MIDTERM EXAM #2
Physics 1A Lecture 2
Instructor: Anton Bondarenko

Friday, May 24rd, 2019
10:00 AM – 10:50 AM

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You will have **50 minutes** to complete this exam. One standard 3" x 5" index note card is permitted. Notes, books, cell phones, calculators, and any other electronics are not allowed. **You must complete the exam using a blue or black ink pen. Exams completed in pencil will not receive credit.**

Please write your answer in the space below the problem. **You must write legibly and demonstrate your reasoning to get full credit**. For quantitative problems, always express your final answer in terms of the variables given in the problem unless otherwise stated. For clarity, please draw a box around your final answer.

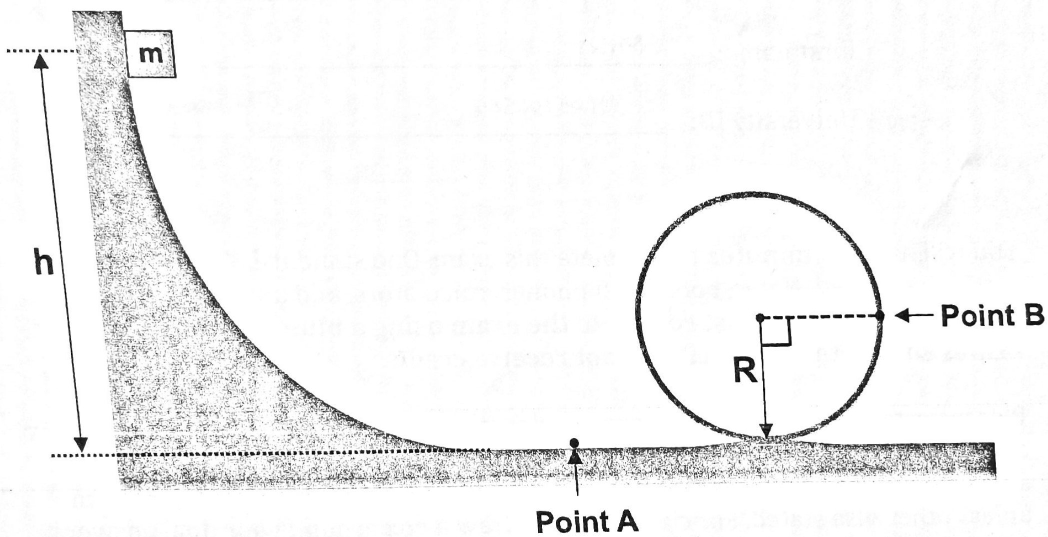
Extra paper is attached at the back of the exam and more is available in the front of the room. If a problem seems confusing or ambiguous, please ask the proctor for clarifications.

DO NOT TURN PAGE UNTIL INSTRUCTED

Problem 1 (40 points):

As shown in **Figure 1** below, a block of mass *m* is released from rest at a height *h*, sliding down a frictionless slope and going around a loop of radius *R*.

Figure 1



Part A (10 points): Determine the minimum height h from which the block must be released so that it completes the loop without falling off. Express your answer in terms of R.

Point A: mech. E is conserved

$$U_{grav\,i} + \cancel{K_i} = \cancel{U_{grav\,f}} + K_f$$

$$mgh = \frac{1}{2} m v_a^2$$

$$v_a = \sqrt{2gh}$$

Top of loop:

$-x$

$\vec{N}$

$\vec{W}$

$$\vec{F}_{net} = \vec{N} + \vec{W}$$

$$\vec{N} \to \vec{0}$$

$$\vec{F}_{net} = \vec{W}$$

$$\frac{m v_b^2}{R} = mg$$

$$v_b^2 = Rg$$

$$v_b = \sqrt{Rg}$$

Point A $\to$ top of loop:

$$\cancel{U_{grav\,i}} + K_i = U_{grav\,f} + K_f$$

$$\frac{1}{2} \cancel{m} v_a^2 = 2\cancel{m} g R + \frac{1}{2} \cancel{m} v_b^2$$

$$gh - 2gR = \frac{1}{2} v_b^2$$

$$v_b = \sqrt{2gh - 4gR}$$

$$Rg = 2gh - 4gR$$

$$5R = 2h$$

$$\boxed{h = \frac{5}{2} R}$$

9

Part B (10 points): A second block of exactly one-fourth the mass of the original block is now placed at rest at Point A. When the original block is released, it slides down the slope and collides with the second block at Point A. After the collision, the two blocks stick together. Determine the minimum height h from which the original block must be released so that the two stuck-together blocks complete the loop without falling off. Express your answer in terms of R.

Point A: $U_{grav\,i} + K_i = \cancel{U_{grav\,f}} + K_f$

$mgh = \frac{1}{2}mv_a^2$

$v_a = \sqrt{2gh}$ ✓ +2

top of loop:

$\vec{F}_{net} = \vec{N} + \vec{w}$

$\vec{N} \to 0$

$\frac{\frac{5}{4}mv_f^2}{R} = \frac{5}{4}mg$

$v_f^2 = Rg$

$v_f = \sqrt{Rg}$ ✓ +1 +2

Collision: momentum conserved

$m_1v_{1i} + m_2v_{2i} = (m_1+m_2)v_f$

$mv_a = \left(\frac{5}{4}m\right)v_b$

$v_b = \frac{4}{5}v_a$ ✓ +2

Point A to top of loop:

$U_{grav\,i} + K_i = U_{grav\,f} + K_f$

$\frac{1}{2}\left(\frac{5}{4}mv_b^2\right) = 2\left(\frac{5}{4}mgR\right) + \frac{1}{2}\left(\frac{5}{4}m\right)v_f^2$ +2

$\frac{1}{2}v_b^2 = 2gR + \frac{1}{2}v_f^2$

$\frac{1}{2}\left(\frac{4}{5}v_a\right)^2 = 2gR + \frac{1}{2}v_f^2$

$\frac{8}{25}(2gh) = 2gR + \frac{1}{2}Rg$ +2

$h = \frac{25}{16}\left(2R + \frac{1}{2}R\right)$ ✓

$\boxed{h = \frac{65}{16}R}$ ×

⑦

Part C (10 points): Now assume that the two blocks from Part B undergo a perfectly elastic collision, such that no kinetic energy is lost. Determine the minimum height *h* from which the original block must be released so that both blocks complete the loop without falling off. Express your answer in terms of *R*.

Collision:

$$\begin{cases} V_{1f} = \left(\frac{2m_2}{m_1+m_2}\right)V_{2i} + \left(\frac{m_1-m_2}{m_1+m_2}\right)V_{1i} \\ V_{2f} = \left(\frac{2m_1}{m_1+m_2}\right)V_{1i} + \left(\frac{m_2-m_1}{m_2+m_1}\right)\cancel{V_{2i}} \end{cases}$$

+2.

$$V_{1f} = \frac{\frac{3}{4}m}{\frac{5}{4}m} V_{1i}$$

$$= \frac{3}{5}\sqrt{2gh} \leftarrow \text{slower}$$

✓ +2.

$$V_{2f} = \frac{2m}{\frac{5}{4}m} V_{1i} = \frac{8}{5}\sqrt{2gh}$$

$$V_{top} = \sqrt{Rg}, \quad \frac{1}{2}mV_a^2 = \frac{1}{2}mV_{top}^2 + 2Rg\left(\frac{5}{4}m\right) \quad m$$

+1 +2

$$\frac{1}{2}\cancel{m}\left(\frac{9}{25}(2gh)\right) = \frac{1}{2}\cancel{m}Rg + \frac{10}{4}Rg$$

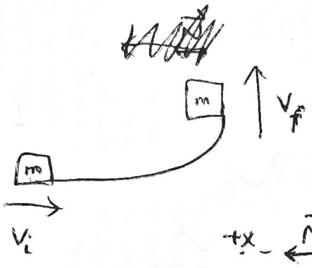
$$\frac{9}{25}h = \frac{1}{2}R + \frac{5}{2}R$$

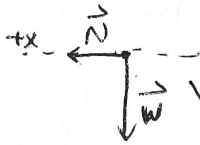
$$= 3R$$

$$\boxed{h = \frac{25}{3}R}$$ ✗

6

Part D (10 points): After the collision described in Part C, what is the magnitude of the normal force exerted by the loop onto the more massive block as it passes Point B, located 90° up the loop with respect to the vertical? Express your answer in terms of m and g.





✓ +1

$v_i = \frac{3}{5}\sqrt{2gh}$ ✓ +1

$= \frac{3}{5}\sqrt{2g\left(\frac{25}{3}R\right)}$

$= 3\sqrt{\frac{2}{3}gR}$

v_f: E_{mech} conserved

+2

$U_{grav\,i} + K_i = U_{grav\,f} + K_f$

$\frac{1}{2}\cancel{m}v_i^2 = \cancel{m}gR + \frac{1}{2}\cancel{m}v_f^2$

$9\left(\frac{2}{3}gR\right) = 2gR + v_f^2$

$4gR = v_f^2$

$v_f = 2\sqrt{gR}$ ✗

$\vec{F}_{net} = \vec{N} + \vec{W}$

$F_{net\,x} = |\vec{N}| = \frac{mv_f^2}{R}$ ✓ +2

$|\vec{N}| = \frac{4mgR}{R}$

$\boxed{|\vec{N}| = 4mg}$ ✗

Problem 2 (40 points):

A particle of mass m is free to move along the y-axis. The particle is subjected to a conservative force described by the potential energy function

$$U(y) = a(y+b)^2 - cy$$

$$= a(y^2+2by+b^2) - cy = ay^2 + (2ab-c)y + ab^2$$

where y is the coordinate of the particle and a, b, and c are positive constants. For this problem, assume that only this conservative force does work on the particle.

Part A (10 points): Find the equilibrium position of this particle in terms of the constants a, b, and c and state whether this is a stable or unstable equilibrium.

$$\frac{dU}{dy} = 2ay + (2ab-c)$$

$$0 = 2ay + 2ab - c$$

$$2ay = -2ab + c$$

$$\boxed{y = \frac{c-2ab}{2a}}$$

$$\frac{d^2U}{dy^2} = 2a > 0 \rightarrow \boxed{\text{stable}}$$

+10

Part B (10 points): What is the work done by the conservative force as the particle travels from position $y = 0$ to position $y = b$? Express your answer in terms of the constants a, b, and c.

$$W_{cons} = -\Delta U$$

$$= (U(b) - U(0))$$

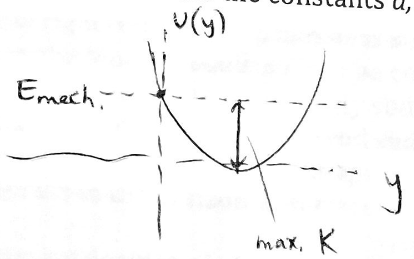
$$= -a(2b)^2 + bc + ab^2$$

$$= \cancel{8ab} - 4ab^2 + bc + ab^2$$

$$= \boxed{-3ab^2 + bc}$$

+10

Part C (10 points): Assume the particle is observed to be instantaneously at rest at $y = 0$. What is the maximum speed that it achieves? Express your answer in terms of the particle mass m and the constants a, b, and c.



$U(0) = ab^2 = E_{mech}$

$U\left(\frac{c-2ab}{2a}\right) = a\left(\frac{c}{2a} - b + b\right)^2 - c\left(\frac{c}{2a} - b\right)$

$= a\left(\frac{c}{2a}\right)^2 - \frac{c^2}{2a} - bc$

$= \frac{c^2}{4a} - \frac{c^2}{2a} - bc$

$= \frac{-c^2}{4a} - bc$

$E_{mech} = K + U$

$K = E_{mech} - U$

$= ab^2 - \left(\frac{-c^2}{4a} - bc\right)$

$= \boxed{ab^2 + \frac{c^2}{4a} + bc}$. $\rightarrow v = ?$

+8

Part D (10 points): Besides $y = 0$, at what other position will the particle be instantaneously at rest? Express your answer in terms of the constants a, b, and c.

$E_{mech} = K + U = ab^2$

$U(y) = ab^2$

$a(y+b)^2 - cy = ab^2$

$ay^2 + (2ab - c)y + \cancel{ab^2} = \cancel{ab^2}$

$y(ay + 2ab - c) = 0$

$ay = -2ab + c$

$$\boxed{y = \frac{-2ab + c}{a}}$$

+10

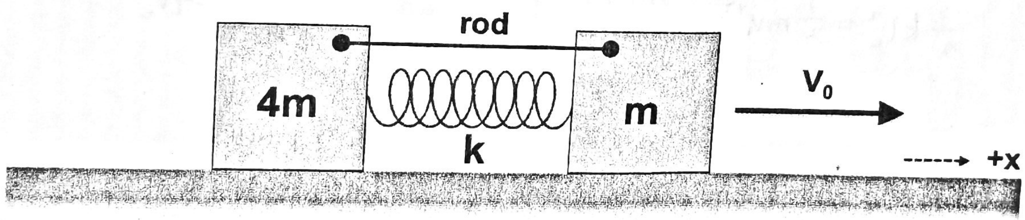
Problem 3 (20 points):

As shown in **Figure 2** below, two blocks of masses m and $4m$ are connected by a massless rigid rod. An ideal, massless spring of spring constant k is inserted between the blocks, requiring it to be compressed by a distance l_0 from its relaxed length. The entire system is initially sliding along a frictionless horizontal surface in the $+x$ direction at speed V_0. The rod suddenly snaps, allowing the spring to decompress and push the blocks apart. The spring is not fixed to the blocks, so it simply falls to the horizontal surface when it returns to its relaxed length.

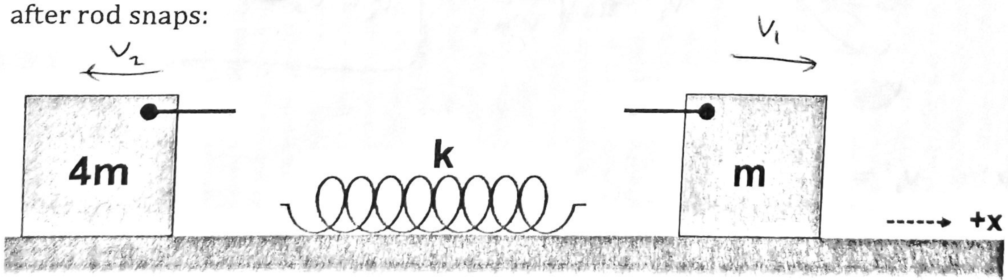
Find the x-velocities of each block after the spring returns to its relaxed length and falls away. Express your answers in terms of m, k, l_0, and V_0. You can assume that the transfer of energy from the spring to the two blocks is perfect.

Figure 2

before rod snaps:



after rod snaps:



Problem 3 (20 points):

w.r.t. center of mass:

$V_{M/E} = V_{M/CM} + V_{CM/E}$

$V_1 = V_1' + V_0$

$V_2 = V_2' + V_0$

mech. energy conserved:

$K_i + U_{el\,i} = K_f + U_{el\,f}$

$\cancel{\tfrac{1}{2}} k l_0^2 = \cancel{\tfrac{1}{2}} m v_1'^2 + \cancel{\tfrac{1}{2}}(4m) v_2'^2$

$k l_0^2 = m v_1'^2 + 4m v_2'^2$

momentum conserved:

$mv_f - mv_i =$ constant

↳ 0 w.r.t center of mass

$\cancel{m} v_1' + 4\cancel{m} v_2' = 0$

$v_1' = -4v_2'$

V_2': $k l_0^2 = m(16 v_2'^2) + 4m v_2'^2$

$= 20 m v_2'^2$

$V_2' = \pm l_0 \sqrt{\frac{k}{20m}} \rightarrow \boxed{V_2 = V_0 \pm l_0 \sqrt{\frac{k}{20m}}}$

V_1': $V_1' = \mp 4 l_0 \sqrt{\frac{k}{20m}}$

$\boxed{V_1 = V_0 \mp 4 l_0 \sqrt{\frac{k}{20m}}}$. +20

SCORING

Problem 1:
Part A: 10 / 10
Part B: 9 / 10
Part C: 7 / 10
Part D: 6 / 10
Total: ~~28~~ / 40
/ 32

Problem 2:
Part A: 10 / 10
Part B: 10 / 10
Part C: 8 / 10
Part D: 10 / 10
Total: 38 / 40

Problem 3:
Total: 20 / 20

Total Midterm #2 Score ~~86~~ 90 / 100