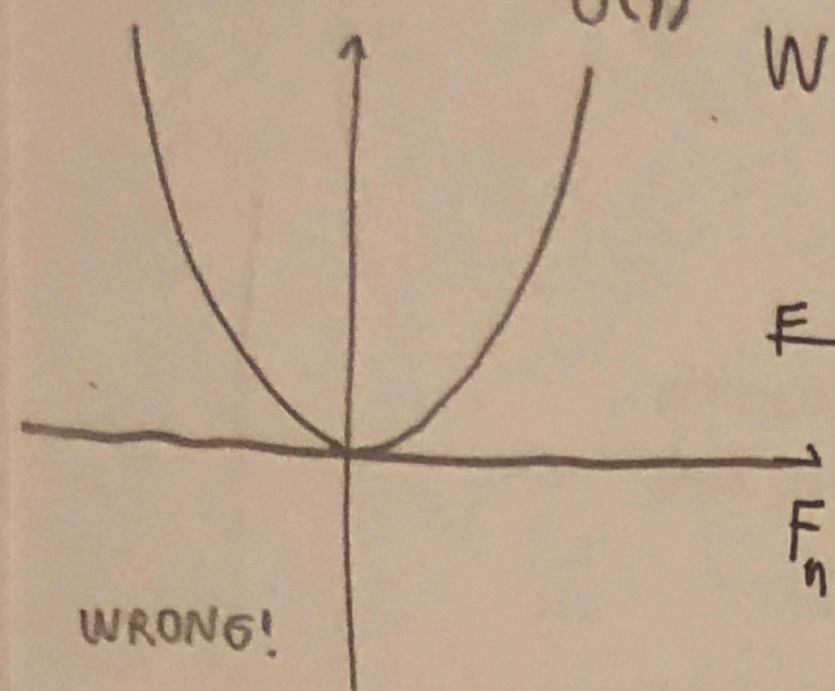
K=0

2 ~~X~~ **Part C (10 points):** Assume the particle is observed to be instantaneously at rest at $y = 0$. What is the maximum speed that it achieves? Express your answer in terms of the particle mass m and the constants a, b, and c.

U(y)



$$W = \int_{y_a}^{y_b} \vec{F}_{net}\, dy \Rightarrow \vec{F}_{net} = \frac{dW}{dy} = -\frac{dU}{dy}$$

$$= -2a(y-b) + c$$

~~$F = ma_x \quad a_x = v_x \frac{dv_x}{dx}$~~

$$F_{net} = ma_y \rightarrow a_y = v_y \frac{dv_y}{dy}$$

WRONG!

$$0 = -2a(y-b) + c$$

$$\frac{c}{2a} + b = y$$

$$m v_y \frac{dv_y}{dy} = -2a(y-b) + c$$

$$\int v_y\, dv_y = \frac{1}{m}\int -2a(y-b) + c\, dy$$

$$\frac{v_y^2}{2} = \frac{1}{2m}\left[-a(y-b)^2 + cy\right] + C$$

$$v_y = \sqrt{\frac{2}{m}\left(-a(y-b)^2 + cy\right)}$$

At $y = 0$ $v_y = 0$ $\therefore C = 0$

~~$v_{y,max} =$~~

Maximum speed is where ~~y~~ particle is at eq. position $(y = b + \frac{c}{2a}$

$$v_y = \sqrt{\frac{2}{m}\left(-a\left(b + \frac{c}{2a} - b\right)^2 + cy\right)}$$

$$= \sqrt{\frac{\cancel{2}}{m}\left(-\cancel{a}\left(\frac{c^2}{\cancel{4}a^{\cancel{2}}}\right) + c\left(b + \frac{c}{2a}\right)\right)}$$

$$\sqrt{\frac{1}{m}\left(-\frac{c^2}{2} + \frac{c^2}{2a} + cb\right)}$$

$$v_y = \sqrt{\frac{1}{m}\left(-\frac{c^2}{2} + \frac{c^2}{2a} + cb\right)} \quad //$$

Solution:

Maximum speed : $U(y) =$ minimum

which is where $y = \frac{c}{2a} + b$

Since at $y=0$, particle is at rest

$E_{mech} = K + U(0) = ab^2$

At $U(\frac{c}{2a} + b) = a\left(\frac{c}{2a} + b - b\right)^2 - c\left(\frac{c}{2a} + b\right)$

$= \frac{c^2}{4a} - \frac{c^2}{2a} + cb = -c\left(\frac{c}{4a} + b\right)$

$\Delta E_{mech} = U_f - U_i = 0$

$U_f = U_i$

$K + U = ab^2$

$\frac{1}{2}mv^2 - c\left(\frac{c}{4a} + b\right) = ab^2$

$\frac{1}{2}mv^2 = ab^2 + \frac{c^2}{4a} + cb$

$$v_{max} = \sqrt{\frac{m}{2}\left[ab^2 + \frac{c^2}{4a} + cb\right]} \quad //$$