

Problem 3 (20 points):

Tension in rod =

$$m_1 = m \quad m_2 = 3m$$

Spring, frictionless rod

Find V of each block

initially Conservation of energy: $\Delta E_{\text{mech}} = 0$

momentum $K_i + U_{\text{el},i} = K_f + U_{\text{el},f}$

$$(4m) V_0^2 + \frac{1}{2} k (l_0)^2 = \frac{1}{2} (m) V_1^2 + \frac{1}{2} (3m) V_2^2$$

$$4 \frac{4m}{2} V_0^2 + \frac{1}{2} k (l_0)^2 = \frac{1}{2} m V_1^2 + \frac{3}{2} m V_2^2 \Rightarrow 4 V_0^2 + \frac{k}{2} (l_0)^2 = V_1^2 + 3 V_2^2$$

Conservation of momentum:

$$(m_1 + m_2) V_{0,i} = m_1 V_{1,i} + m_2 V_{2,i}$$

$$4m V_0 = m V_1 + 3m V_2$$

$$4V_0 = V_1 + 3V_2$$

$$V_0 = 0$$

Use c.o.m. reference frame $\rightarrow V_{\text{cm}} = V_0$

$$0 = V_1' + 3V_2'$$

$$k l_0^2 = V_1'^2 + 3V_2'^2$$

$$V_1' = -3V_2'$$

$$4V_0^2 = 3V_2'^2, \quad 4V_0^2 + k l_0^2 = 3V_2'^2$$

$$k l_0^2 = 3V_2'^2$$

~~$$V_{1,f} = \frac{2m_2}{m_1+m_2} V_{2,i} + \frac{m_1-m_2}{m_1+m_2} V_{1,i}$$~~

~~$$V_{1,f} = \frac{6m}{4m} V_0 + \frac{-2m}{4m} V_0$$~~

~~$$= \frac{3}{2} V_0 - \frac{1}{2} V_0 = V_0$$~~

~~$$V_{2,f} = \frac{2m_1}{m_1+m_2} V_{1,i} + \frac{m_2-m_1}{m_1+m_2} V_{2,i}$$~~

~~$$= \frac{2m}{4m} V_0 + \frac{2m}{4m} V_0 = V_0$$~~

Solution:

Equation is, now:

$$\frac{1}{2} k (l_0)^2 = \frac{1}{2} m V_1'^2 + \frac{1}{2} (3m) V_2'^2$$

Substitute $V_1' = -3V_2'$

$$k (l_0)^2 = m (-3V_2')^2 + 3m (V_2')^2$$

$$k (l_0)^2 = 12m V_2'^2$$

$$V_2' = \sqrt{\frac{k l_0^2}{12m}} = \frac{l_0}{2} \sqrt{\frac{k}{3m}}, \quad V_1' = -3V_2' = -\frac{3l_0}{2} \sqrt{\frac{k}{3m}}$$

Return to the reference

frame of figure 2, add V_0

$$V_1 = V_0 - \frac{3l_0}{2} \sqrt{\frac{k}{3m}}$$

$$V_2 = V_0 + \frac{l_0}{2} \sqrt{\frac{k}{3m}}$$