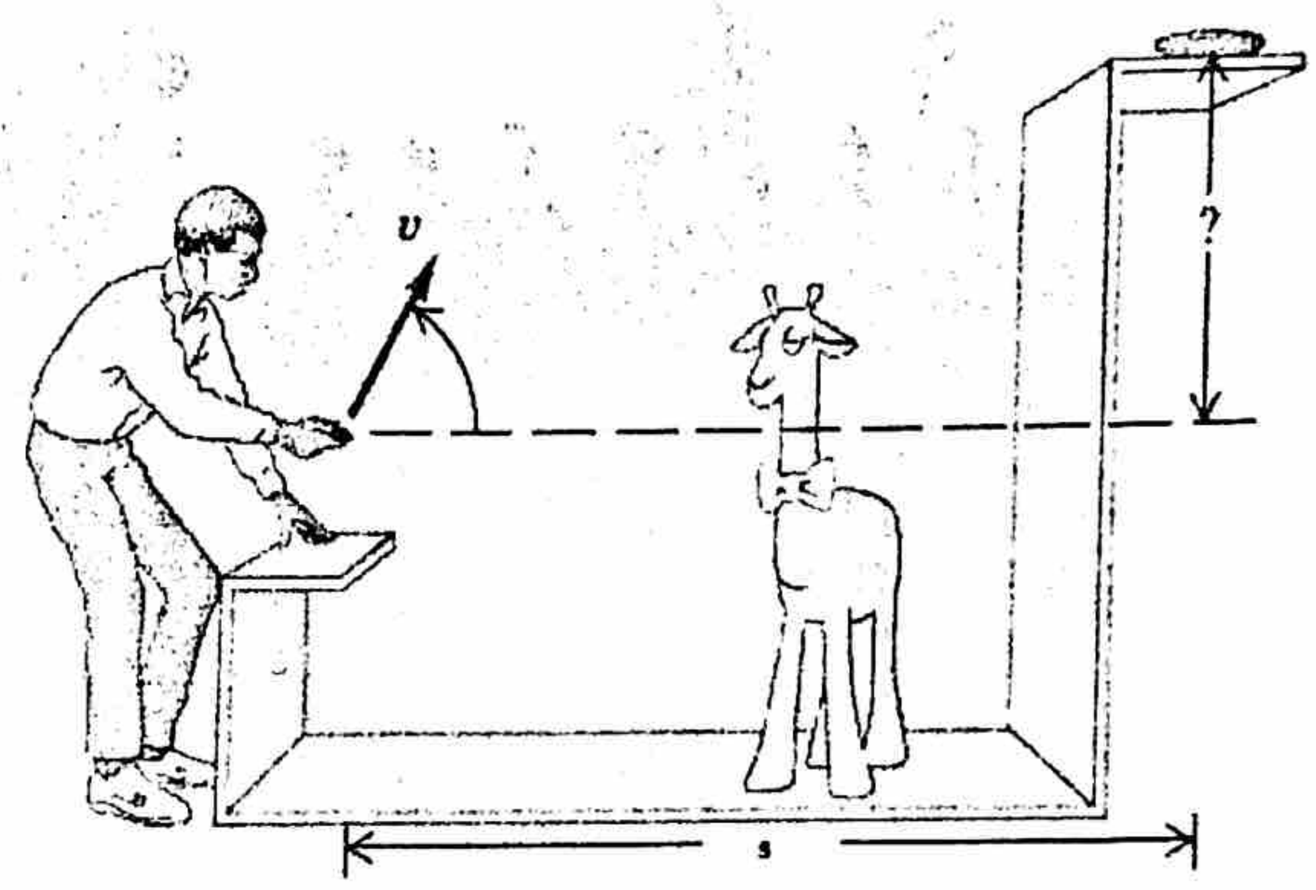
Problem 1

In a carnival booth, you win a stuffed giraffe if you toss a quarter into a small dish. The dish is on a shelf above the point where the quarter leaves your hand and is a horizontal distance of s meters from this point (see figure). If you toss the coin with a velocity of $\vec{v}$ at an angle of α degrees above the horizontal, the coin lands in the dish. You can ignore air resistance.

a) What is the height of the shelf above the point where the quarter leaves your hand?

b) What is the vertical component of the velocity of the quarter just before it lands in the dish?



$h = ?$

a. $\Delta x = vt$

$\Rightarrow t = \frac{\Delta x}{v} = \frac{s}{v\cos\alpha}$

$\Delta y = vt + \frac{1}{2}at^2$

$= v\sin\alpha\, t + \frac{1}{2}(g)t^2$

$\Rightarrow \Delta y = v\sin\alpha\left(\frac{s}{v\cos\alpha}\right) + \frac{1}{2}g\left(\frac{s}{v\cos\alpha}\right)^2$

$\boxed{\Delta y = s v\tan\alpha + \frac{1}{2}g\left(\frac{s}{v\cos\alpha}\right)^2}$ 7

b. $v_f = v_0 + at$

$0 = v\sin\alpha + gt$ (time to peak)

$\Rightarrow t = \frac{-v\sin\alpha}{g}$

$t_{Total} = t_{peak} + t_{dish}$

$\Rightarrow t_d = t_T - t_p$

$= \frac{s}{v\cos\alpha} + \frac{v\sin\alpha}{g}$

$v_f = v_0 + at$

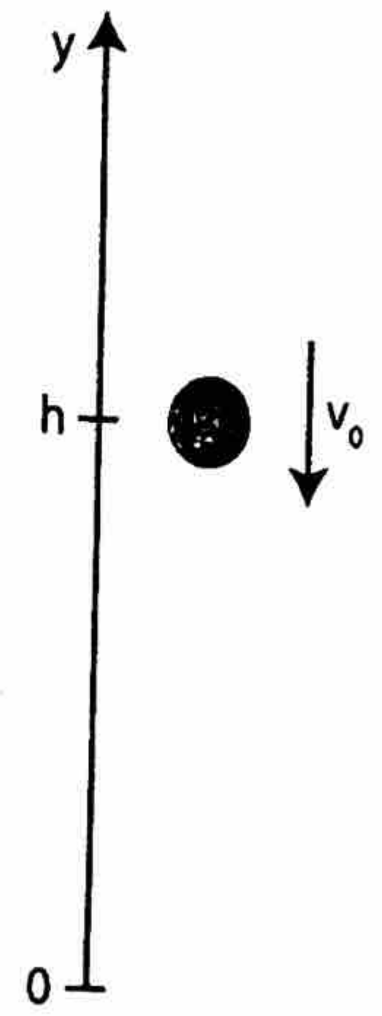
$= 0 + g\left(\frac{s}{v\cos\alpha} + \frac{v\sin\alpha}{g}\right)$

$\Rightarrow \boxed{v_f = \frac{-sg}{v\cos\alpha} + v\sin\alpha}$ 1

Problem 2

A particle of mass m is falling vertically in a gravitational field. The retarding force applied on the particle is *proportional* to its velocity. Given that the particle's initial position and velocity (at $t = 0$) are h and v_0, find the velocity and displacement (at $t \neq 0$). You might find the following integrals useful: $\int \frac{1}{a+x}dx = \ln(a+x) + \text{const.}$ and $\int e^{ax}dx = \frac{1}{a}e^{ax} + \text{const.}$

a_r = acceleration due to retarding force



F_r = ret. force

F_g

+3

$\Sigma F_y = F_r - F_g = ma$

$ma = ma_r v - mg$

$\Rightarrow a = a_r v - g$

$v(t) = \int a(t)\, dt = \int a_r v - g \, dt$

$v = a_r v t - gt + C$

$v - a_r v t = -gt + C$

$v(1 - a_r t) = -gt + c$

$C = v(1 - a_r t) + gt$

@$t=0$, $v = v_0$

$\Rightarrow C = v_0(1) + 0$

$\Rightarrow \boxed{v = \frac{v_0 - gt}{1 - a_r t}}$ X

+1

$x(t) = \int v(t)\, dt$

$= \int a_r v t - gt + v_0 \, dt$

$= \frac{a_r v t^2}{2} - \frac{gt^2}{2} + v_0 t + C$

@ $t=0$, $x(t) = h$

$h = c$

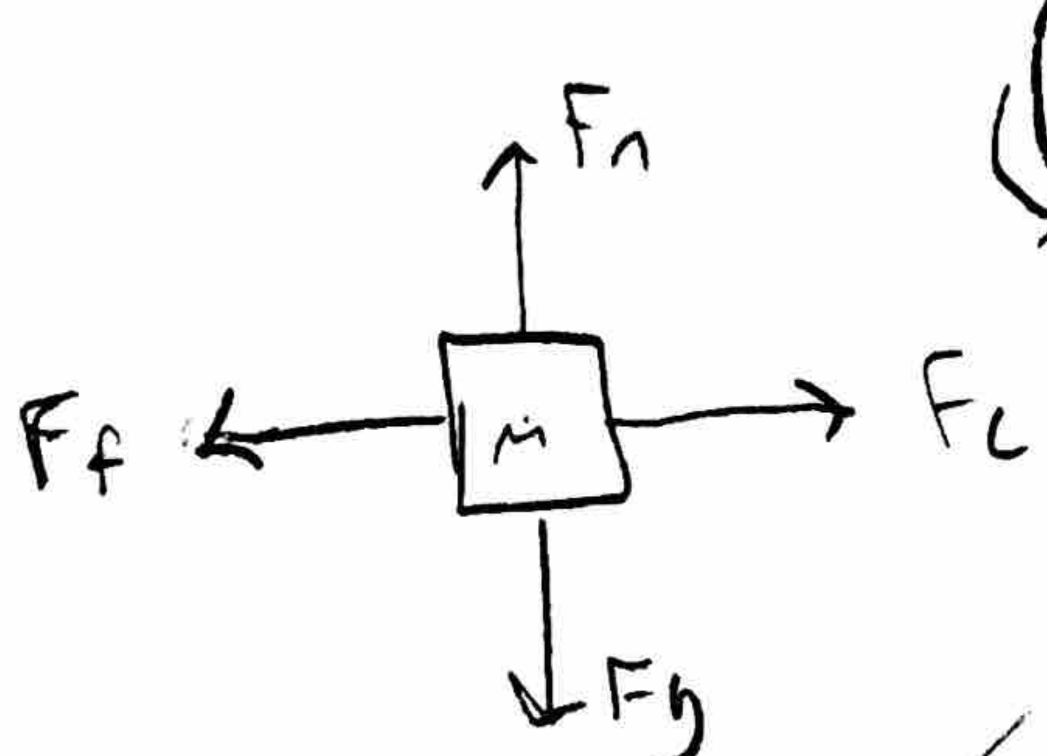
$\Rightarrow \boxed{x(t) = \frac{a_r v t^2}{2} - \frac{gt^2}{2} + v_0 t + h}$ X

+1

Problem 3

A car of mass m is traveling around a flat circular race track of radius R. The static coefficient of friction between the tire and the road (against transverse motion) is μ. (a) How fast can the car travel before it starts to skid? (b) What is the angular velocity ω of the car at the speed calculated in (a).

$F_c = \frac{mv^2}{r}$ $\quad a_{rad} = \frac{4\pi r^2}{T^2}$



$\Sigma F_x = F_c - F_f = ma = 0$

a. $F_c = F_f$ $\quad \mu \quad m$

m → $\frac{mv^2}{r} = \mu mg$

$\Rightarrow$ $\boxed{v = \sqrt{mgR}}$ X

$v = \sqrt{\mu g R}$ $\quad$ not $\sqrt{mgR}$

m

b. $v = \omega r$ ✓

$\omega = \frac{v}{r} = \boxed{\frac{\sqrt{mgR}}{R}}$ X

+9

Write more Clearly